Table of Contents

Ratios and Proportional Relationships

Module Overview .................................................................................................................................................. 3

Topic A: Proportional Relationships (7.RP.2a) .................................................................................................. 8
   Lesson 1: An Experience in Relationships as Measuring Rate ................................................................. 9
   Lesson 2: Proportional Relationships ........................................................................................................ 17
   Lessons 3–4: Identifying Proportional and Non-Proportional Relationships in Tables ...................... 24
   Lessons 5–6: Identifying Proportional and Non-Proportional Relationships in Graphs .................... 38

Topic B: Unit Rate and the Constant of Proportionality (7.RP.2b, 7.RP.2c, 7.RP.2d, 7.EE.4a) .................. 57
   Lesson 7: Unit Rate as the Constant of Proportionality ........................................................................... 59
   Lessons 8–9: Representing Proportional Relationships with Equations ................................................ 66
   Lesson 10: Interpreting Graphs of Proportional Relationships ............................................................... 84

Mid-Module Assessment and Rubric .............................................................................................................. 93

Topics A through B (assessment 1 day, return 1 day, remediation or further applications 2 days)

Topic C: Ratios and Rates Involving Fractions (7.RP.1, 7.RP.3, 7.EE.4a) .......................................................... 101
   Lessons 11–12: Ratios of Fractions and Their Unit Rates ...................................................................... 103
   Lesson 13: Finding Equivalent Ratios Given the Total Quantity ............................................................ 116
   Lesson 14: Multistep Ratio Problems ........................................................................................................ 126
   Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions .............................. 132

Topic D: Ratios of Scale Drawings (7.RP.2b, 7.G.1) .......................................................................................... 139
   Lesson 16: Relating Scale Drawings to Ratios and Rates .................................................................... 140
   Lesson 17: The Unit Rate as the Scale Factor ......................................................................................... 154
   Lesson 18: Computing Actual Lengths from a Scale Drawing .............................................................. 165
   Lesson 19: Computing Actual Areas from a Scale Drawing ................................................................. 175
   Lesson 20: An Exercise in Creating a Scale Drawing ............................................................................. 185
   Lessons 21–22: An Exercise in Changing Scales .................................................................................... 194

1 Each lesson is ONE day and ONE day is considered a 45 minute period.
End-of-Module Assessment and Rubric ................................................................. 209

Topics A through D (assessment 1 day, return 1 day, remediation or further applications 2 days)
Module Overview

In Module 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (6.RP.1, 6.RP.2, 6.RP.3) to formally define proportional relationships and the constant of proportionality (7.RP.2). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (7.RP.2a).

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the constant of proportionality and can be used to represent proportional relationships with equations of the form \( y = kx \), where \( k \) is the constant of proportionality (7.RP.2b, 7.RP.2c, 7.EE.4a). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (7.RP.2d).

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of \( \frac{1}{2} \) mile per \( \frac{1}{4} \) hour (7.RP.1). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multistep ratio word problems (7.RP.3, 7.EE.4a).

In the final topic of this module, students bring the sum of their experience with proportional relationships to the context of scale drawings (7.RP.2b, 7.G.1). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (6.G.1, 6.G.3) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.

Later in the year, in Module 4, students will extend the concepts of this module to percent problems.

The module is comprised of 22 lessons; 8 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.
Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.

7.RP.2 Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. For example, if total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed at t = pn.
   d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1,r), where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

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2 In this module, the equations are derived from ratio problems. 7.EE.4a is returned to in Modules 2 and 3.
Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

6.RP.2 Understand the concept of a unit rate \(\frac{a}{b}\) associated with a ratio \(a:b\) with \(b \neq 0\), and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

3 Expectations for unit rates in this grade are limited to non-complex fractions.
Focus Standards for Mathematical Practice

**MP.1** Make sense of problems and persevere in solving them. Students make sense of and solve multistep ratio problems, including cases involving pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant proportionality in proportional relationships, the importance of $(0,0)$ and $(1,r)$ on graphs and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing.

**MP.2** Reason abstractly and quantitatively. Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations including, $y = kx$, where $k$ is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, $distance = rate \times time$. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.

Terminology

New or Recently Introduced Terms

- **Proportional To** (Measures of one type of quantity are proportional to measures of a second type of quantity if there is a number $k > 0$ so that for every measure $x$ of a quantity of the first type the corresponding measure $y$ of a quantity of the second type is given by $kx$, i.e., $y = kx$.)

- **Proportional Relationship** (A one-to-one matching between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.)

- **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where $k$ is a positive constant, then $k$ is called the constant of proportionality; e.g., If the ratio of $y$ to $x$ is 2 to 3, then the constant of proportionality is $2/3$ and $y = 2/3 \times x$.)

- **One-to-One Correspondence** (Two figures in the plane, $S$ and $S'$, are said to be in one-to-one correspondence if there is a pairing between the points in $S$ and $S'$, so that, each point $P$ of $S$ is paired with one and only one point $P'$ in $S'$ and likewise, each point $Q'$ in $S'$ is paired with one and only one point $Q$ in $S$.)

- **Scale Drawing and Scale Factor** (For two figures in the plane, $S$ and $S'$, $S'$ is said to be a scale drawing of $S$ with scale factor $r$ if there exists a one-to-one correspondence between $S$ and $S'$ so that, under the pairing of this one-to-one correspondence, the distance $|PQ|$ between any two points $P$ and $Q$ of $S$ is related to the distance $|P'Q'|$ between corresponding points $P$ and $Q$ of $S$ by $|P'Q'| = r \cdot |PQ|$.)

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4 These terms will be formally defined in Grade 8. A description is provided in Grade 7.
Familiar Terms and Symbols$^5$

- Ratio
- Rate
- Unit Rate
- Equivalent Ratio
- Ratio Table

Suggested Tools and Representations

- Ratio Table (See example below)
- Coordinate Plane (See example below)
- Equations of the form $y = kx$

<table>
<thead>
<tr>
<th>Ratio Table</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sugar</strong></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coordinate Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Coordinate Plane" /></td>
</tr>
</tbody>
</table>

Assessment Summary

<table>
<thead>
<tr>
<th>Assessment Type</th>
<th>Administered</th>
<th>Format</th>
<th>Standards Addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mid-Module Assessment Task</td>
<td>After Topic B</td>
<td>Constructed response with rubric</td>
<td>7.RP.2</td>
</tr>
<tr>
<td>End-of-Module Assessment Task</td>
<td>After Topic D</td>
<td>Constructed response with rubric</td>
<td>7.RP.1, 7.RP.2, 7.RP.3, 7.EE.4a, 7.G.1</td>
</tr>
</tbody>
</table>

$^5$ These are terms and symbols students have seen previously.
Topic A: Proportional Relationships

**7.RP.2a**

<table>
<thead>
<tr>
<th>Focus Standard:</th>
<th>7.RP.2a Recognize and represent proportional relationships between quantities.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Instructional Days:</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson 1:</strong></td>
<td>An Experience in Relationships as Measuring Rate (P)&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Lesson 2:</strong></td>
<td>Proportional Relationships (P)</td>
</tr>
<tr>
<td><strong>Lessons 3–4:</strong></td>
<td>Identifying Proportional and Non-Proportional Relationships in Tables (P)</td>
</tr>
<tr>
<td><strong>Lessons 5–6:</strong></td>
<td>Identifying Proportional and Non-Proportional Relationships in Graphs (E)</td>
</tr>
</tbody>
</table>

In Lesson 1 of Topic A, students are reintroduced to the meanings of value of a ratio, equivalent ratios, rate, and unit rate through a collaborative work task where they record their rates choosing an appropriate unit of rate measurement. In Lesson 2, students conceptualize that two quantities are proportional to each other when there exists a constant such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity (7.RP.2). They then apply this basic understanding in Lessons 3–6 by examining situations to decide whether two quantities are in a proportional or non-proportional relationship by first checking for a constant multiple between measures of the two quantities, when given a table, and then by graphing on a coordinate plane. Students recognize that the graph of a proportional relationship must be a straight line through the origin (7.RP.2a).

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<sup>1</sup> Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
Lesson 1: An Experience in Relationships as Measuring Rate

Student Outcomes

- Students compute unit rates associated with ratios of quantities measured in different units. Students use the context of the problem to recall the meaning of value of a ratio, equivalent ratios, rate and unit rate, relating them to the context of the experience.

Classwork

Example 1 (15 minutes): How fast is our class?

To start this first class of the school year, conduct an exercise in how to pass out papers. The purpose of the task is not only to establish a routine at the start of the school year, but also to provide a context to discuss ratio and rate.

Determine how papers will be passed out in class depending upon seating arrangement. For this task, it is best to divide the original stack so that one student (in each row or group) has a portion of the original stack. Based upon this determination, explain the system to students. A brief demonstration may help to provide a visual.

For example: If the room is arranged in rows, pass across the rows. Have students start on command and perhaps require that only the current paper-passing student may be out of his or her seat. If the room is arranged in groups or at tables, have the students pass papers to their left, on command, until everyone has a paper. Note: this procedure is highly customizable for use in any classroom structure.

Begin the task by handing a stack of papers to a starting person. Secretly start a stopwatch as the start command is given. Once every student has a paper, report the paper-passing time out loud – for example, “Twelve seconds. Not bad, let’s see if we can get these papers passed out in eleven seconds next time.”

Tell students to begin returning papers back in to the original stack and then report the time upon completion. “Excellent job, now pass them back out in ten seconds. Excellent, now pass them back in in eight seconds.”

Pose the following questions to the students as a whole group, one question at a time.

Questions to discuss:

- How will we measure our rate of passing out papers?
  - Using a stopwatch or similar tool to measure the number of seconds taken to pass out papers.

- What quantities will we use to describe our rate?
  - Number of papers passed out and time that it took

Complete the 2nd and 3rd columns (number of papers and time) on the table as a class.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Number of Papers Passed</th>
<th>Time (in seconds)</th>
<th>Ratio of Number of Papers Passed to Time</th>
<th>Rate</th>
<th>Unit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 1

An Experience in Relationships as Measuring Rate

Date: 8/8/13

Key Terms from Grade 6 Ratios and Unit Rates:

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A : B$ to indicate the order of the numbers: the number $A$ is first and the number $B$ is second.

Two ratios $A : B$ and $C : D$ are equivalent ratios if there is a positive number, $c$, such that $C = cA$ and $D = cB$.

A ratio of two quantities, such as 5 miles per 2 hours, can be written as another quantity called a rate. The numerical part of the rate is called the unit rate and is simply the value of the ratio, in this case 2.5. This means that in 1 hour the car travels 2.5 miles. The unit for the rate is miles/hour, read miles per hour.

Guide students to complete the ratio column in the table as shown below.

Example 1: How Fast is our Class?

Record the results from the paper passing exercise in the table below.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Number of Papers Passed</th>
<th>Time (in seconds)</th>
<th>Ratio of Number of Papers Passed to Time</th>
<th>Rate</th>
<th>Unit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>12</td>
<td>24: 12 or 24 to 12 or equivalent ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>11</td>
<td>24: 11 or 24 to 11 or equivalent ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>10</td>
<td>24: 10 or 24 to 10 or equivalent ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>8</td>
<td>24: 8 or 24 to 8 or equivalent ratio</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When we started passing papers, the ratio was 24 papers in 12 seconds, and by the end the ratio was 24 papers in 8 seconds. Are these two ratios equivalent? Explain why or why not. Guide students in a discussion about the fact that the number of papers was constant, and the time decreased with each successive trial. See if students can relate this to rate and ultimately determine which rate is greatest.

The ratios are not equivalent since we passed the same number of papers in a shorter time. We passed 2 papers per second at the beginning and 3 papers per second by the end. Equivalent ratios must have the same value.

The following questioning is meant to guide students into the realization that unit rate can act as an equalizer, allowing us to make comparisons between a variety of ratios and helping us to compare different data points.
In another class period, students were able to pass 28 papers in 15 seconds, then 28 papers in 12 seconds. A third class period passed 18 papers in 10 seconds. How do these compare to our class? (The teacher may use sample data used here or could use real data collected from other classes prepared in advance.)

- We could find how many papers per second to make these comparisons. Answers on how they compare would vary depending on class results in table.

Review meaning of rate and unit rate in the key ideas box and complete the last two columns of the table, modeling how to find both rate and unit rate. The associated unit rate is the numerical value $A/B$ when there are $A$ units of one quantity for every $B$ units of another quantity.

### Example 1: How Fast is our Class?

Record the results from the paper passing exercise in the table below.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Number of Papers Passed</th>
<th>Time (in seconds)</th>
<th>Ratio of Number of Papers Passed to Time</th>
<th>Rate</th>
<th>Unit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>12</td>
<td>24: 12 or 24 to 12 or equivalent ratio</td>
<td>2 papers per second</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>11</td>
<td>24 :11 or 24 to 11 or equivalent ratio</td>
<td>Approximately 2.2 papers per second</td>
<td>2.2</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>10</td>
<td>24:10 or 24 to 10 or equivalent ratio</td>
<td>2.4 papers per second</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>8</td>
<td>24: 8 or 24 to 8 or equivalent ratio</td>
<td>3 papers per second</td>
<td>3</td>
</tr>
</tbody>
</table>

### Example 2 (15 minutes): Our Class by Gender

Let’s make a comparison of two quantities that are measured in the same units by comparing the ratio of boys to girls in this class to the ratio for different classes (and the whole grade).

- Sample discussion: In this class, we have 14 boys and 12 girls. In another class, there are 7 boys and 6 girls. Note: Any class may be used for comparison; the ratios do not need to be equivalent.

Guide students to complete the table accordingly, pausing to pose questions below.

### Example 2: Our Class by Gender

<table>
<thead>
<tr>
<th></th>
<th>Number of boys</th>
<th>Number of girls</th>
<th>Ratio of boys to girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>14</td>
<td>12</td>
<td>7 to 6</td>
</tr>
<tr>
<td>Class 2</td>
<td>7</td>
<td>6</td>
<td>7 to 6</td>
</tr>
<tr>
<td>Whole 7th grade</td>
<td>Answers vary</td>
<td>Answers Vary</td>
<td></td>
</tr>
</tbody>
</table>
Create a pair of equivalent ratios by making a comparison of quantities discussed in this Example.

- Are the ratios of boys to girls in the two classes equivalent?
- What could these ratios tell us?
- What does the ratio of boys to girls in class 1 to the entire 7th grade tell us? The teacher would need this information in advance. Are they equivalent?
- If there is a larger ratio of boys to girls in one class than in the grade as a whole, what does that mean must be true about the boy/girl ratio in other classes? (It may be necessary to modify this question based upon real results or provide additional examples where this is true.)

- Provide ratios from four classes and the total number of students in 7th grade; challenge students to determine 5th class ratio and derive a conclusion. (See detailed explanation in chart below.)

  If the total students is 55 boys + 65 girls, or 120, then the missing number of boys (for Class 5) is 8, and the missing number of girls (for Class 5) is 16, giving a boy/girl ratio that is smaller than the whole grade ratio. In this situation, the boy/girl ratio in other classes would have to be less than in the grade as a whole.

  This extension would also allow for students to see the usefulness of using unit rate when making comparisons.

- How do we compare the ratios when we have varying sizes of quantities?

  Finding unit rate may help. In the data given here, the unit rate for classes 1 and 2 is 1.16, and the unit rate for whole grade is approximately 0.85. The unit rate for class 4 is approximately 0.53, and the unit rate for class 5 is 0.5.

<table>
<thead>
<tr>
<th>Number of boys</th>
<th>Number of girls</th>
<th>Ratio of boys to girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Class 2</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Class 3</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Class 4</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>Class 5</td>
<td>? = 8</td>
<td>? = 16</td>
</tr>
<tr>
<td>Whole 7th grade</td>
<td>55</td>
<td>65</td>
</tr>
</tbody>
</table>

Larger than whole grade ratio
Smaller than whole grade ratio

The total number of students in the entire grade is 120, which can be used to find class 5 numbers.

Review the key idea box focusing on the meaning of equivalent ratios and give student 2 minutes to write down a pair of equivalent ratios comparing boys to girls, or a similar comparison from their class. Discuss responses as a whole class.
Exercise 1 (8 minutes): Which is the Better Buy?

Read the problem as a class, and then allow time for students to solve independently. Ask students to share responses regarding how to determine if the ratios are equivalent. Reinforce key vocabulary from Grade 6.

Exercise 1: Which is the Better Buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for $7.97 whereas a 12-pack of the same brand cost for $4.77. Which is the better buy? How do you know?

The better buy is the pack of 30. The pack of 30 has a smaller unit rate, approximately 0.27, as compared to the pack of 12 with a unit price of 0.40. You would pay $0.27 per pencil in the pack of 30 whereas you would pay $0.40 per pencil in the pack of 12. Students may instead choose to compare the costs for every 60 pencils, or every 360 pencils, etc. Facilitate a discussion of the different methods students may have used to arrive at their decision. See calculations below:

<table>
<thead>
<tr>
<th>Pack of 30</th>
<th>Pack of 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio:</td>
<td></td>
</tr>
<tr>
<td>$7.97 for every 30 pencils</td>
<td>$4.77 for every 12 pencils</td>
</tr>
<tr>
<td>$15.94 for every 60 pencils</td>
<td>$23.85 for every 60 pencils</td>
</tr>
<tr>
<td>Or equivalent comparison</td>
<td>Or equivalent comparison</td>
</tr>
<tr>
<td>Rate:</td>
<td></td>
</tr>
<tr>
<td>7.97/30, or approx. 0.27 dollars per pencil</td>
<td>4.77/12, or approx. 0.40 dollars per pencil</td>
</tr>
<tr>
<td>Unit Rate:</td>
<td></td>
</tr>
<tr>
<td>0.27</td>
<td>0.40</td>
</tr>
<tr>
<td>Unit of Measure:</td>
<td>dollars per pencil</td>
</tr>
</tbody>
</table>

Closing (2 minutes)

- How is finding an associated rate or unit rate helpful when making comparisons between quantities?
  - The unit rate tells the quantity of one unit required for just one of another unit. For example, a unit price of .4 means 1 juice box from a six-pack costs $0.40.

Lesson Summary

Unit Rate is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per 1 unit of the second quantity. This value of the ratio is the unit rate.

Exit Ticket (5 minutes)

http://www.youtube.com/watch?feature=player_embedded&v=tCKstDXMsIQ

Students may need to see the video more than once. After watching the video the first time, it might be helpful for students to know that 100 meters is just a little longer than a football field (which measures 100 yards), and this record was recorded in 2009. Tillman the English Bulldog, covered a 100-meter stretch of a parking lot in a time of 19.678 seconds during he X Games XV in Los Angeles, California, USA.
Lesson 1: An Experience in Relationships as Measuring Rate

Exit Ticket

Watch the video clip of Tillman the English Bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. At the conclusion of the video, your classmate takes out his or her calculator and says, “Wow that was amazing! That means the dog went about 5 meters in 1 second!” Is your classmate correct, and how do you know?

2. After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman’s skateboarding record. Lightning’s fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on this data, did Lightning break Tillman’s record for fastest dog on a skateboard? Explain how you know.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Watch the video clip of Tillman the English Bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. At the conclusion of the video, your classmate takes out his or her calculator and says, “Wow that was amazing! That means the dog went about 5 meters in 1 second!” Is your classmate correct, and how do you know?

   Yes, the classmate is correct. The dog traveled at an average rate of 100 meters in 19.678 seconds, or an associated rate of \((100/19.678)\) meters per second, giving a unit rate of approximately 5.08.

2. After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman’s skateboarding record. Lightning’s fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on this data, did Lightning break Tillman’s record? Explain how you know.

   No, Lightning did not break Tillman’s record. Tillman traveled at an average rate of 5.08 meters per second (calculated from an associated rate of \(75/15.5\) meters per second), and Lightning traveled at an average rate of 4.84 meters per second (about \(\frac{3}{4}\) of a meter slower per second), making Tillman the faster skateboarder.

Problem Set Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. Find each rate and unit rate.
   a. 420 miles in 7 hours
      
      \[
      \text{Rate} = 60 \text{ miles per hour; Unit Rate} = 60
      \]
   b. 360 customers in 30 days
      
      \[
      \text{Rate} = 12 \text{ customers per day; Unit Rate} = 12
      \]
   c. 40 meters in 16 seconds
      
      \[
      \text{Rate} = 40/16, \text{ or 2.5 meters per second; Unit rate} = 2.5
      \]
   d. $7.96 for 5 pounds
      
      \[
      \text{Rate} = 7.96/5, \text{ or approximately 1.59 dollars per pound; Unit rate} = 1.592
      \]

2. Write three ratios that are equivalent to the one given: 18 right-handed students for every 4 left-handed students.

   Sample response: 9 right-handed students for every 2 left-handed students, 36 right-handed students for every 8 left-handed students, 27 right-handed students for every 6 left-handed students
3. Mr. Rowley has 16 homework papers and 14 exit tickets to return. Ms. Rivera has 64 homework papers and 60 exit tickets to return. For each teacher, write a ratio to represent the number of homework papers to number of exit tickets they have to return. Are the ratios equivalent? Explain.

Mr. Rowley has 16 homework papers to 14 exit tickets; Ms. Rivera has 64 homework papers to 60 exit tickets. No, the ratios are not equivalent because Mr. Rowley’s unit rate is 8/7, or approximately 1.14, and Ms. Rivera’s unit rate is 16/15, or approximately 1.07.

4. Jonathan’s parents told him that for every 5 hours of homework or reading he completes, he will be able to play 3 hours of video games. His friend Lucas’s parents told their son that he can play 30 minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games and how do you know?

If both boys spend 5 hours on homework and reading, Jonathan will be able to play 3 hours of video games and Lucas will be able to play 2.5 hours of video games. Jonathan gets more time playing video games. Jonathan gets 0.6 hours (36 minutes) for every 1 hour of homework or reading time, whereas Lucas gets only 30 minutes for every hour of homework or reading time.

5. At Euclid Middle School, of the 30 girls who tried out for the lacrosse team, 12 were selected and of the 40 boys who tried out, 16 were selected. Are the ratios of number of students on the team to number of student trying out the same for both boys and girls? How do you know?

Yes, the ratios are the same: girls – 12 to 30 or 2 to 5; boys–16 to 40 or 2 to 5. The value of each ratio is 2/5.

6. Devon is trying to find the unit price on a 6-pack of energy drinks on sale for $2.99. His sister says that at that price, each energy drink would cost just over $2.00. Is she correct and how do you know? If she is not, how would Devon’s sister find the correct price?

Devon’s sister is not correct. His sister divided the number of drinks by the cost, and to correctly find the unit price, you need to divide the price by the number of drinks. 2.99/6, or approximately 0.50, is the correct unit price. The energy drinks cost approximately 0.50 dollars per drink.

7. Each year Lizzie’s school purchases student agenda books, which are sold in the school store. This year, the school purchased 350 books at a cost of $1,137.50. If the school would like to make a profit of $1,500 to help pay for field trips and school activities, what is the least amount they can charge for each agenda book? Explain how you found your answer.

The school paid a unit price ($) per book of 3.25. In order to make a profit of $1,500, the agenda books need to be sold for a minimum of $7.54 per book. $7.54•350 brings revenue of $2,639, and $2,639 less $1,137.50 (expense) gives $1,501.50 of profit.
Lesson 2: Proportional Relationships

Student Outcomes
- Students understand that two quantities are proportional to each other when there exists a constant (number) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- When students identify the measures in the first quantity with \( x \) and the measures in the second quantity with \( y \), they will recognize that the second quantity is proportional to the first quantity if \( y = kx \) for some positive number \( k \). They apply this same relationship when using variable choices other than \( x \) and \( y \).

Classwork

Example 1 (10 minutes): Pay by the Ounce Frozen Yogurt!

The purpose of this example is for students to understand when measures of one type of quantity are proportional to measures of another type of quantity.

<table>
<thead>
<tr>
<th>Weight (ounces)</th>
<th>12.5</th>
<th>10</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Cost _______________________________ Weight.

Discussion

Note: The time designated for this discussion is included in previous heading.
- Does everyone pay the same cost per ounce? How do you know?
  - Yes, it costs $0.40 per ounce. If we divide each cost value by its corresponding weight, it will give the same unit price (or unit rate) of 0.40. Since we want to compare cost per ounce, we can use the unit (cost per ounce) to determine that we want to divide each cost value by each corresponding weight value.
- Isabelle’s brother takes an extra-long time to create his dish. When he puts it on the scale, it weighs 15 ounces. If everyone pays the same rate in this store, how much will his dish cost? How did you calculate this cost?
  - $6. Guide students to notice that if you multiply the number of ounces by the constant (cost per ounce), it will give you the total cost. Take a moment to have students confer that this would be true for the values they found.
Lesson 2: Proportional Relationships

Since this is true, we say “cost is proportional to weight”. Have students complete the statement in their materials.

What happens if you don’t serve yourself any yogurt or toppings, how much do you pay?
- $0.

And does the relationship above still hold true? In other words, if you buy 0 ounces of yogurt, then multiply by the cost per ounce, do you get 0?
- Even for 0, you can still multiply by this constant value to get the cost (not that you would do this but we can examine this situation for the sake of developing a rule that is always true).

You always multiply the number of ounces, \( x \), by the constant that represents cost per ounce, to get the total cost, \( y \). Pause with students to note that any variables could be chosen but for the sake of this discussion, they are \( x \) and \( y \).

Teacher should label the table with the indicated variables and guide student to do the same.

- For any measure \( x \), how do we find \( y \)?
  - Multiply it by 0.40 (unit price). Indicate this on the given chart as done below. Be sure students do the same.

<table>
<thead>
<tr>
<th>( x ), Weight (ounces)</th>
<th>( y ), Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>3.20</td>
</tr>
</tbody>
</table>

\( y = 0.40x \).

Example 2 (5 minutes): A Cooking Cheat Sheet!

In the back of a recipe book, a diagram provides easy conversions to use while cooking.

- What does the diagram tell us? The number of ounces in a given number of cups.
  - More specifically, each pair of numbers indicates the correct matching of ounces to cups.
- Is the number of ounces proportional to the number of cups? How do you know?
There are 8 ounces for every cup and to get the number of ounces. You can always multiply the number of cups by 8.

Have students complete statement on their materials, ounces is proportional to cups, and note how they can tell. It is important to acknowledge that you could also divide by 8 if you know the number ounces and are trying to find the number of cups. This discussion should lead to the importance of defining the quantities clearly.

- How many ounces are there in 4 cups? 5 cups? 8 cups? How do you know?
  - 32, 40, 64; each time the number of cups is multiplied by 8 to get the number of ounces.

- For the sake of this discussion (and to provide continuity from between examples), let’s represent the cups with $x$, and the ounces with $y$.

Teacher should label the diagram with the indicated variables and guide student to do the same.

- For any number of cups $x$, how do we find the number of ounces, $y$?
  - Multiply it by 8.

- So $y = 8x$.

- If we want to verify that our equation is $y = 8x$, which $x$ and $y$ values can we use to see if it is true? How do you know?
  - We can choose any pair of given $x$, $y$ values since the equation should model the relationship for every pair of values.

It is a good idea to check more than one pair. Guide students to substitute the pairs of values into the equation to prove that for each one, the equation is true.

**Exercise 1 (5 minutes)**

Have students complete the following example independently, then discuss responses as a class.

**Exercise 1:**

During Jose’s physical education class today, students visited activity stations. Next to each station was a chart depicting how many Calories (on average) would be burned by completing the activity.

<table>
<thead>
<tr>
<th>Calories Burned while Jumping Rope</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Calories Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
</tr>
</tbody>
</table>

a. Is the number of Calories burned proportional to time? How do you know?

*Yes, the time is always multiplied by the same number, 11, to find the calories burned.*

b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

*71.5 since 6.5 times 11 is 71.5.*
Example 3 (15 minutes): Summer Job

Read through Example 3 aloud. Allow for brief discussion (2 minutes) of summer jobs or ways students may have earned money over the summer. Pose the following questions:

- How much do you think Alex had earned by the end of 2 weeks?
  - He probably earned twice what he had earned by in week 1.

- How will a table help us to check Alex’s prediction?
  - It will help to see how his earnings grow over time and whether he will have enough money by the end of the summer. A table may also help to check calculations for reasonableness.

- Where did the two given pairs of data come from?
  - He earned $112 after working 4 weeks, therefore his unit rate was $28 for every 1 week OR the total earnings is 28 times the week number.

- Is this reasonable?
  - Yes, could include a brief discussion of minimum wage for part time workers or babysitting rates so that students have some sense of reasonable earning amounts.

- What other pair could we complete fairly easily?
  - At 0 weeks, he has earned $0.

- How will we find out his earnings after 2 weeks? 3 weeks?
  - Since the rate will be the same, we could multiply each number of weeks by 28 to get the corresponding total earnings.

Allow students time (3 minutes) to answer part a, complete remaining values if needed. Give students time to share responses to part a.

Example 3: Summer Job

Alex spent the summer helping out at his family’s business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112.

Alex wonders, “If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?”

To check his assumption, he decided to make a table. He entered his total money earned at the end of week 1 and his total money earned at the end of week 4.

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Earnings</td>
<td>$28</td>
<td>$112</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Work with a partner to answer Alex’s question.

Yes, Alex will have earned enough money to buy the $220 gaming system by the end of the summer since he will have earned 8 · 28, or 224 dollars for the 8 weeks he worked. A sample table is shown below.

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Earnings</td>
<td>$0</td>
<td>$28</td>
<td>$56</td>
<td>$84</td>
<td>$112</td>
<td>$140</td>
<td>$168</td>
<td>$196</td>
<td>$224</td>
</tr>
</tbody>
</table>
Allow for students to share responses with the class for part b, then record in their student pages.

b. Are Alex’s total earning proportional to the number of weeks he worked? How do you know?

Alex’s total earnings are proportional to the number of weeks he worked. There exists a constant value, 28, that can be multiplied by the number of weeks to determine the corresponding earnings for that week. The table shows an example of a proportional relationship.

Closing (2 minutes)

- How do we know if two quantities are proportional to each other?
  - Two quantities are proportional to each other if there is one constant number that is multiplied by each measure in the first quantity to give the corresponding measure in the second quantity.

- How can we recognize a proportional relationship when looking at a table or a set of ratios?
  - If each of the measures in the second quantity is divided by its corresponding measure in the first quantity and it produces the same number, called a constant, then the two quantities are proportional to each other.

Lesson Summary

Measures in one quantity are proportional to measures of a second quantity if there is a positive number $k$ so that for every measure $x$ of the first quantity, the corresponding quantity $y$ is given by $kx$. The equation $y = kx$ models this relationship.

A proportional relationship is one in which the measures of one quantity are proportional to the measures of the second quantity.

In the example given below, the distance is proportional to time since each measure of distance, $y$, can be calculated by multiplying each corresponding time, $t$, by the same value, 10. This table illustrates a proportional relationship between time, $t$, and distance, $y$.

<table>
<thead>
<tr>
<th>Time (h), $t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (km), $y$</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Exit Ticket (8 minutes)
Lesson 2: Proportional Relationships

Exit Ticket

Ms. Albero decided to make juice to serve along with the pizza at the Student Government party. The directions said to mix 2 scoops of powdered drink mix with a half a gallon of water to make each pitcher of juice. One of Ms. Albero’s students said she will mix 8 scoops with 2 gallons of water to get 4 pitchers. How can you use the concept of proportion to decide whether the student is correct?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Ms. Albero decided to make juice to serve along with the pizza at the Student Government party. The directions said to mix 2 scoops of powdered drink mix with a half gallon of water to make each pitcher of juice. One of Ms. Albero’s students said she will mix 8 scoops with 2 gallons of water to get 4 pitchers. How can you use the concept of proportion to decide whether the student is correct?

<table>
<thead>
<tr>
<th>Amount of powdered drink mix</th>
<th>1 scoop</th>
<th>2 scoops</th>
<th>4 scoops</th>
<th>8 scoops</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of water (gallons)</td>
<td>1/4</td>
<td>1/2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

As long as the amount of water is proportional to the number of scoops of drink mix, then the second quantity, amount of water, can always be determined by multiplying the first quantity by the same constant. In this case, if the amount of powdered drink mix is represented by $x$, and the gallons of water is represented by $y$, then $y = \frac{1}{4}x$. To determine any of the measures of water, you will always multiply the number of scoops by $\frac{1}{4}$.

Problem Set Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. A cran-apple juice blend is mixed in a ratio of cranberry to apple of 3 to 5.
   a. Complete the table to show different amounts that are proportional.

<table>
<thead>
<tr>
<th>Amount of Cranberry</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Apple</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

   b. Why are these quantities proportional?

   The amount of apple is proportional to the amount of cranberry since there exists a constant number, $\frac{5}{3}$, that when multiplied by any of the given measures for the amount of cranberry always produces the corresponding amount of apple. If the amount of cranberry is represented by $x$, and the amount of apple is represented by $y$, then each pair of quantities satisfies the equation $y = \frac{5}{3}x$. A similar true relationship could be derived by comparing the amount of cranberry to the amount of apple. In the case where $x$ is the amount of apple and $y$ is the amount of cranberry, the equation would be $y = \frac{5}{3}x$.

2. John is filling a bathtub that is 18 inches deep. He notices that it takes two minutes to fill the tub with three inches of water. He estimates it will take ten more minutes for the water to reach the top of the tub if it continues at the same rate. Is he correct? Explain.

   Yes. In 10 more minutes, the tub will reach 18 inches. At that time the ratio of time to height may be expressed as 12 to 18, which is equivalent to 3 to 2. The height of the bathtub increases $1\frac{1}{2}$ inches every minute.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>1</th>
<th>2</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bathtub Water Height (inches)</td>
<td>1 1/2</td>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>
Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

Student Outcomes

- Students examine situations to decide whether two quantities are proportional to each other by checking for a constant multiple between measures of $x$ and measures of $y$ when given in a table.
- Students study examples of relationships that are not proportional in addition to those that are.

Classwork (20 minutes)

You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>4.5</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>6.5</td>
<td>52</td>
</tr>
</tbody>
</table>

Based on the table above, is pay proportional to hours worked? How do you know?

Yes pay is proportional to hours because every ratio of pay to hours is the same. The ratio is 8 and every measure of hours worked multiplied by 8 will result in the corresponding measure of pay.

\[
\begin{align*}
8 & = 8 \\
16 & = 8 \\
24 & = 8 \\
32 & = 8 \\
36 & = 8 \\
40 & = 8 \\
48 & = 8 \\
52 & = 8
\end{align*}
\]
Discussion

- Explain how you completed the table.
- How did you determine the pay for 4 ½ hours?
  - Multiply the hours by the constant multiple of 8 that relates hours to pay. It is also halfway between 4 and 5; or the pay for ½ an hour would be ½ an hour’s pay, or $4.
- How could you use the information to determine the pay for a week in which you worked 20 hours?
  - Multiply 20 by 8 dollars an hour or continue to extend the table.
- How many other ways can the answer be determined?
  - You have taken the amount for 4 hours worked and multiplied by 5.
- If the quantities in the table were graphed, would the point (0,0) be on that graph? What would it mean in the context of the problem?
  - Yes, the point (0,0) could be a point in the table because if you multiply 0 by any constant, you would get 0. For this problem the point (0,0) represents 0 hours earning $0.
- Describe the relationship between the amount of money earned and the number of hours worked in this example.
  - The two quantities are in a proportional relationship. A proportional relationship exists since a number exists such that when every measure of time is multiplied by the same number the corresponding measures of pay are obtained.
- How can multiplication and division be used to show the earnings are proportional to the number of hours worked?
  - Every measure of time (hours) can be multiplied by the constant 8 to get each measure of pay. Division can be used by dividing each measure of y (pay) by 8 to get the corresponding x (hours) measure.

Guide students to write a response to the question in the student materials.

- In this example, is pay proportional to the hours worked? How do you know?
  - Yes, the amount of money is proportional to the number of hours worked because there is a number, 8, such that each measure of the first quantity multiplied by this same number, 8, gives the corresponding measure of the second quantity.
Examples 1–4 (15 minutes)

Examples 1–4

For Examples 1–3, determine if $y$ is proportional to $x$. Justify your answer.

1. The table below represents the amount of snowfall in 5 counties (in inches) to hours of a recent winter storm.

<table>
<thead>
<tr>
<th>$x$ (Time (h))</th>
<th>$y$ (Snowfall (in))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

$\frac{10}{2} = 5, \frac{12}{6} = 2, \frac{16}{8} = 2, \frac{5}{2.5} = 2, \frac{14}{7} = 2$

$y$ (snowfall) is not proportional to $x$ (time) because all of the ratios comparing snowfall to time are not equivalent. All of the ratios must be the same for the relationships to be proportional. There is NOT one number such that each measure of the $x$ (time) multiplied by the number gives the corresponding measure of $y$ (snowfall).

2. The table below shows the relationship between cost of renting a movie to the number of days on rent.

<table>
<thead>
<tr>
<th>$x$ (Number of Days)</th>
<th>$y$ (Cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

$\frac{2}{6} = \frac{1}{3}, \frac{3}{9} = \frac{1}{3}, \frac{8}{24} = \frac{1}{3}, \frac{1}{3} = \frac{1}{3}$

$y$ (cost) is proportional to $x$ (number of days) because all of the ratios comparing cost to days are equivalent. All of the ratios are equal to $\frac{1}{3}$. Therefore, every measure of $x$ (days) can be multiplied by the number $\frac{1}{3}$ to get each corresponding measure of $y$ (cost).

3. The table below shows the relationship between the amount of candy (pounds) bought and the total cost.

<table>
<thead>
<tr>
<th>$x$ (Pounds)</th>
<th>$y$ (Cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

$\frac{10}{5} = 2, \frac{8}{4} = 2, \frac{12}{6} = 2, \frac{16}{8} = 2, \frac{20}{10} = 2$

$y$ (cost) is proportional to $x$ (pounds) because all of the ratios comparing cost to days are equivalent. All of the ratios are equal to 2. Therefore, every measure of $x$ (pounds) can be multiplied by the number 2 to get each corresponding measure of $y$ (cost).

Possible questions asked by teacher or students:

- When looking at ratios that describe two quantities that are proportional in the same order, do the ratios always have to be equivalent?
  - Yes, all the ratios are equivalent and a constant exists that can be multiplied by the measure of the first quantity to get the measure of the second quantity for every ratio pair.

- For each example, if the quantities in the table were graphed, would the point (0,0) be on that graph? Describe what the point (0,0) would represent in each table.
  - Example 1: 0 inches snowfall in 0 hours
  - Example 2: 0 days out on rent costs $0
  - Example 3: 0 pounds of candy cost $0
- Do the x and y values need to go up at a constant rate? In other words, when the x and y values both go up at a constant rate, does this always indicate that the relationship is proportional?
  - Yes, if a constant exists such that each measure of the x when multiplied by the constant gives the corresponding y-value

Complete Example 4 as a class, pausing to allow students to explain how they arrived at their answers.

4. Randy is planning to drive from New Jersey to Florida. Randy recorded the distance traveled and the total number of gallons used every time he stopped for gas. Assume miles driven is proportional to Gallons Consumed in order to complete the table.

<table>
<thead>
<tr>
<th>Gallons Consumed</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven</td>
<td>54</td>
<td>108</td>
<td>189</td>
<td>216</td>
<td>270</td>
<td>324</td>
</tr>
</tbody>
</table>

Since the quantities are proportional, then every ratio comparing miles driven to gallons consumed must be equal. Using the given values for each quantity, the ratio is

\[
\frac{54}{2} = 27 \quad \frac{216}{8} = 27
\]

If gallons consumed is given and miles driven is the unknown, then multiply the gallons consumed by 27 to get the miles driven.

\[
4(27) = 108 \quad 10(27) = 270 \quad 12(27) = 324
\]

If miles driven is given and gallons consumed is the unknown, then divide the miles driven by 27 to get the gallons consumed.

- Why is it important for you to know that miles are proportional to the gallons used?
  - Without knowing this proportional relationship exists, just knowing how many gallons you consumed will not allow you to determine how many miles you traveled. You would not know if the same relationship exists for each pair of numbers.

Scaffolding:
Some students may not be familiar with cars, distance, miles per gallon relationships. Students may select the car with the lower unit rate because they may be confused with the better buy and lower unit prices. Further clarification may be needed to explain that more miles per gallon is better.
Describe the approach you used to complete the table.

Since miles driven is proportional to gallons consumed, a constant exists such that every measure of gallons used can be multiplied by the constant to give the corresponding amount of miles driven. Once this constant is found to be 27, continue to use it to fill in the missing parts by multiplying each number of gallons by 27.

What is the value of the constant? Explain how the constant was determined.

The value of the constant is 27, and it was determined by dividing the given miles driven by the given gallons consumed.

Explain how to use multiplication and division to complete the table.

If the gallons consumed was given then the number represented the gallons consumed is to be multiplied by the constant of 27 to obtain the amount of the miles driven. If the miles driven were given, then that number needs to be divided by the constant of 27 to get the number of gallons consumed.

Follow – Up Activity: Have students work with a partner. Give each pair 2 3x5 index cards. On one index card, the students work together to create a table of two quantities that are proportional to one another. On the other index card, the students create a “story problem” that would generate the table. Once complete, the teacher collects all the table cards and all the story cards. The teacher displays the table cards around the room and randomly passes out story cards. Students are to match their story to the correct table representation.

Closing (2 minutes)

Describe additional questions.

How can you use a table to determine whether the relationship between two quantities is proportional? The quantities are proportional if a constant number exists such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

Lesson Summary

One quantity is proportional to a second if a constant (number) exists such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

Steps to determine if two quantities in a table are proportional to each other:

For each given measure of Quantity $A$ and Quantity $B$, find the value of $\frac{B}{A}$.

If the value of $\frac{B}{A}$ is the same for each pair of numbers, then the quantities are proportional to each other.

Exit Ticket (5 minutes)
Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

Exit Ticket

The table below shows the price for the number of roses indicated.

<table>
<thead>
<tr>
<th>Number of Roses</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (Dollars)</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
</tr>
</tbody>
</table>

Is the price proportional to the number of roses? How do you know?

Find the cost of purchasing 30 roses.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

The table below shows the price for the number of roses indicated.

<table>
<thead>
<tr>
<th>Number of Roses</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (Dollars)</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
</tr>
</tbody>
</table>

Is the price proportional to the number of roses? How do you know?

The quantities are proportional to one another because there is a constant of 3 such that every measure of the number of roses when multiplied by the constant of 3 results with the corresponding measure of price.

Find the cost of purchasing 30 roses.

If there are 30 roses, then the cost would be $30 \times 3 = $90.

Problem Set Sample Solutions

In each table determine if \( y \) is proportional to \( x \). Explain why or why not.

1. \[
\begin{array}{c|c}
   x & y \\
   3 & 12 \\
   5 & 20 \\
   8 & 32 \\
\end{array}
\]

   Yes, \( y \) is proportional to \( x \) because all ratios of \( y/x \) are equivalent to 4. Each measure of \( x \) multiplied by this constant of 4 gives the corresponding measure in \( y \).

2. \[
\begin{array}{c|c}
   x & y \\
   3 & 15 \\
   4 & 17 \\
   5 & 19 \\
\end{array}
\]

   No, it is not proportional because all the ratios of \( y/x \) are not equivalent. There is not a constant where every measure of \( x \) multiplied by the constant gives the corresponding measure in \( y \). The values of the ratios are 5, 4.25, 3.8, 3.5.

3. \[
\begin{array}{c|c}
   x & y \\
   6 & 4 \\
   9 & 6 \\
   12 & 8 \\
   3 & 2 \\
\end{array}
\]

   Yes, \( y \) is proportional to \( x \) because a constant value of 2/3 exists where each measure of \( x \) multiplied by this constant gives the corresponding measure in \( y \).

4. Kayla made observations about the selling price of a new brand of coffee that sold in three different sized bags. She recorded those observations in the following table:

<table>
<thead>
<tr>
<th>Ounces of Coffee</th>
<th>6</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price in Dollars</td>
<td>$2.10</td>
<td>$2.80</td>
<td>$5.60</td>
</tr>
</tbody>
</table>

Is the price proportional to the amount of coffee? Why or why not?

Yes, the price is proportional to amount of coffee because a constant value of 0.35 exists where each measure of \( x \) multiplied by this constant gives the corresponding measure in \( y \).
Use the relationship to predict the cost of a 20 oz. bag of coffee.

20 ounces would cost $7.

5. You and your friends go to the movies. The cost of admission is $9.50 per person. Create a table showing the relationship between number of people going to the movies and the total cost of admission. Explain why the cost of admission is proportional to the amount of people.

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.50</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>28.50</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
</tr>
</tbody>
</table>

The cost is proportional to the number of people because a constant value of 9.50 exists where each measure of the number of people multiplied by this constant gives the corresponding measure in y.

6. For every 5 pages Gil can read, his daughter can read 3 pages. Let \( g \) equal the number of pages Gil reads and let \( d \) equal the number of pages his daughter reads. Create a table showing the relationship between the number of pages Gil reads and the number of pages his daughter reads. Is the number of pages Gil’s daughter reads proportional to the number of pages he reads? Explain why or why not.

<table>
<thead>
<tr>
<th>( g )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

Yes, the number of pages read by Gil’s daughter is proportional to the number of pages read by Gil because all the ratios of Gil’s daughter to Gil are equivalent to 0.6. Whenever I divide the number of pages read by Gil’s daughter by the number of pages Gil read, I always get the same quotient. Therefore, every measure of Gil’s number of pages multiplied by the constant 0.6 gives the corresponding values of Gil’s daughter’s pages.

7. The table shows the relationship between the number of parents in a household and the number of children in the same household. Is the number of children proportional to the number of parents in the household? Explain why or why not.

<table>
<thead>
<tr>
<th>Number of Parents</th>
<th>Number of Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
No, because there is no constant such that every measure of the number of parents multiplied by the constant would result in the corresponding values of the number of children. When I divide the number of children by the corresponding number of parents, I do not get the same quotient every time. Therefore, the ratios of children to parents are not equivalent. They are 3, 5, 2, 0.5.

8. The table below shows the relationship between the number of cars sold and money earned for a car salesperson. Is the money earned proportional to the number of cars sold? Explain why or why not.

<table>
<thead>
<tr>
<th>Number of Cars Sold</th>
<th>Money Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>950</td>
</tr>
<tr>
<td>4</td>
<td>1076</td>
</tr>
<tr>
<td>5</td>
<td>1555</td>
</tr>
</tbody>
</table>

No, there is no constant such that every measure of number of cars sold multiplied by the constant would result in the corresponding values of the earnings because the ratios of money earned to number of cars sold are not equivalent; the ratios are 250, 300, 316 2/3, 269, 311.

9. Make your own example of a relationship between two quantities that are NOT proportional. Describe the situation and create a table to model it. Explain why one quantity is not proportional to the other. Answers will vary but should include pairs of numbers that do not always have the same value $B/A$. 
Lesson 4: Identifying Proportional and Non-Proportional Relationships in Tables

Student Outcomes

- Students examine situations to decide whether two quantities are proportional to each other by checking for a constant multiple between measures of x and measures of y when given in a table or when required to create a table.
- Students study examples of relationships that are not proportional in addition to those that are.

Classwork

Opening Exercise

Students will work on the following activity independently for 10 minutes. Then students may collaborate with a partner or small group of classmates to discuss answers for 5 minutes. During this time students are to compare, critique the work that was done individually, and work together to come up with a presentable solution. If all students completed the task individually, then they should check each other’s work for accuracy and completeness. Lastly, students share their solutions with the class for 5 minutes. Many times there are multiple ways that the problem may have been completed or explained, so teacher should circulate during the collaboration time and select students with different approaches by asking students who have come up with one way to solve to find another. If the same approach was used throughout, select different students for different parts of the problem to present.

Example 1 (25 minutes): Which Team Will Win the Race?

Example 1: Which Team Will Win the Race?

You have decided to run in a long distance race. There are two teams that you can join. Team A runs at a constant rate of 2.5 miles per hour. Team B runs 4 miles the first hour and then 2 miles per hour after that.

Task: Create a table for each team showing the distances that would be run for times of 1, 2, 3, 4, 5, and 6 hours. Using your tables, answer the questions that follow:

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>
Lesson 4
Identifying Proportional and Non-Proportional Relationships in Tables

Date: 8/8/13

Scaffolding:
The teacher may be able to extend the concept to derive the formula \( d = rt \) followed by a class discussion on how to transform the formula where rate can be found in terms of distance and time. Lastly, distance and time can be related to \( y \) and \( x \) variables.

a. For which team is distance proportional to time? Explain your reasoning.
   Team A since all the ratios comparing distance to time are equivalent. The equivalent ratio is 2.5. Every measure of time can be multiplied by 2.5 to give the corresponding measures of distance.

b. Explain how you know the distance for the other team is not proportional to time.
   In team B the ratios are not equivalent. The ratios are 4, 3, 8/3, 2.5, 12/5, and 14/6. Therefore, every measure of time cannot be multiplied by a constant to give each corresponding measure of distance.

c. If the race were 2.5 miles long, which team would win? Explain.
   Team B would win because more distance was covered in less time.

   If the race were 3.5 miles long, which team would win? Explain.
   Team B would win because more distance was covered in less time.

   If the race were 4.5 miles long, which team would win? Explain.
   Team A would win because more distance was covered in less time

d. For what length race would it be better to be on Team B than Team A? Explain.
   If the race were less than 10 miles, Team B is faster because more distance would be covered in less time.

e. Using this relationship, if the members on the team ran for 10 hours, how far would each member run on each team?
   Team A = 25 miles
   Team B = 22 miles

f. Will there always be a winning team, no matter what the length of the course? Why or why not?
   No, there would be a tie (both teams win) at 4 hours.

g. If the race were 12 miles long, which team should you choose to be on if you wish to win? Why would you choose this team?
   Team A because they would finish in 4.8 hours compared to Team B in 5 hours

h. How much sooner would you finish on that team compared to the other team?
   2 hours or \( \frac{2}{10} \times 60 = 12 \) minutes

Closing (5 minutes)

- How does knowing two quantities are proportional help answer questions about the quantities? For example, if we know 1 cup = 8 oz, what does that allow us to do?
  - Understanding the relationship can allow you to find missing quantities. For the example mentioned, one cup can be substituted for 8 ounces or vice versa.

Exit Ticket (10 minutes)
Lesson 4: Identifying Proportional and Non-Proportional Relationships in Tables

Exit Ticket

The table below shows the relationship between the side lengths of a regular octagon and its perimeter.

<table>
<thead>
<tr>
<th>Side Lengths, s (inches)</th>
<th>Perimeter, P (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Complete the table.

If Gabby wants to make an octagon with a side length of 20 inches using wire, how much wire does she need? Justify your reasoning with an explanation of whether perimeter is proportional to the side length.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

The table below shows the relationship between the side lengths of a regular octagon and its perimeter.

<table>
<thead>
<tr>
<th>Side Lengths, s (inches)</th>
<th>Perimeter, P (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>72</td>
</tr>
<tr>
<td>12</td>
<td>96</td>
</tr>
</tbody>
</table>

Complete the table.

If Gabby wants to make an octagon with a side length of 20 inches using wire, how much wire does she need? Justify your reasoning with an explanation of whether perimeter is proportional to the side length.

Gabby would need 20(8) = 160 inches of wire to make an octagon out of wire with a side length of 20. This table shows perimeter is proportional to the side length since the constant = 8, and when all side lengths are multiplied by the constant, the corresponding perimeter is obtained. Since the perimeter is found by adding all 8 side lengths together or multiplying the length of 1 side by 8, the two numbers must always be proportional.

Problem Set Sample Solutions

1. Joseph earns $15 for every lawn he mows. Is the amount of money he earns proportional to the number of lawns he mows? Make a table to help you identify the type of relationship.

<table>
<thead>
<tr>
<th>Number of Lawns Mowed</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings ($)</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

The table shows earning proportional to the number of lawns mowed. The equivalent ratio and constant is 15.

2. At the end of the summer, Caitlin had saved $120 from her summer job. This was her initial deposit into a new savings account at the bank. As the school year starts, Caitlin is going to deposit another $5 each week from her allowance. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

<table>
<thead>
<tr>
<th>Time (in weeks)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Account Balance ($)</td>
<td>120</td>
<td>125</td>
<td>130</td>
<td>135</td>
</tr>
</tbody>
</table>

Caitlin’s account balance is not proportional to the number of weeks of deposits since the ratio does not remain the same, and when every time is multiplied by the constant, it does not give the corresponding balance values.

120 125 130 135
0 ≠ 1 ≠ 2 ≠ 3

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3. Lucas and Brianna read three books each last month. The table shows the number of pages in each book and the length of time it took to read the entire book.

<table>
<thead>
<tr>
<th>Pages Lucas Read</th>
<th>208</th>
<th>156</th>
<th>234</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>8</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pages Brianna Read</th>
<th>168</th>
<th>120</th>
<th>348</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>6</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

a. How many observations can you make about any similarities or differences that exist between the reading rates of the two students.

The table shows Lucas’s number of pages read to be proportional to the time because when the constant of 26 is multiplied by each measure of time, it gives the corresponding values for the number of pages read.

b. Both Lucas and Brianna had specific reading goals they needed to accomplish. What different strategies did each person employ in reaching those goals?

Lucas read at a constant rate throughout the summer, 26 pages, where Brianna’s reading rate was not the same throughout the summer.
Lesson 5: Identifying Proportional and Non-Proportional Relationships in Graphs

Student Outcomes

- Students decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Students study examples of quantities that are proportional to each other as well as those that are not.

Classwork

Opening Exercise (5 minutes)

- Give students the ratio table and ask them to identify if the two quantities are proportional to each other and to give reasoning for their answer. The two quantities are not proportional to each other because a constant describing the proportion does not exist.

<table>
<thead>
<tr>
<th>x</th>
<th>Candy Bars Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Example 1 (7 minutes): From a Table to Graph

Prompt students to create another ratio table that contains two sets of quantities that are proportional to each other using the first ratio on the table.
Present a coordinate grid and ask students to recall standards in grade 5 and 6 on coordinate plane, x-axis, y-axis, origin, quadrants, plotting points, and ordered pairs.

As a class, ask students to express the ratio pairs as ordered pairs.

Questions to discuss:

- What is the origin and where is it located?
  - The intersection of the x and y axis, at the ordered pair (0,0).
- Why are we going to focus on quadrant 1?
  - We are using numbers that are positive. Since we are measuring or counting quantities (number of candy bars sold and amount of money), the numbers in our ratios will be positive.
- What should we label the x-axis and y-axis?
  - The x-axis should be the chocolate bars, and the y-axis should be the amount of money received.
- Could it be the other way around?
  - No, the amount of money received should depend on the candy bars being sold, so the amount of money should be y, the dependent variable.
- How should we note that on the table?
  - The first values of the table should be the x-coordinates (the independent variable) and the second values of the pairs should be the y-coordinate (the dependent variable).
- How do we plot the first ratio pair?
  - If the relationship is 3:2, where 3 represents 3 chocolate bars, and 2 is 2 dollars, then from the first point, we go 3 to the right on the x axis and go up to 2 on the y axis.
- When we are plotting a point, where do we count from?
  - The origin, (0,0)

Have students plot the rest of the points and use a ruler to join the points and ask them:

- What observations can you make about the arrangement of points?
  - The points all fall on a straight line.
- Do we extend the line in both directions? Explain why or why not.
  - Technically, the line for this situation should start at (0,0) to represent 0 dollars for 0 chocolate bars and extend infinitely in the positive direction because the more chocolate bars Isaiah sells, the more he makes.
- Would all proportional relationships pass through the origin? Think back to those discussed in previous lessons. Take a few minutes for students to share some of the context of previous examples and whether (0,0) would always be included on the line that passes through the pairs of points in a proportional relationship.
  - Yes, it should always be included for proportional relationships. For example, if a worker works zero hours, then he or she would get paid zero dollars, or if a person drives zero minutes, the distance covered is zero miles.
- What can you infer about graphs of two quantities that are proportional to each other?
  - The graph will be a straight line and go through the origin.
Why is it a straight line?
- Each chocolate bar is being sold for $1.50 each, which is the unit rate and also the constant of the proportion. This means that for every increase of 1 on the x-axis, there will be an increase of the same amount (the constant) on the y-axis. This creates a straight line. Each point may not be part of the set of ratios; however, the line would pass through all of the points that do exist in the set of ratios.

Complete “Important Note:” with class. In a proportional relationship, the points will all fall on a straight line going through the origin.

Example 1: From a Table to Graph

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

Important Note:
Characteristics of graphs of proportional relationships:
1. Points lie in a straight line.
2. Line goes through the origin.

Example 2 (7 minutes)

Have students plot ordered pairs for all the values of the Opening Exercise.

- Does the ratio table represent quantities that are proportional to each other?
  - No, not all the quantities are proportional to each other.
- What can you predict about the graph of this ratio table?
  - The points will not lie on a straight line and will not go through the origin.
- Was your prediction correct?
  - Partly, the majority of the points lie in a straight line that goes through the origin, but because of one point, the line is not completely straight.
Lesson 5
Identifying Proportional and Non-Proportional Relationships in Graphs

From this example, what is important to note about graphs of two quantities that are not proportional to each other?

- The graph could go through the origin, but if it does not lie in a straight line, it does not represent two quantities that are proportional to each other.

Example 2

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

Example 3 (7 minutes)

Have students plot the points from Example 3.

- How are the graphs of the data in examples 1 and 3 similar? How are they different?
  - In both graphs, the points lie on a line. One graph is steeper than the other. Graph 1 begins at the origin, but Graph 3 does not.

- What do you know about the ratios before you graph them?
  - The quantities are not proportional to each other.

- What can you predict about the graph of this ratio table?
  - The points will not lie in a straight line that goes through the origin

- Was your prediction correct?
  - No, the graph is a straight line, but it does not go through the origin.

- What are the similarities of the graphs of two quantities that are proportional to each other and graphs of two quantities that are not shared?
  - Both graphs can have points that lie in a line but the graph of the quantities that are proportional to each other must also go through the origin.
Example 3

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

Similarities with Example 1:
The points of both graphs fall in a line.

Differences from Example 1:
The points of Graph 1 fall in a line that pass through the origin. The points of Graph 3 fall in a line that do not pass through the origin.

Closing Questions (5 minutes)

- How are proportional quantities represented in a graph?
  - They are represented in a graph where the points lie on a straight line that passes through the origin.

- What is a common mistake a student might make when deciding whether a graph of two quantities shows that they are proportional to each other?
  - Both graphs can have points that lie on a straight line, but the graph of the quantities that are proportional to each other also goes through the origin. In addition the graph could go through the origin, but the points do not lie on a straight line.

Lesson Summary:
When two proportional quantities are graphed on a coordinate plane, the points lie on a straight line that passes through the origin.

Exit Ticket (5 minutes)
Lesson 5: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. The following table gives the number of people picking strawberries in a field and the corresponding number of hours that these people worked picking strawberries. Graph the table. Does the graph represent two quantities that are proportional to each other? Explain why or why not.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Fill in the table and given values to create quantities proportional to each other and graph them.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
3.
   a. What are the differences between the graphs in Problem 1 and 2?

   b. What are similarities in the graphs in Problem 1 and 2?

   c. What makes one graph represent quantities that are proportional to each other and one graph that does not represent quantities that are proportional to each other in Problems 1 and 2?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. The following table gives the number of people picking strawberries in a field and the corresponding number of hours that these people worked picking strawberries. Graph the table. Does the graph represent two quantities that are proportional to each other? Why or why not?

   Although the points fall on a line, the line does not pass through the origin, so the graph does not represent two quantities that are proportional to each other.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Fill in the table and given values to create quantities proportional to each other and graph.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

3. a. What are the differences between the graphs in Problem 1 and 2?

   The graph in Problem 1 forms a line that slopes downward while the one in Problem 2 slopes upward.

   b. What are similarities in the graphs in Problem 1 and 2?

   Both graphs form straight lines and both graphs include the point (4,2).

   c. What makes one graph represent quantities that are proportional to each other and one graph that does not represent quantities that are proportional to each other in Problems 1 and 2?

   Although both graphs form straight lines, the graph that represents quantities that are proportional to each other needs to pass through the origin.
Problem Set Sample Solutions

1. Determine whether or not the following graphs represent two quantities that are proportional to each other. Give reasoning.
   a. This graph represents two quantities that are proportional to each other because the points lie on a line, and the line that passes through the points would also pass through the origin.

   ![Graph 1](image1)
   **Donated Money vs. Donations Matched by Benefactor**
   - **Donations Matched by Benefactor ($)**
   - **Money Donated**
   - **This graph represents two quantities that are proportional to each other because the points lie on a line, and the line that passes through the points would also pass through the origin.**

   b. This graph does not because although the points lie on a line, the line does not go through the origin.

   ![Graph 2](image2)
   **Age Versus Admission Price**
   - **Admission Price ($)**
   - **Age (years)**
   - **This graph does not because although the points lie on a line, the line does not go through the origin.**

   c. This graph does not because although it goes through the origin, the points do not lie on one line.

   ![Graph 3](image3)
   **Extra Credit vs. Problems Solved**
   - **Extra Credit Points**
   - **Problems Solved**
   - **This graph does not because although it goes through the origin, the points do not lie on one line.**
2. Create a table and a graph for the ratios 2:22, 3 to 15 and 1/11. Does the graph show that the two quantities are proportional to each other? Explain why or why not.

This graph does not because the points do not lie on a line that goes through the origin.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>

3. Graph the following tables and identify if the two quantities are proportional to each other on the graph.

a. yes

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

b. no

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Student Outcomes

- Students examine situations carefully to decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether all the points would fall on a line that passes through the origin.
- Students study examples of relationships that are not proportional as well as those that are.

Today’s activity is an extension of Lesson 5. You will be working in groups to table, graph and identify whether or not the two quantities are proportional to each other.

Classwork

Preparation (7 minutes)

Place students in groups of four. Hand out markers, poster paper, graph paper, and envelopes containing 5 ratios each. (Each group will have identical contents.)

- Have recorder fold the poster paper in quarters and label as follows: Quad 1- Table, Quad 2- Problem, Quad 3- Graph, Quad 4- Proportional or Not?, Explain.
- Instruct the reader to take out the contents of the envelopes and the group to arrange them on a table and graph.
- Instruct the reader to state the problem. Students use multiple methods to show whether the quantities represented in the envelope are proportional to each other or not.

Collaborative Work (20 minutes)

Within the groups, give students 15 minutes to discuss the problem and record their responses onto the poster paper. For the last 5 minutes, have groups adhere their posters on the wall and circulate around the room looking for the group that has the same ratios. Have groups with the same ratios identify and discuss the differences of their posters.

Art Gallery (8 minutes)

In groups, have students observe each poster, write any thoughts on sticky notes and adhere them to the posters. Also, have students answer the following questions on their worksheets:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Was there a group that stood out by representing their problems and findings exceptionally clearly?
Closing (10 minutes)

- Why make posters with others? Why not do this activity in your student books?
  - We can dialogue with others and learn from their thought processes. When we share information with others, our knowledge is tested and questioned.

- What does it mean for a display to be both visually appealing and informative?
  - For a display to be both visually appealing and informative, the reader should be able to find data and results fairly quickly and somewhat enjoyably.

- How much time did your group spend on the content of your poster, and how much time was spent making it visually appealing? What factors determined these time lengths?
  - The discussion and dialogue take the most time and then the outline of the poster the next.

- Suppose we invited people from another school, state or country to walk through our gallery. Would they be able to learn about ratio and proportion from our posters?
  - Hopefully, after looking through the series of posters, people can learn and easily determine for themselves if graphs represent proportional and non-proportional relationships.

Lesson Summary

Graphs of Proportional Relationships: The graph of two quantities that are proportional fall on a straight line that passes through the origin.

Exit Ticket (5 minutes)
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

**Ratios for Groups**

Cut and place in labeled envelopes prior to instructional time.

<table>
<thead>
<tr>
<th><strong>Group 1</strong></th>
<th><strong>Group 2</strong></th>
<th><strong>Group 3</strong></th>
<th><strong>Group 4</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A local frozen yogurt shop is known for their monster sundaes to be shared by a group. The ratios represent the number of toppings to total cost. Create a table then graph and explain if the quantities are proportional to each other or not.</td>
<td>The school library receives money for every book sold at the school’s book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table then graph and explain if the quantities are proportional to each other or not.</td>
<td>Your uncle just bought a hybrid car and wants to take you and your siblings camping. The ratios represent the number of gallons remaining to hours of driving. Create a table then graph and explain if the quantities are proportional to each other or not.</td>
<td>For a Science project Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to colonies of mold on the bread. Create a table then graph and explain if the quantities are proportional to each other or not.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 to 0</th>
<th>1 to 5</th>
<th>8 to 0</th>
<th>1 to 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:3</td>
<td>2 to 10</td>
<td>After 1 hour of driving, there are 6 gallons of gas left in the tank.</td>
<td>2 to 4</td>
</tr>
<tr>
<td>8/6</td>
<td>The library received $15 for selling 3 books.</td>
<td>4 : 4</td>
<td>3:9</td>
</tr>
<tr>
<td>The total cost of a 10-topping sundae is $9.</td>
<td>4:20</td>
<td>2 to 7</td>
<td>4/16</td>
</tr>
<tr>
<td>12 to 12</td>
<td>5: 25</td>
<td>0/8</td>
<td>Twenty five colonies were found on the fifth day.</td>
</tr>
</tbody>
</table>
### Group 5
For a Science project Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to colonies of mold on the bread. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 1</td>
<td>0/8</td>
</tr>
<tr>
<td>2 to 4</td>
<td>After 1 hour of driving, there are 6 gallons of gas left in the tank.</td>
</tr>
<tr>
<td>3:9</td>
<td>The library received $15 for selling 3 books.</td>
</tr>
<tr>
<td>4/16</td>
<td>Twenty five colonies were found on the fifth day.</td>
</tr>
</tbody>
</table>

### Group 6
Your uncle just bought a hybrid car and wants to take you and your siblings camping. The ratios represent the number of gallons remaining to hours of driving. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 to 0</td>
<td>2 to 10</td>
</tr>
<tr>
<td>1 to 5</td>
<td>After 1 hour of driving, there are 6 gallons of gas left in the tank.</td>
</tr>
<tr>
<td>4 : 4</td>
<td>The library received $15 for selling 3 books.</td>
</tr>
<tr>
<td>2 to 7</td>
<td>4:20</td>
</tr>
</tbody>
</table>

### Group 7
The school library receives money for every book sold at the school’s book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 5</td>
<td>4/16</td>
</tr>
<tr>
<td>4/16</td>
<td>After 1 hour of driving, there are 6 gallons of gas left in the tank.</td>
</tr>
<tr>
<td>5: 25</td>
<td>The library received $15 for selling 3 books.</td>
</tr>
<tr>
<td>12 to 12</td>
<td>Twenty five colonies were found on the fifth day.</td>
</tr>
</tbody>
</table>

### Group 8
A local frozen yogurt shop is known for their monster sundaes to be shared by a group. The ratios represent the number of toppings to total cost. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 to 0</td>
<td>8/6</td>
</tr>
<tr>
<td>4 to 0</td>
<td>6:3</td>
</tr>
<tr>
<td>8/6</td>
<td>4:20</td>
</tr>
<tr>
<td>12 to 12</td>
<td>After 1 hour of driving, there are 6 gallons of gas left in the tank.</td>
</tr>
</tbody>
</table>
Collaborative Work Sample Solutions

Group 1 and 8

**Problem:** A local frozen yogurt shop is known for their monster sundaes. Create a table then graph and explain if the quantities are proportional to each other or not.

**Table:**

<table>
<thead>
<tr>
<th>Number of Toppings</th>
<th>Cost of Toppings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

**Graph:**

Explanation: Although the graph lies in a straight line, the quantities are not proportional to each other because the line does not go through the origin. Each topping does not have the same unit cost.

Group 2 and 7

**Problem:** The school library receives money for every book sold at the school’s book fair. Create a table then graph and explain if the quantities are proportional to each other or not.

**Table:**

<table>
<thead>
<tr>
<th>Number of Books Sold</th>
<th>Donations per Sponsor ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

**Graph:**

Explanation: The quantities are proportional to each other because the points lie in a straight line and go through the origin. Each book sold brings in $5.00 no matter how many books are sold.
Group 3 and 6

Problem: Your uncle just bought a hybrid car and wants to take you and your siblings camping. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Gallons of Gas left in tank</th>
<th>Hours of Driving</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Graph:

Explanation: The graph is not a straight line passing through the origin so the quantities are not proportional to each other. The number of gallons of gas vary depending on how fast or slow the car is driven.

Group 4 and 5

Problem: For a Science project Eli decided study colonies of mold. He observed a piece of bread that was molding. Create a table then graph and explain if the quantities are proportional to each other or not.

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>Colonies of Mold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

Graph:

Explanation: Although the graph looks as though it goes through the origin, the quantities are not proportional to each other because the points do not lie on a straight line. Each day does not produce the same amount of colonies as the other days.
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. Which graphs in the art gallery walk represented proportional relationships and which did not? List the group number.

<table>
<thead>
<tr>
<th>Proportional Relationship</th>
<th>Non-proportional Relationship</th>
</tr>
</thead>
</table>

2. What are the characteristics of the graphs that represent proportional relationships?

3. For the graphs representing proportional relationships, what does (0,0) mean in the context of the given situation?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. Which graphs in the art gallery walk represented proportional relationships and which did not? List the group number.

<table>
<thead>
<tr>
<th>Proportional Relationship</th>
<th>Non-proportional Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2</td>
<td>Group 1</td>
</tr>
<tr>
<td>Group 7</td>
<td>Group 3</td>
</tr>
<tr>
<td></td>
<td>Group 4</td>
</tr>
<tr>
<td></td>
<td>Group 5</td>
</tr>
<tr>
<td></td>
<td>Group 6</td>
</tr>
<tr>
<td></td>
<td>Group 8</td>
</tr>
</tbody>
</table>

2. What are the characteristics of the graphs that represent proportional relationships?

*Graphs of groups 2 and 7 fall in a straight line and go through the origin.*

3. For the graphs representing proportional relationships, what does (0,0) mean in the context of the situation?

*For zero books sold, the library received zero dollars of donations.*
1. Sally’s aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent years to amount of money in her savings account.

- After one year, the interest had accumulated, and the total was $312 in Sally’s account.
- After three years, the total was $340. After six years, the total was $380.
- After nine years, the total was $430. After 12 years, the total amount in Sally’s savings account was $480.

Using the same four-fold method from class, create a table then graph, and determine whether the amount of money accumulated and time elapsed are proportional to each other or not. Use your table and graph to support your reasoning.

Table:

<table>
<thead>
<tr>
<th>Years</th>
<th>Savings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>312</td>
</tr>
<tr>
<td>3</td>
<td>340</td>
</tr>
<tr>
<td>6</td>
<td>380</td>
</tr>
<tr>
<td>9</td>
<td>430</td>
</tr>
<tr>
<td>12</td>
<td>480</td>
</tr>
</tbody>
</table>

Graph:

Explanation: The graph is not proportional because although it seems to pass through a line, it is not a line that goes through the origin. The amount of interest collected is not the same every year.
Topic B:
Unit Rate and Constant of Proportionality

**Focus Standard:**

7.RP.2b, 7.RP.2c, 7.RP.2d, 7.EE.4a

- **7.RP.2b**
  - Recognize and represent proportional relationships between quantities.

- **7.RP.2c**
  - Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **7.RP.2d**
  - Represent proportional relationships by equations. For example, if total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).

- **7.EE.4a**
  - Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

  - **a.** Solve word problems leading to equations of the form \( px + q = r \) and \( p(x + q) = r \), where \( p, q, \) and \( r \) are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

**Instructional Days:** 4

- **Lesson 7:** Unit Rate as the Constant of Proportionality (P)
- **Lessons 8–9:** Representing Proportional Relationships with Equations (P)
- **Lesson 10:** Interpreting Graphs of Proportional Relationships (P)

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1 Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson
In Topic B, students learn to identify the constant of proportionality by finding the unit rate in the collection of equivalent ratios. They represent this relationship with equations of the form \( y = kx \), where \( k \) is the constant of proportionality (7.RP.2, 7.RP.2c). In Lessons 8 and 9, students derive the constant of proportionality from the description of a real-world context and relate the equation representing the relationship to a corresponding ratio table and/or graphical representation (7.RP.2b, 7.EE.4). Topic B concludes with students consolidating their graphical understandings of proportional relationships as they interpret the meanings of the points \((0,0)\) and \((1, r)\), where \( r \) is the unit rate, in terms of the situation or context of a given problem (7.RP.2d).
Lesson 7: Unit Rate as the Constant of Proportionality

Student Outcomes

- Students identify the same value relating the measures of x and the measures of y in a proportional relationship as the constant of proportionality and recognize it as the unit rate in the context of a given situation.
- Students find and interpret the constant of proportionality within the contexts of problems.

Classwork

Example 1 (20 minutes): National Forest Deer Population in Danger?

Begin this lesson by presenting the following situation: Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 24 deer in a 216 square acre plot of the forest. Do conservationists need to be worried?

Questions for Discussion: Guide students to complete necessary information on student handout.

Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 24 deer in a 216 square acre plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the national forest?
   - Have students generate as many theories as possible (e.g., food supply, overpopulation, damage to land, etc.).

b. What is the population density of deer per square mile?
   - See chart below.

Encourage students to make a chart to organize the data from the problem and then explicitly model finding the constant of proportionality. Students have already found unit rate in earlier lessons but have not identified it as the constant of proportionality. Remember that the constant of proportionality is also like a scalar and will be used in an equation as the constant.

- When we find the number of deer per 1 square mile, what is this called?
  - Unit rate.
- When we look at the relationship between square miles and number of deer in the table below, how do we know if the relationship is proportional?
The square miles is always multiplied by the same value, 9 in this case.

<table>
<thead>
<tr>
<th>Square Miles</th>
<th>Number of Deer</th>
<th>( \frac{144}{16} = 9 )</th>
<th>( \frac{117}{13} = 9 )</th>
<th>( \frac{216}{24} = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>144</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>117</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>216</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- We call this constant (or same) value the “constant of proportionality”.
- So deer per square mile is 9, and the constant of proportionality is 9. Is that a coincidence or will that always be the case: that the unit rate and the constant of proportionality are the same?

Allow for comments or observations but leave a lingering question for now.

- We could add the unit rate to the table so that we have 1 square mile in the first column and 9 in the second column? (Add this to table for students to see). Does that help to guide your decision about the relationship between unit rate and COP? We will see if your hypothesis holds true as we move through more examples.

**The Unit Rate of deer per 1 square mile is 9.**

**Constant of Proportionality:**  
\( k = 9 \)

**Meaning of Constant of Proportionality in this problem:** *There are 9 deer for every 1 square mile of forest.*

(Could be completed later after formalizing this concept in a few more examples)

c. Use the unit rate of deer per square mile to determine how many deer are there for every 207 square miles.  
\[ 9(207) = 1863 \]

d. Use the unit rate to determine the number of square miles in which you would find 486 deer?  
\[ \frac{486}{9} = 54 \]

Based upon the discussion of the questions above, answer the question: Do conservationists need to be worried? Be sure to support your answer with mathematical reasoning about rate and unit rate.

Review vocabulary box with students.
Remind students that in the example with the deer population, we are looking for deer per square mile, so the number of square miles could be defined as \( x \), and the number of deer could be defined as \( y \), so the unit rate in deer per square mile is \( \frac{144}{16} \), or 9. The constant of proportionality, \( k \), is 9. The meaning in the context of Example 1 is: There are 9 deer for every 1 square mile of forest.

**Discussion**

- How are the constant of proportionality and the unit rate alike?
  - They are the same. They both represent the same number that is the value of the ratio of \( y \) to \( x \).

**Example 2 (18 minutes): You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needed 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needed the cookies for an event at school on the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets. Encourage students to make a chart to organize the data from the problem.

a. Is the number of cookies proportional to the number of sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies needed.

<table>
<thead>
<tr>
<th># of cookie sheets</th>
<th># of cookies baked</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>72</td>
</tr>
<tr>
<td>10</td>
<td>180</td>
</tr>
<tr>
<td>16</td>
<td>288</td>
</tr>
</tbody>
</table>

\[
\frac{36}{2} = 18
\]
\[
\frac{72}{4} = 18
\]
\[
\frac{180}{10} = 18
\]
\[
\frac{288}{16} = 18
\]

**Scaffolding:**

For students who need more challenge, have them create a problem in which the constant rate is a fraction.
The unit rate is 18.  
The constant of proportionality is 18.  
Meaning of Constant of Proportionality in this problem: There are 18 cookies per 1 sheet.

b. It took 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 pm, when will they finish baking the cookies?

96 students (3 cookies each) = 288 cookies

\[
\frac{288 \text{ cookies}}{18 \text{ cookies per sheet}} = 16 \text{ sheets of cookies}
\]

If it takes 2 hours to bake 8 sheets, it will take 4 hours to bake 16 sheets of cookies. They will finish baking at 8:00 pm.

Example 3: French Class Cooking

Example 3: French Class Cooking

Suzette and Margo want to prepare crepes for all of the students in their French class. A recipe makes 20 crepes with a certain amount of flour, milk, and 2 eggs. The girls know that they already have plenty of flour and milk but need to determine the number of eggs needed to make 50 crepes because they are not sure they have enough eggs for the recipe.

a. Considering the amount of eggs necessary to make the crepes, what is the constant of proportionality?

\[
\frac{2 \text{ eggs}}{20 \text{ crepes}} = \frac{1 \text{ egg}}{10 \text{ crepes}}; \text{The constant of proportionality is } \frac{1}{10}.
\]

b. What does the constant or proportionality mean in the context of this problem?

One egg is needed to make 10 crepes.

c. How many eggs will be needed for 50 crepes?

\[
50 \left( \frac{1}{10} \right) = 5; \text{ Five eggs will be needed to make 50 crepes.}
\]

Closing Question (2 minutes)

- What is another name for the constant that relates the measures of two quantities?
  - Constant of Proportionality

- How is the Constant of Proportionality related to the unit rate?
  - They are the same.

Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality.
Lesson 7: Unit Rate as the Constant of Proportionality

Exit Ticket

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will be able to determine the total cost of the sodas. Who is right and why?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will be able to determine the total cost of the sodas. Who is right and why?

Susan is correct. The table below shows that if you multiply by the unit price, say 0.50, by the number of people, say 12, you will determine the total cost of the soda. I created a table to model the proportional relationship. I used a unit price of 0.50 to make the comparison.

Susan

<table>
<thead>
<tr>
<th># of people</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost of soda (in $)</td>
<td>1</td>
<td>1.50</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

I used the same values to compare to John.

\[
\frac{\text{John total cost}}{12 \text{ people}} = ?
\]

If the total cost is $24, then \( \frac{24}{12} = 2 \). This would mean that $2 will be needed for the 12 pack which is not correct. This amount would represent the unit price or cost for 1 soda.

Problem Set Sample Solutions

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. Bananas are $0.59/pound.
   a. What is the constant of proportionality, k?
      \[ K = 0.59 \]
   b. How much do 25 pounds of bananas cost?
      \[ 25(0.59) = $14.75 \]

2. The dry cleaning fee for 3 pairs of pants is $18.
   a. What is the constant of proportionality?
      \[ \frac{18}{3} = 6 \text{ so } K=6 \]
   b. How much will the dry cleaner charge for 11 pairs of pants?
      \[ 6(11) = $66 \]

3. For every $5 that Micah saves, his parents give him $10.
   a. What is the constant of proportionality?
      \[ \frac{10}{5} = 2 \text{ so } k=2 \]
b. If Micah saves $150, how much money will his parents give him?

\[ 2(150) = 300 \]

4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $1260 for 84 students to enter the zoo. In 2011, the school paid $1050 for 70 students to enter the zoo. In 2012, the school paid $1395 for 93 students to enter the zoo.

a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?

<table>
<thead>
<tr>
<th># of students</th>
<th>Price</th>
<th>[ \frac{1260}{84} = 15 ]</th>
<th>[ \frac{1050}{70} = 15 ]</th>
<th>[ \frac{1395}{93} = 15 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>1,260</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>70</td>
<td>1,050</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>93</td>
<td>1,395</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

b. Explain why or why not?

Because the ratio of the entrance fee paid per student was the same

\[ \frac{1260}{84} = 15 \]

c. Identify the constant of proportionality and explain what it means in the context of this situation.

\[ K = 15. \text{ the price per student} \]

d. What would the school pay if 120 students entered the zoo?

\[ 120(15) = 1,800 \]

e. How many students would enter the zoo if the school paid $1,425?

\[ \frac{1,425}{15} = 95 \text{ students} \]
Lesson 8: Representing Proportional Relationships with Equations

Student Outcomes

- Students use the constant of proportionality to represent proportional relationships by equations in real world contexts as they relate the equations to a corresponding ratio table and/or graphical representation.

Classwork

Discussion (5 minutes)

Points to remember:

- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate, can also be called the constant of proportionality.

Discussion Notes

How could we use what we know about the constant of proportionality to write an equation?

Discuss important facts.

Encourage students to begin to think about how we can model a proportional relationship by using an equation by framing with the following probing questions:

- If we know that the constant of proportionality, \( k \), to be equal to \( y/x \) for a given set of ordered pairs, \( x \) and \( y \), then we can write \( k = y/x \). How else could we write this equation? What if we know the \( x \)-values, and the constant of proportionality, but do not know the \( y \)-values. Could we rewrite this equation to solve for \( y \)?

Elicit ideas from students. Apply ideas in examples below. Provide the context of the examples below to encourage students to test their thinking.

Students should note the following in their materials: \( k = y/x \) and eventually \( y = kx \) (may need to add this second equation after Example 1).
Examples 1 and 2 (33 minutes)

MP.2 Write an equation that will model the real world situation.

Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother’s Gas Record

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
</tbody>
</table>

a. Find the constant of proportionality and explain what it represents in this situation.

\[
\begin{array}{c|c}
\text{Gallons} & \text{Miles Driven} \\
\hline
8 & 224 \\
10 & 280 \\
4 & 112 \\
\end{array}
\]

Constant of proportionality is \( k = 28 \). The car travels 28 miles for every one gallon of gas.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

\[ y = 28x \text{ or } m = 28g \]

c. Knowing that there is a half gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

\text{No, she will not make it because she gets 28 miles to one gallon. Since she has ½ gallon remaining in the gas tank, she can travel 14 miles. Since the nearest gas station is 26 miles away, she will not have enough gas.}

d. Using the equation found in part b, determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways.

\text{Using arithmetic: } 28(18) = 504

\text{Using Algebra: } m = 28g \quad \text{— Use substitution to replace the } g \text{ (gallons of gas) with 18.}

\[
m = 28(18) \quad \text{— This is the same as multiplying by the constant of proportionality.}
\]

\[
m = 504
\]

Your mother can travel 504 miles on 18 gallons of gas.
Lesson 8

NYS COMMON CORE MATHEMATICS CURRICULUM

7•1

Lesson 8

Representing Proportional Relationships with Equations

Date: 8/8/13

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80 gallons of gas would be needed to travel 750 miles.

Using arithmetic:

\[
\frac{750}{28} = 26.8
\]

Using algebra:

\[
m = 28g
\]

− Use substitution to replace the \( m \) (miles driven) with 750.

\[
750 = 28g
\]

− This equation demonstrates the same as dividing by the Constant of Proportionality or algebraically, use the multiplicative inverse (making one) to solve the equation.

\[
\left( \frac{1}{28} \right) 750 = \left( \frac{1}{28} \right) 28g
\]

\[
26.8 = 1g
\]

26.8 (rounded to the nearest tenth) gallons would be needed to drive 750 miles.

Have students write the pairs of numbers in the chart as ordered pairs. Explain that in this example \( x \) = gallons and \( y \) = miles driven. Remind students to think of the constant of proportionality as \( k = \frac{y}{x} \). The ratio is a certain number of miles divided by a certain number of gallons. This constant is the same as the unit rate of miles per gallon. Remind students that you will use the constant of proportionality (or unit rate) as a multiplier in your equation.

- Write equation(s) that will relate the miles driven to the number of gallons of gas.
- In order to write the equation to represent this situation, direct students to think of the independent and dependent variables that are implied in this problem.
- Which part depends on the other for its outcome?
  - The number of miles driven depends on the number of gallons of gas that are in the gas tank.
- Which is the dependent variable — gallons of gas or miles driven?
  - The number of miles is the dependent variable while the number of gallons is the independent variable.
- Tell students that \( x \) is usually known as the independent variable, and \( y \) is known as the dependent variable.
- Remind students the constant of proportionality can also be expressed as \( \frac{y}{x} \) from an ordered pair. It is the ratio of the dependent variable to the independent variable.
- Ask, when \( x \) and \( y \) are graphed on a coordinate grid, which axis would show the values of the dependent variable?
  - \( y \)-axis
- The independent variable?
  - \( x \)-axis
- Tell students that any variable may be used to represent the situation as long as it is known that in showing a proportional relationship in an equation that the constant of proportionality is multiplied by the independent variable. In this problem, students can write \( y = 28x \) or \( m = 28g \). We are substituting the 28 for \( k \) in the equation \( y = kx \) or \( m = kg \).
- Tell students that this equation models the situation and will provide us with a way to determine either variable when the other is known. If the equation is written so that the known information is substituted into the equation, then students can use algebra to solve the equation.
Example 2: Andrea’s Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits of tourists). People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours needed to draw the portraits.

a. Write several ordered pairs from the graph and explain what each coordinate pair means in the context of this graph.

(4, 6) means that in 4 hours she can draw 6 portraits
(6, 9) means that in 6 hours she can draw 9 portraits
(2, 3) means that in 2 hours she can draw 3 portraits
(1, 1 ½) means that in 1 hour she can draw 1 ½ portraits

b. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

\[ T = \frac{3}{2} N \]
\[ T = \frac{6}{4} N \]
\[ T = \frac{9}{6} N \]

\[
\begin{align*}
\frac{3}{2} &= \frac{6}{4} = \frac{9}{6} = \frac{1}{2} \quad \text{(equal)}
\end{align*}
\]

c. Determine the constant of proportionality and explain what it means in this situation.

The constant of proportionality is \( \frac{3}{2} \) which means that Andrea can draw 3 portraits in 2 hours or can complete 1 \( \frac{1}{2} \) portrait in 1 hour.

- Tell students these ordered pairs can be used to generate the constant of proportionality and write the equation for this situation. Remember that \( y = \frac{y}{x} \).

Closing (2 minutes)

- How can unit rate be used to write an equation relating two variables that are proportional?
  - The unit rate is the constant of proportionality, \( k \). After computing the value for \( k \), it may be substituted in place of \( k \) in the equation \( y = kx \). The constant of proportionality can be multiplied by the independent variable to find the dependent variable, and the dependent variable can be divided by the constant of proportionality to find the dependent variables.

Lesson Summary:

If a proportional relationship is described by the set of ordered pairs that satisfies the equation \( y = kx \), where \( k \) is a positive constant, then \( k \) is called the constant of proportionality. The constant of proportionality expresses the multiplicative relationship between each \( x \)-value and its corresponding \( y \)-value.

Exit Ticket (5 minutes)
Lesson 8: Representing Proportional Relationships with Equations

Exit Ticket

John and Amber work at an ice cream shop. The hours worked and wages earned are given for each person.

<table>
<thead>
<tr>
<th>John’s wages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>Wages ($)</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
</tbody>
</table>

a. Determine whether John’s wages are proportional to time. If they are, determine the unit rate. If not, explain why not.

b. Determine whether Amber’s wages are proportional to time. If they are, determine the unit rate. If not, explain why not.
c. Write an equation to model the relationship between each person’s wages. Identify constant of proportionality for each. Explain what it means in the context of the situation.

d. How much would each worker make after working 10 hours? Who will earn more money?

e. How long will it take each worker to earn $50?
Exit Ticket Sample Solutions
The following solutions indicate an understanding of the objectives of this lesson:

John and Amber work at an ice cream shop. The hours worked and wages earned are given for each person.

<table>
<thead>
<tr>
<th>John’s wages</th>
<th>Time (h)</th>
<th>Wages ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>36</td>
</tr>
</tbody>
</table>

a. Determine whether John’s wages are proportional to time. If they are, determine the unit rate. If not, explain why not.

Yes, the unit rate is 9. The collection of ratios is equivalent.

b. Determine whether Amber’s wages are proportional to time. If they are, determine the unit rate. If not, explain why not.

Yes, the unit rate is 8. The collection of ratios is equivalent.

c. Write an equation to model the relationship between each person’s wages. Identify constant of proportionality for each. Explain what it means in the context of the situation.

John: \( w = 9h; \) constant of proportionality is 9; John earns $9 for every hour he works.

Amber: \( w = 8h; \) constant of proportionality is 9; Amber earns $8 for every hour she works.

d. How much would each worker make after working 10 hours? Who will earn more money?

After 10 hours John will earn $90 because 10 hours is the value of the independent variable which should be multiplied by \( k \) the constant of proportionality. \( w = 9h; w = 9(10); w = 90. \) After 10 hours Amber will earn $80 because her equation is \( w = 8h; w = 8(10); w = 80. \) John will earn more money than Amber in the same amount of time.

e. How long will it take each worker to earn $50?

To determine how long it will take John to earn $50, the dependent value will be divided by 9, the constant of proportionality. Algebraically, this can be shown as a one-step equation: \( 50 = 9h; \) \( \frac{1}{9} 50 = \frac{1}{9} 9h; \)

\[
\frac{50}{9} = 1 \text{ h; } 5.56 = h \text{ (round to the nearest hundredth). It will take John nearly 6 hours to earn $50. To find out how long it will take Amber to earn $50 divide by 8, her constant of proportionality. } 50 = 8h;
\]

\[
\frac{1}{8} 50 = \frac{1}{8} 8h; \frac{50}{8} = 1h/6.25 = h. \text{ It will take Amber 6.25 hours to earn $50.}
\]
Write an equation that will model the proportional relationship given in each real world situation.

1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.
   a. Find the constant of proportionality for this situation.
      \[
      \frac{9 \text{ balls} (B)}{3 \text{ cans} (C)} = 3
      \]
   b. Write an equation to represent the relationship.
      \[
      B = 3C
      \]

2. In 25 minutes Li can run 10 laps around the track. Consider the number of laps she can run per minute.
   a. Find the constant of proportionality in this situation.
      \[
      \frac{10 \text{ laps} (L)}{25 \text{ minutes} (M)} = \frac{2}{5}
      \]
   b. Write an equation to represent the relationship.
      \[
      L = \frac{2}{5}M
      \]

3. Jennifer is shopping with her mother. They pay $2 per pound for tomatoes at the vegetable stand.
   a. Find the constant of proportionality in this situation.
      \[
      \frac{2 \text{ $ (D)}}{1 \text{ pound} (P)} = 2
      \]
   b. Write an equation to represent the relationship.
      \[
      D = 2P
      \]

4. It cost $5 to send 6 packages through a certain shipping company. Consider the number of packages per dollar.
   a. Find the constant of proportionality for this situation.
      \[
      \frac{6 \text{ pkg} (P)}{5 \text{ $ (D)}} = \frac{6}{5}
      \]
   b. Write an equation to represent the relationship.
      \[
      P = \frac{6}{5}D
      \]
5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded on to personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $58.00 for the month offered by another company. Which is the better buy?

<table>
<thead>
<tr>
<th>S = # of songs purchased</th>
<th>C = total cost</th>
<th>Constant of proportionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>36</td>
<td>( \frac{36}{40} = 0.90 )</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>( \frac{18}{20} = 0.90 )</td>
</tr>
<tr>
<td>12</td>
<td>10.80</td>
<td>( \frac{10.80}{12} = 0.90 )</td>
</tr>
<tr>
<td>5</td>
<td>4.50</td>
<td>( \frac{4.50}{5} = 0.90 )</td>
</tr>
</tbody>
</table>

a. Find the constant of proportionality for this situation.

\( k = 0.9 \)

b. Write an equation to represent the relationship.

\( C = 0.9S \)

c. Use your equation to find the answer to Susan’s question above. Justify your answer with mathematical evidence and a written explanation.

Compare the flat fee of $58 per month for songs to $.90 per song. If \( C = 0.9S \) and we substitute 60 for \( S \) (number of songs), then the result is \( C = 0.9(60) = 54 \). She would spend $54 on songs when she bought 60 songs. If she maintains the same number of songs, the $0.90 cost per song would be cheaper than the flat fee of $58 per month.

6. Allison’s middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T’s and More charges $8 per shirt. Which company should they use?

<table>
<thead>
<tr>
<th># Shirts (S)</th>
<th>Total Cost (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>25</td>
<td>375</td>
</tr>
<tr>
<td>50</td>
<td>375</td>
</tr>
<tr>
<td>75</td>
<td>375</td>
</tr>
<tr>
<td>100</td>
<td>375</td>
</tr>
</tbody>
</table>

Print-o-Rama

Not Proportional

Value T's and More

Not Proportional
a. Does either pricing model represent a proportional relationship between quantity of t-shirts and total cost? Explain.

The unit rate for Print-o-Rama is not constant. The graph for Value T’s and More is proportional since the ratios are equivalent (8) and the graph shows a straight line through the origin.

b. Write an equation relating cost and shirts for Value T’s and More.

\[ C = 8S \text{ for Value T's and More} \]

c. What is the constant of proportionality of Value T’s and More? What does it represent?

8; the cost of one shirt is $8.

d. How much is Print-o-Rama’s set up fee?

\[ \text{Guess and Check: } C = \text{price of a shirt (} \# \text{ of shirts)} + \text{set up fee} \]

\[
\begin{align*}
95 &= \underline{10} + \underline{15} \quad \text{or} \quad 375 = \underline{50} + \underline{15} \\
\text{Attempt #1} \quad 95 &= (8) \quad 10 + 15 \\
95 &= 95 \\
375 &= 800 + 15 \\
95 &= 95 \\
375 &= 375 \\
\text{Set up fee} &= \$25
\end{align*}
\]

e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.

Since we plan on a purchase of 90 shirts, we should choose Print-o-Rama.

Print-o-Rama: \[ C = 7S + 25; C = 7(90) + 25; C = 655 \]

Value T’s and More: \[ C = 8S; C = 8(90); C = 720 \]
Lesson 9: Representing Proportional Relationships with Equations

Student Outcomes

- Students use the constant of proportionality to represent proportional relationships by equations in real world contexts as they relate the equations to a corresponding ratio table and/or graphical representation.

Classwork (35 minutes)

Students will begin to write equations in two variables. They will analyze data that will help them understand the constant of proportionality and write the equation with two variables. The teacher may need to explicitly connect the graphical and tabular representations by modeling them side-by-side.

Example 1: Jackson’s Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to fill all of the requests for birdhouses. If Jackson can build 7 birdhouses in 5 hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.

\[ H = \frac{5}{7} B \]

Define the variables. \( B \) = # of birdhouses and \( H \) = number of hours (time constructing birdhouses)

- Ask the students: Does it matter which of these variables is independent or dependent?
  - No
- If it is important to determine the number of birdhouses that can be built in one hour, what is the constant of proportionality?
  - # of birdhouses/hours is \( \frac{7}{5} \) or 1.4.
- What does that mean in the context of this situation?
  - It means that Jackson can build 1.4 birdhouses in one hour or one entire house and part of a second birdhouse in one hour.
- If it is important to determine the number of hours it takes to build one birdhouse, what is the constant of proportionality?
  - # of hours / # of birdhouses is \( \frac{5}{7} \) or 0.71, which means that it takes him \( \frac{5}{7} \) of an hour to build one birdhouse or \( \frac{5}{7} \times 60 = 43 \) minutes to build one birdhouse.
This part of the problem asks you to write an equation that will let him determine how long it will take him to build any number of birdhouses, so we want to know \( H \). This forces \( H \) to be the dependent variable and \( B \) to be the independent variable. Our constant of proportionality will be dependent/independent which is \( H / B \) or \( y / x \) which is \( 7 / 5 \) so use the equation \( H = 5/7 B \).

Use the equation above to determine the following:

### b. How many birdhouses can Jackson build in 40 hours?

If \( H = 5/7 B \) and \( H = 40 \), then substitute 40 in the equation for \( H \) and solve for \( B \) since the question asks for the number of birdhouses.

\[
40 = (5/7) B \\
(7/5) 40 = (7/5)(5/7) B \\
56 = 1B
\]

Jackson can build 56 birdhouses in 40 hours.

### c. How long will it take Jackson to build 35 birdhouses? Use the equation from part a to solve the problem.

If \( H = 5/7 B \) and \( B = 35 \) then substitute 35 into the equation for \( B \); \( H = (5/7)(35) \); \( H = 25 \). It will take Jeff 25 hours to build 35 birdhouses.

### d. How long will it take to build 71 birdhouses? Use the equation from part a to solve the problem.

If \( H = 5/7 B \) and \( B = 71 \), then substitute 71 for \( B \) into the equation; \( H = (5/7)(71) \); \( H = 50.7 \) (rounded to the nearest tenth). It will take Jeff 50 hours and 42 minutes (60.7) to build 71 birdhouses.

Remind students that while you may work for a fractional part of an hour, a customer will not want to buy a partially built birdhouse. Tell students that some numbers can be left as non-integer answers (e.g., parts of an hour that can be written as minutes), but others must be rounded to whole numbers (e.g., the number of birdhouses completed or sold), all of this depends on the context. We must consider the real-life context, and we must consider the real-life situation before we determine if and how we round.

### Example 2: Al’s Produce Stand

Let students select any two pairs of numbers from either Al’s Produce stand or Barbara’s Produce stand to calculate the constant of proportionality \( (k = \text{dependent/independent}) \). In order to determine the unit price, students need to divide the cost (dependent variable) by the number of ears of corn (independent variable). Lead them through the following questions to organize their thinking.

- Which makes more sense: to use a unit rate of “ears of corn per dollar” or of “dollars/cents per ear of corn”?
  - Cost per ear of corn makes more sense because corn is sold as an entire ear of corn not part of an ear of corn.
- Based on the previous question, which would be the independent variable?
  - number of ears of corn
Lesson 9: Representing Proportional Relationships with Equations

- Which would be the dependent variable and why?
  - Cost, because the cost depends on the number of ears of corn purchased

- Have students volunteer to share the pair of numbers they used to determine their unit rate or constant of proportionality and compare the values for Al’s and for Barbara’s.
  - Al’s = 0.21 and Barbara’s = 0.22

- How do you write an equation for a proportional relationship?
  - \( y = kx \)

- Write the equation for Al’s Produce Stand:
  - \( y = 0.21x \)

- Write the equation for Barbara’s Produce Stand:
  - \( y = 0.22x \)

### Example 2: Al’s Produce Stand

Al’s Produce Stand sells 7 ears of corn for $1.50. Barbara’s Produce stand sells 13 ears of corn for $2.85. Write two equations, one for each produce stand that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

\[
\begin{align*}
\text{Al’s Produce Stand:} & \quad y = 0.21x \\
\text{Barbara’s Produce Stand:} & \quad y = 0.22x
\end{align*}
\]

<table>
<thead>
<tr>
<th>Ears</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>238</th>
<th>Ears</th>
<th>13</th>
<th>14</th>
<th>21</th>
<th>227</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.50</td>
<td>$2.94</td>
<td>$4.41</td>
<td>$50.00</td>
<td>Cost</td>
<td>$2.85</td>
<td>$3.08</td>
<td>$4.62</td>
<td>$50.00</td>
</tr>
</tbody>
</table>

- If you used \( E \) = number of ears of corn and \( C \) = cost for the variables instead of \( x \) and \( y \), how would you write the equations?
  - \( C = 0.21E \) and \( C = 0.22E \)

### Closing Questions (5 minutes)

- What type of relationship can be modeled using an equation in the form \( y = kx \), and what do you need to know to write an equation in this form?
  - A proportional relationship can be modeled using an equation in the form \( y = kx \). You need to know the constant of proportionality, which is represented by \( k \) in the equation.

- Give an example of a real-world relationship that can be modeled using this type of equation and explain why.
  - Distance equals rate multiplied by time. If the rate of a vehicle is going at an unchanging speed (constant), then the distance will depend on time elapsed.

- How do you determine which value is \( x \) (independent) and which value is \( y \) (dependent)?
  - The value that is determined by multiplying a constant to a value is the dependent variable.
Lesson 9: Representing Proportional Relationships with Equations

- Give an example of a real-world relationship that cannot be modeled using this type of equation and explain why.
  - The distance is the dependent variable, and the time the independent variable because time is being multiplied by the rate.

Lesson Summary:

How do you find the constant of proportionality? Divide to find the unit rate, \( y/x = k \).

How do you write an equation for a proportional relationship? \( y = kx \), substituting the value of the constant of proportionality in place of \( k \).

What is the structure of proportional relationship equations, and how do we use them? \( x \) and \( y \) values are always left as variables, and when one of them is known, they are substituted into \( y = kx \) to find the unknown using algebra.

Exit Ticket (5 minutes)

**Problem Set Directions:** Print each question on a sheet of paper. Post the questions around the room. Have students work in groups of four and progress from problem to problem. Give students a student record sheet as a place to record their solutions. Use a timer to help students pace the activity and stay on task from one problem to the next. Students within a group should discuss the problem and its solution and agree on the solution as they move from problem to problem.
Lesson 9: Representing Proportional Relationships with Equations

Exit Ticket

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in kilometers and distance in miles. One entry in the table paired 150 km with 93 miles. If \( k = \) number of kilometers and \( m = \) number of miles, who wrote the correct equation that would relate miles to kilometers? Explain why.

a. Oscar wrote the equation \( k = 1.61m \), and he said that the rate \( 1.61/1 \) represents miles per km.

b. Maria wrote the equation \( k = 0.62m \) as her equation, and she said that 0.62 represents miles per km.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in kilometers and distance in miles. One entry in the table paired 150 km with 93 miles. If \( k \) = number of kilometers and \( m \) = number of miles, who wrote the correct equation that would relate miles to kilometers and why?

- Oscar wrote the equation \( k = 1.61m \), and he said that the rate 1.61/1 represents miles per km.
- Maria wrote the equation \( k = 0.62m \) as her equation, and she said that 0.62 represents miles per km.

Maria is correct. Maria found the unit rate to be 0.62 by dividing miles by km. The rate that Michael used represents km per mile. However, it should be noted that the variables were not well-defined. Since we do not know which values are independent or dependent, each equation should include a definition of each variable. For example, Maria should have stated that \( k \) represents number of km and \( m \) represents number of miles.

Problem Set Sample Solutions

1. A person who weighs 100 pounds on Earth weighs 16.6 lb. on the moon.
   a. Which variable is the independent variable? Explain why.
      
      Weight on the earth is the independent variable because most people do not fly to the moon to weigh themselves first. The weight on the moon depends on a person’s weight on the earth.

   b. What is an equation that relates weight on Earth to weight on the moon?
      
      \[ M = (16.6/100)E \]

   c. How much would a 185 pound astronaut weigh on the moon?
      
      30.71 lb.

   d. How much would a man that weighed 50 pounds on the moon weigh back on Earth?
      
      301 lb.

2. Use this table to answer the following questions.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
</tr>
<tr>
<td>4</td>
<td>124</td>
</tr>
<tr>
<td>10</td>
<td>310</td>
</tr>
</tbody>
</table>

   a. Which variable is the dependent variable and why?
      
      The number of miles driven is the dependent variable because the number of miles you can drive depends on the number of gallons of gas you have in your tank.
b. Is miles driven proportionally related to gallons? If so, what is the equation that relates miles driven to gallons?

Yes, miles driven is proportionally related to gallons because every measure of gallons can be multiplied by 31 to get every corresponding measure of miles driven. \( M = 31G \)

c. In any ratio relating gallons and miles driven, will one of the values always be larger, if so, which one?

Yes, miles

d. If the number of gallons is known, can you find the miles driven? Explain how this value would be calculated?

Yes, multiply the constant of proportionality (31 mpg) by the number of gallons.

e. If the number of miles driven is known, can you find the number of gallons consumed?

Explain how this value would be calculated? Yes, divide the number of miles driven by constant of proportionality (31 mpg).

f. How many miles could be driven with 18 gallons of gas?

558 miles

g. How many gallons are used when the car has been driven 18 miles?

18/31 of a gallon

h. How many miles have been driven when \( \frac{3}{2} \) of a gallon is used?

\( \frac{31}{2} = 15.5 \) miles

i. How many gallons have been used when the car has been driven \( \frac{1}{2} \) mile?

1/62 of a gallon

3. Suppose that the cost of renting a snowmobile is $37.50 for 5 hours.

a. If the \( c = \) cost and \( h = \) hours, which variable is the dependent variable? Explain why.

\( C \) is the dependent variable because the cost of using the snowmobile depends on the number of hours you use it. \( c = 7.5h \)

b. What would be the cost of renting 2 snow mobiles for 5 hours each?

$75
4. In mom’s car, the number of miles driven is proportional to the number of gallons of gas used.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
<tr>
<td>6</td>
<td>168</td>
</tr>
<tr>
<td>8</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>280</td>
</tr>
</tbody>
</table>

a. Write the equation that will relate the number of miles driven to the gallons of gas.

\[ M = 28G \]

b. What is the constant of proportionality?

28

c. How many miles could you go if you filled your 22-gallon tank?

616 miles

d. If your family takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?

21 \frac{3}{7} \text{ gallons}

e. If you drive 224 miles during one week of commuting to school and work, how many gallons of gas would you use?

8 gallons
Lesson 10: Interpreting Graphs of Proportional Relationships

Student Outcomes

- Students consolidate their understanding of equations representing proportional relationships as they interpret what points on the graph of a proportional relationship mean in terms of the situation or context of the problem, including the point \((0, 0)\).
- Students are able to identify and interpret in context the point \((1, r)\) on the graph of a proportional relationship where \(r\) is the unit rate.

Classwork

Example 1–2 (15 minutes)

Example 1 is a review of previously taught concepts, but lesson will be built upon this example. Pose the challenge to the students to complete the table.

Have students work individually and then compare and critique each other’s work with a partner.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Grandma’s Special Chocolate-Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour to make 4 dozen cookies. Using this information, complete the chart:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table</strong></td>
<td>Create a chart comparing the amount of flour used to the amount of cookies.</td>
</tr>
<tr>
<td><strong>Flour (cups)</strong></td>
<td><strong>Cookies (Dozen)</strong></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td><strong>Table</strong></td>
<td>Is the number of cookies proportional to the amount of flour used? Explain.</td>
</tr>
<tr>
<td>Yes, because there exists a constant = (4/3) or (1 \frac{1}{3}) such that each measure of the cups of flour multiplied by the constant gives the corresponding measure of cookies</td>
<td></td>
</tr>
<tr>
<td><strong>Unit Rate</strong></td>
<td>What is the unit rate, and what is the meaning in the context of the problem?</td>
</tr>
<tr>
<td>1 (\frac{1}{3})</td>
<td>(1 \frac{1}{3}) dozen cookies or 16 cookies for 1 cup of flour</td>
</tr>
</tbody>
</table>
Lesson 10
Interpreting Graphs of Proportional Relationships

Graph – Model the relationship on a graph.

Does the graph show the two quantities being proportional to each other? Explain.

The points lie on a straight line that passes through the origin (0,0).

Equation – Write an equation that can be used to represent the relationship.

\[ D = 1 \frac{1}{3} F \]
Or
\[ D = 1. \frac{3}{1} F \]

\( D = \text{Number of Dozen Cookies} \)
\( F = \text{Number of cups of Flour} \)

Example 2

Below is a graph modeling the amount of sugar required to make Grandma’s Chocolate-Chip Cookies.

Record the coordinates of flour of the points from the graph in a table. What do these ordered pairs (values) represent?

(0, 0); 0 cups of sugar will give 0 dozen cookies
(2, 3); 2 cups of sugar yields 3 dozen cookies
(4, 6); 4 cups of sugar yields 6 dozen cookies
(8, 12); 8 cups of sugar yields 12 dozen cookies
(12, 18); 12 cups of sugar yields 18 dozen cookies
(16, 24); 16 cups of sugar yields 24 dozen cookies

Grandma has 1 remaining cup of sugar, how many dozen cookies will she be able to make? Plot the point on the graph above.

1.5 dozen cookies

How many dozen cookies can grandma make if she has no sugar? Can you graph this on the grid provided above? What do we call this point?

(0, 0); 0 cup of sugar = 0 dozen cookies, point is called the origin
Generate class discussion using the following questions to lead to the conclusion the point \((1, r)\) must be on the graph and what it means.

- How is the unit rate related to the graph?
  - The unit rate must be the value of the \(y\)-coordinate of the point on the graph, which has an \(x\)-coordinate of one.

- What quantity is measured along the horizontal axis?
  - sugar

- When you plot the ordered pair \((A, B)\), what does \(A\) represent?
  - The amount of sugar in cups

- What quantity is measured along the vertical axis?
  - The amount of cookies (dozens)

- When you plot the point \((A, B)\), what does \(B\) represent?
  - The total amount of cookies

- What is the unit rate for this proportional relationship?
  - 1.5

- Starting at the origin, if you move one unit along the horizontal axis, how far would you have to move vertically to reach the line you graphed?
  - 1.5 units

- Why are we always moving 1.5 units vertically?
  - The unit rate is 1.5 dozen cookies for every 1 cup of sugar. The vertical axis or \(y\) value represents the number of cookies. Since the unit rate is 1.5, every vertical move would equal the unit rate of 1.5 units.

- Continue moving one unit at a time along the horizontal axis. What distance vertically do you move?
  - 1.5 units

- Does this number look familiar? Is it the unit rate? Do you think this will always be the case, whenever two quantities that are proportional are graphed?
  - The vertical distance is the same as the unit rate. Yes, the vertical distance will always be the equal to the unit rate when moving one unit horizontally on the axis.

- Graphs of different proportional relationship have different points, but what point must be on every graph of a proportional relationship? Explain why.
  - The point \((1, r)\) or unit rate must be on every graph because the unit rate describes the change in the vertical distance for every 1 unit change in the horizontal axis.
Exercises (15 minutes)

1. The graph below shows the amount of time a person can shower with a certain amount of water.

   ![Graph](image)

   a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.
   
   Yes, the quantities are proportional to each other since all points lie on a straight line that passes through the origin (0, 0).

   b. How long can a person shower with 15 gallons of water and with 60 gallons of water?
   
   5 minutes, 20 minutes

   c. What are the coordinates of point A? Describe point A in the context of the problem.
   
   (30, 10) If there are 30 gallons of water, then a person can shower for 10 minutes.

   d. Can you use the graph to identify the unit rate?
   
   Since the graph is a line that passes through (0, 0) and (1, r), you can take a point on the graph, such as (15, 5) and get $\frac{1}{3}$.

   e. Plot the unit rate on the graph. Is the point on the line of this relationship?
   
   Yes, the unit rate is a point on the graph of the relationship.

   f. Write the equation to represent the relationship between the number of gallons used and the length of a shower.
   
   $m = \frac{1}{3}g$ where $m$ is minutes and $g$ is gallons

2. Your friend uses the equation $C = 50P$ to find the total cost of $P$ people entering the local Amusement Park.

   a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
</tbody>
</table>
b. Is the cost of admission proportional to the amount of people entering the Amusement Park? Explain why or why not.
   
   Yes, because there exists a constant = 50 such that each measure of the amount of people multiplied by the constant gives the corresponding measures of cost.

   c. What is the unit rate, and what does it represent in the context of the situation?
   
   50, 1 person costs $50.

   d. Sketch a graph to represent this relationship.

   ![Graph](graph.png)

   e. What point(s) MUST be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe this point in the context of the problem.

   (0, 0), (1, 50).
   
   If 0 people enter the park, then the cost would be $0.
   If 1 person enters the park, the cost would be $50.
   
   For every 1-unit increase along the horizontal axis, the change in the vertical distance is 50 units.

   f. Would the point (5, 250) be on the graph? What does this point represent in the context of the situation?

   Yes the point (5, 250) would be on the graph because 5(50) = 250. The meaning is that it would cost a total of $250 for 5 people to enter an Amusement Park.

---

Closing (5 minutes)

- What points are always on the graph of two quantities that are proportional to each other?
  - The points (0,0) and (1, r), where r is the unit rate, are always on the graph.

- How can you use the unit rate to create a table, equation, or graph of a relationship of two quantities that are proportional to each other?
  - In a table you can multiply each x value by the unit rate to obtain the corresponding y-value, or you can divide every y value by the unit rate to obtain the corresponding x-value. In an equation, you can use the equation y = kx and replace the k with the value of the unit rate. In a graph, the point (1, r) and (0,0) must be on the straight line of the proportional relationship.

- How can you identify the unit rate from a table, equation, or graph?
  - From a table, you can divide each y value by the corresponding x value. If the ratio y/x is equivalent for the entire table, then the ratio y/x is the unit rate, and the relationship is proportional. In an equation in the form y = kx, the unit rate is the number represented by the k. If a graph of a straight line that passes through the origin and contains the point (1, r), r representing the unit rate, then the relationship is proportional.
Lesson Summary:

The points \((0, 0)\) and \((1, r)\), where \(r\) is the unit rate, will always fall on the line representing two quantities that are proportional to each other.

The unit rate \(r\) in the point \((1, r)\) represents the amount of vertical increase for every horizontal increase of 1 unit on the graph.

The point \((0, 0)\) indicates that when there is zero amount of one quantity, there will also be zero amount of the second quantity.

These two points may not always be given as part of the set of data for a given real-world or mathematical situation, but they will always fall on the line that passes through the given data points.

Exit Ticket (5 minutes)
Lesson 10: Interpreting Graphs of Proportional Relationships

Exit Ticket

Great Rapids White Watering Company rents rafts for $125 per hour. Explain why the point (0,0) and (1,125) are on the graph of the relationship, and what these points mean in the context of the problem.
Exit Ticket Sample Solutions
The following solutions indicate an understanding of the objectives of this lesson:

Great Rapids White Watering Company rents rafts for $125 per hour. Explain why the point $(0,0)$ and $(1,125)$ are on the graph of the relationship, and what these points mean in the context of the problem.

Every graph of a proportional relationship must include the points $(0,0)$ and $(1,r)$. The point $(0,0)$ is on the graph because $0$ can be multiplied $y$ constant to get the corresponding value of $0$. The point $(1,125)$ is on the graph because it is the unit rate. On the graph for every 1 unit change on the horizontal axis, the vertical axis will change by 125 units. The point $(0,0)$ means 0 hours of renting a raft would cost $0$, and $(1,125)$ means 1 hour of renting the raft would cost $125$.

Problem Set Sample Solutions
The problem set requires students to have a full understanding of proportional relationships, their tables, equations and graphs. Within each problem, students are given the information in a different format, sometimes table, equation or graph and students have to connect unit rate and other points to the equation and graph.

1. The graph to the right shows the distance (ft.) run by a Jaguar.
   a. What does the point $(5, 280)$ represent in the context of the situation?
      In 5 seconds, a jaguar can run 280 feet.
   b. What does the point $(3,174)$ represent in the context of the situation?
      A jaguar can run 174 feet in 3 hours.
   c. Is the distance run by the Jaguar proportional to the time? Explain why or why not.
      Yes, because it is a straight line that passes through the origin $(0,0)$
   d. Write an equation to represent the distance ran by the Jaguar. Explain or model your reasoning.
      \[ y = 58x \]
      The constant of proportionality, or unit rate, is 58 and can be substituted into the equation \( y = kx \) in place of $k$.

2. Championship T-shirts sell for $22 each.
   a. What point(s) MUST be on the graph for the quantities to be proportional to each other?
      $(0,0), (1, 22)$
   b. What does the ordered pair $(5, 110)$ represent in the context of this problem?
      5 T-shirts would cost $110
   c. How many T-shirts were sold if you spent a total of $88?
      \[ 4; 88/22 = 4 \]
3. The following graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day, regardless of how many miles the car is driven. It does not matter how many miles you drive; you just pay an amount per day.
   a. What does the ordered pair (4, 250) represent?
      It would cost $250 to rent a car for 4 days.
   b. What would be the cost to rent the car for a week? Explain or model your reasoning.
      Since the unit rate is 62.5, the cost for a week would be
      \( 62.5 \times 7 = 437.50 \)

4. Jackie is making a snack mix for a party. She is using M&M’s and peanuts. The table below shows how many packages of M&M’s she needs to how many cans of peanuts she needs to make the mix.
   a. What points MUST be on the graph for the number of cans of peanuts to be proportional to the packages of M&M’s? Explain why.
      (0,0) \( (1,2) \), All graphs of proportional relationships are straight lines that passes through the origin (0,0) and the unit rate \((1,r)\).
   b. Write an equation to represent this relationship.
      \( y = 2x \)
   c. Describe the ordered pair \((12,24)\) in the context of the problem.
      In the mixture you will need 12 packages of M&M’s to 24 cans of peanuts

5. The following table shows the amount of candy and price paid.

<table>
<thead>
<tr>
<th>Amount of Candy (pounds)</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (Dollars)</td>
<td>5</td>
<td>7.5</td>
<td>12.5</td>
</tr>
</tbody>
</table>

   a. Is the cost of candy proportional to the amount of candy?
      Yes, because there exists a constant = 2.5 such that each measure of the amount of candy multiplied by the constant gives the corresponding measure of cost
   b. Write an equation to illustrate the relationship between the amount of candy and the cost.
      \( y = 2.5x \)
   c. Using the equation, predict how much it will cost for 12 pounds of candy?
      \( 2.5 \times 12 = 30 \)
   d. What is the maximum amount of candy you can buy with $60?
      \( 60 / 2.5 = 24 \) pounds
   e. Graph the relationship
1. Josiah and Tillery have new jobs at YumYum’s Ice Cream Parlor. Josiah is Tillery’s manager. In their first year, Josiah will be paid $14 per hour and Tillery will be paid $7 per hour. They have been told that after every year with the company, they will each be given a raise of $2 per hour. Is the relationship between Josiah’s pay and Tillery’s pay rate proportional? Explain your reasoning using a table.

2. A recent study claimed that in any given month, for every 5 text messages a boy sent or received, a girl sent or received 7 text messages. Is the relationship between number of text messages sent or received by boys proportional to the number of text messages sent or received by girls? Explain your reasoning using a graph on the coordinate plane.
3. When a song is sold by an online music store, the store takes some of the money and the singer gets the rest. The graph below shows how much money a pop singer makes given the total amount of money brought in by one popular online music store from sales of the song.

a. Identify the constant of proportionality between dollars earned by the pop singer and dollars brought in by sales of the song.

b. Write an equation relating dollars earned by the pop singer, \( y \), to dollars brought in by sales of the song, \( x \).
c. According to the proportional relationship, how much money did the song bring in from sales in the first week, if the pop star earned $800 that week?

d. Describe what the point (0, 0) on the graph represents in terms of the situation being described by the graph.

e. Which point on the graph represents the amount of money the pop singer gets for $1 in money brought in from sales of the song by the store?
## A Progression Toward Mastery

<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
<th>STEP 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 7.RP.2a</strong></td>
<td>Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</td>
<td>Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
</tr>
<tr>
<td>Student answered incorrectly. Student was unable to complete at least two correct pairs of values in the table. Student was unable to respond or reason out their answer.</td>
<td>Student may or may not have answered that the relationship was not proportional. Student was able to complete at least two correct pairs of values in the table. Student provided a limited expression of reasoning.</td>
<td>Student correctly answered that the relationship was not proportional. The table was correctly set up with at least two correct entries. Student’s reasoning may have contained a minor error.</td>
<td>Student correctly answered that the relationship was not proportional. Student provided correct set-up and values of table with two or more correct entries. Student reasoned AND demonstrated that there was no constant of proportionality or that the constant of proportionality changes for each pair of values.</td>
<td></td>
</tr>
<tr>
<td><strong>2 7.RP.2a</strong></td>
<td>Student answered incorrectly. Student was unable to give a complete graph AND/OR was unable to relate the proportional relationship to the graph.</td>
<td>Student may or may not have answered that the relationship was proportional. Student provided a graph with mistakes (unlabeled axis, incorrect points). Student provided a limited expression of reasoning.</td>
<td>Student correctly answered that the relationship was proportional. Student labeled the axis AND plotted points with minor error. Student explanation was slightly incomplete.</td>
<td>Student correctly answered that the relationship was proportional. Student correctly labeled the axis AND plotted the graph on the coordinate plane. Student explained that the proportional relationship was confirmed by the fact that the graph was a straight line going through the origin.</td>
</tr>
<tr>
<td></td>
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<tr>
<td>---</td>
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<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>7.RP.2b</td>
<td>Student was unable to answer ( k = \frac{1}{5} ) AND no work was shown.</td>
<td>Student was unable to answer ( k = \frac{1}{5} ). Concept of constant of proportionality was used incorrectly.</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>7.RP.2c</td>
<td>Student was unable to write an equation or wrote an equation that was not in the form ( y = kx ) or even ( x = ky ) for any value ( k ).</td>
<td>Student wrote an incorrect equation, such as ( y = 5x ), or ( x = \frac{1}{5}y ), AND/OR used an incorrect value of ( k ) from part (a) to write the equation in the form ( y = kx ).</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>7.RP.2d</td>
<td>Student answered incorrectly and shows no or little understanding of analyzing graphs.</td>
<td>Student answered incorrectly, but shows some understanding of analyzing graphs AND/OR solving equations.</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>7.RP.2d</td>
<td>Student was unable to describe the situation correctly.</td>
<td>Student was able to explain that the zero was the dollar amount to either the singer’s earnings or sales, but was unable to describe the relationship.</td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>7.RP.2d</td>
<td>Student was unable to identify either of the ( x )- or ( y )-coordinate of the point.</td>
<td>Student answers only one of the ordered pair values correctly.</td>
</tr>
</tbody>
</table>
1. Josiah and Tillery have new jobs at YumYum’s Ice Cream Parlor. Josiah is Tillery’s manager. In their first year, Josiah will be paid $14 per hour and Tillery will be paid $7 per hour. They have been told that after every year with the company, they will each be given a raise of $2 per hour. Is the relationship between Josiah’s pay and Tillery’s pay rate proportional? Explain your reasoning using a table. (7.RP.2a)

<table>
<thead>
<tr>
<th>Year</th>
<th>Josiah (J)</th>
<th>Tillery (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>15</td>
</tr>
</tbody>
</table>

No, the relationship between Josiah’s pay rate and Tillery’s pay rate is not proportional because the constant of proportionality changes for each pair of numbers.

2. A recent study claimed that in any given month, for every 5 text messages a boy sent or received, a girl sent or received 7 text messages. Is the relationship between number of text messages sent or received by boys proportional to the number of text messages sent or received by girls? Explain your reasoning using a graph on the coordinate plane. (7.RP.2a)

Yes, the number of text messages sent/received by boys is proportional to the number of text messages sent/received by girls because the pairs of values make a graph that is a straight line going through the origin.
3. When a song is sold by an online music store, the store takes some of the money and the singer gets the rest. The graph below shows how much money a pop singer makes given the total amount of money brought in by one popular online music store from sales of the song.

![Graph showing the relationship between sales and earnings](image)

a. Identify the constant of proportionality between dollars earned by the pop singer and dollars brought in by sales of the song.

\[
\frac{40}{200} = \frac{1}{5} = k
\]

b. Write an equation relating dollars earned by the pop singer, \( y \), to dollars brought in by sales of the song, \( x \).

\[
y = \frac{1}{5} x
\]
c. According to the proportional relationship, how much money did the song bring in from sales in the first week, if the pop star earned $800 that week?

\[
800 = \frac{1}{3} x \\
\frac{800}{\frac{1}{3}} = x \\
4000 = x
\]

The sales for that week were $4,000

---

d. Describe what the point (0, 0) on the graph represents in terms of the situation being described by the graph.

When the sales of the song brings in zero dollars, then the singer earns zero dollars.

---

e. Which point on the graph represents the amount of money the pop singer gets for $1 in money brought in from sales of the song by the store?

\[(1, \frac{1}{3})\]
Topic C:
Ratios and Rates Involving Fractions

Focus Standard:

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

7.EE.4a Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Days: 5

Lessons 11–12: Ratios of Fractions and Their Unit Rates (P)
Lesson 13: Finding Equivalent Ratios Given the Total Quantity (P)
Lesson 14: Multistep Ratio Problems (P)
Lesson 15: Equations and Graphs of Proportional Relationships Involving Fractions (P)
In the first two lessons of Topic C, students’ knowledge of unit rates for ratios and rates is extended by considering applications involving fractions, such as a speed of ½ mile per ¼ hour. Students continue to use the structure of ratio tables to reason through and validate their computations of rate. In Lesson 13, students continue to work with ratios involving fractions as they solve problems where a ratio of two parts is given along with a desired total quantity. Students can choose a representation that most suits the problem and their comfort levels, such as tape diagrams, ratio tables, or possibly equations and graphs, as they solve these problems, reinforcing their work with rational numbers. In Lesson 14, students solve multistep ratio problems, which include fractional markdowns, markups, commissions, and fees. In the final lesson of the topic, students focus their attention on using equations and graphs to represent proportional relationships involving fractions, reinforcing the process of interpreting the meaning of points on a graph in terms of the situation or context of the problem.
Lesson 11: Ratios of Fractions and Their Unit Rates

Student Outcomes

- Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities such as recipes, lengths, areas, and speed.
- Students work together and collaboratively to solve a problem while sharing their thinking process, strategies, and solutions with the class.

Classwork

Example 1 (25 minutes): Who is Faster?

During their last workout, Izzy ran 2 ¼ miles in 15 minutes, and her friend Julia ran 3 ¾ miles in 25 minutes. Each girl thought she were the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

Even if one of the approaches were not taken or a student took a different approach go through ALL possible ways as a class as shown below (bar models, Equations, Number Line, Clocks). Each approach reviews and teaches different concepts that are needed for the “big” picture. Starting with tables will not only reinforce all of the previous material but also will review and address concepts required for the other possible approaches.

Note: Time can be represented in either hours or minutes; the solutions show both.

Example 1: Who is Faster?

During their last workout, Izzy ran 2 ¼ miles in 15 minutes, and her friend Julia ran 3 ¾ miles in 25 minutes. Each girl thought she were the faster runner. Based on their last run, which girl is correct?

Izzy

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>¼</td>
<td>2 ¼</td>
</tr>
<tr>
<td>30</td>
<td>½</td>
<td>4 ½</td>
</tr>
<tr>
<td>45</td>
<td>¾</td>
<td>6 ¾</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>75</td>
<td>1 ¼</td>
<td>11 ¼</td>
</tr>
</tbody>
</table>

Julia

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5/12</td>
<td>3 ¾</td>
</tr>
<tr>
<td>50</td>
<td>5/6</td>
<td>7 ½</td>
</tr>
<tr>
<td>75</td>
<td>1/4</td>
<td>11 ¼</td>
</tr>
<tr>
<td>100</td>
<td>2/3</td>
<td>15</td>
</tr>
</tbody>
</table>

Scaffolding:

- It may be helpful to draw a clock or continually refer to a clock. Many students have difficulty telling time with the new technology available to them.
- Also, it may be helpful to do an example similar to the first example but uses whole numbers.
Lesson 11: Ratios of Fractions and Their Unit Rates

When looking and comparing the tables, it appears that Julia went further, so this would mean she ran faster. Is that assumption correct? Explain your reasoning.

- By creating a table of equivalent ratios for each runner showing the elapsed time and corresponding distance run, it may be possible to find a time or distance that is common to both tables. The student can see if one girl had a greater distance for a given time, or if one girl had a lesser time for a given distance. In this case, at \( t = 75 \text{ min} \) both girls would have run 11 \( \frac{3}{4} \) miles assuming constant speed.

How can we use the tables produced to determine the unit rate?

- Since we assumed distance is proportional to time, you can find the unit rate or constant of proportionality by dividing the distance by the time. If you used time in hours, then you will find the unit rate, in miles per hour, to be 9. If you used time in minutes, then the unit rate, in miles per minute, would be 3/20.

Discuss: Some students may have chosen to calculate the unit rates for each of the girls. To calculate the unit rate for Izzy, students divided the distance run by the elapsed time which gives us the ratio \( 2 \frac{3}{4} / (15/60) \), which is 9. To find the unit rate for Julia, students divided \( 3 \frac{3}{4} \) by \( (25/60) \) and arrived at a unit rate of 9 as well, leading the students to conclude that neither girl was the fastest.

We all agree that the girls ran at the same rate; however, some members of the class identified the unit rate as 9 while others gave a unit rate of 3/20. How can both groups of students be correct?

- Time can be represented in minutes; however, in the real-world, most people are comfortable with distance measured by hours. It is easier for a person to visualize 9 miles per hour compared to 3/20 miles per minute although it is an acceptable answer.

Scaffolding:
Review how to divide fractions using a bar model.

- How can you divide fractions with a picture, using a bar model?
  
  Make 2 whole blocks and a third whole block broken into fourths. Then, divide the wholes into fourths and count how many fourths there are in the original 2 \( \frac{3}{4} \) blocks. The answer would be 9.

1. Green Blocks are the original 2 \( \frac{3}{4} \) blocks (1st diagram).
2. Divide the whole blocks into \( \frac{1}{4} \) (2nd diagram).
3. How many blocks are there? (green blocks = 9).

More practice, if needed, with bar models:

- \( \frac{3}{4} \div \frac{1}{2} \)
  1. Make 1 \( \frac{3}{4} \) blocks, represented by green blocks.
  2. Divide the blocks into groups of \( \frac{1}{2} \).
  3. The number of \( \frac{1}{2} \) that are shaded are 3 \( \frac{1}{2} \).

- \( \frac{2}{3} \div \frac{1}{6} \)
  4. Make 2 \( \frac{1}{3} \) blocks, represented by green blocks.
  5. Divide the whole blocks into groups of \( \frac{1}{6} \).
  6. The number of 1/6 blocks that are shaded is 14.

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Equations:

Izzy

\[ d = rt \]

\[ 2 \frac{1}{4} = r \cdot \frac{1}{4} \]

\[ 4 \left( 2 \frac{1}{4} \right) = \left( r \cdot \frac{1}{4} \right) \left( 4 \right) \]

\[ 9 = r \]

9 miles/hour

Julia

\[ d = rt \]

\[ 3 \frac{3}{4} = r \cdot \frac{25}{60} \]

\[ 60 \left( 3 \frac{3}{4} \right) = \left( r \cdot \frac{25}{60} \right) \left( 60 \right) \]

\[ 9 = r \]

9 miles/hour

What assumptions are made when using the formula \( d = rt \) in this problem?

- We are assuming the distance is proportional to time, and that Izzy and Julia run at a constant rate. This means they run the same speed the entire time, not slower at one point and faster at another.

Picture:

- Some students may decide to draw a clock.
- Possible student explanation:

  - For Izzy, every 15 minutes will give 2 ¼ miles. Therefore, if you divide the clock into 15 minute intervals, then you can add the distance for each 15 minute interval until you get 60 minutes, which is your unit rate since 60 minutes = 1 hour. For Julia, you are given 3 ¾ miles for 25 minutes, so I divided the clock into 25 minute intervals. This will give 50 minutes. For the remaining 10 minutes, I broke it up into 5-minute intervals. To figure out the amount for each 5 minute interval, I found how much one 5-minute interval was when given 25 minutes = 3 ¾. I found one 5-minute interval to be ¾ mile.

Total Distance for 1 hour (unit rate)

Izzy: \[ 2 \frac{1}{4} + 2 \frac{1}{4} + 2 \frac{1}{4} + 2 \frac{1}{4} = 9 \]

Julia: \[ 3 \frac{3}{4} + 3 \frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 9 \]
Double Number Line Approach:

- How do you find the value of a 5-minute time increment? What are you really finding?
  - To find the value of a 5-minute increment, you need to divide $3\frac{3}{4}$ by 5 since 25 minutes is five 5-minute increments. This is finding the unit rate for a 5-minute increment.

- Why were 5-minute time increments chosen?
  - 5-minute time increments were chosen for a few reasons. First, a clock is broken into 5-minute intervals, so it may be easier to visualize what fractional part of an hour one has when given in a 5-minute interval. Also, 5 is the greatest common factor of the two given times.

- What if the times had been 24 and 32, or 18 and 22?
  - If it were 24 and 32 minutes, then the time increment would be 8-minute intervals. This is because 8 is the greatest common factor of 24 and 32.
  - If the times were 18 and 22, then the comparison should be broken into 2-minute intervals since the greatest common factor of 18 and 22 is 2.

Exercises (10 minutes)

Exercises

1. A turtle walks $\frac{3}{4}$ of a mile in 50 minutes. What is the unit rate expressed in miles per hour?
   a. To find the turtle’s unit rate, Meredith wrote and simplified the following complex fraction. Explain how the fraction $\frac{5}{6}$ was obtained.

   $$\frac{\frac{7}{8}}{\frac{5}{6}} \cdot \frac{24}{24} = \frac{7 \cdot 3}{5 \cdot 4} = \frac{21}{20}$$

   Since the unit rate is expressed in miles per hour, the 50 minutes needs to be converted to hours. Since 60 minutes is equal to 1 hour, 50 minutes can be written as $\frac{50}{60} = \frac{5}{6}$. 

   $$\frac{\frac{3}{4}}{\frac{24}{24}} = \frac{21}{20}$$
b. Did Meredith simplify the complex fraction correctly? Explain how you know.

Yes, she multiplied the fraction by 24/24, which does not change the value but simplifies to 21/20.

2. For Anthony's birthday his mother is making cupcakes for his 12 friends at his daycare. The recipe calls for 3 1/2 cups of flour. This recipe makes 2 1/2 dozen cookies. Anthony's mother has only 1 cup of flour. Is there enough flour for each of his friends to get a cupcake? Explain and show your work.

\[
\frac{\text{cups}}{\text{dozen}} = \frac{3 \frac{1}{2}}{2 \frac{1}{2}} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{5} = 1 \frac{2}{5} \text{ cups/dozen}
\]

\[
\text{OR}
\frac{\text{cups}}{\text{dozen}} = \frac{3 \frac{1}{2}}{2 \frac{1}{2}} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{5} = 1 \frac{2}{5} \text{ cups/dozen}
\]

No, since Anthony has 12 friends, he would need 1 dozen cupcakes. This means you need to find the unit rate. Finding the unit rate will tell us how much flour his mother needs for 1 dozen cupcakes. Upon finding the unit rate, Anthony's mother would need 1 2/3 cups of flour, so she does not have enough flour to make cupcakes for all his friends.

3. Sally is making a painting for which she is mixing red paint and blue paint. The table below shows the different mixtures being used.

<table>
<thead>
<tr>
<th>Red Paint (Quarts)</th>
<th>Blue Paint (Quarts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1/2</td>
<td>2 1/2</td>
</tr>
<tr>
<td>2 2/5</td>
<td>4</td>
</tr>
<tr>
<td>3 3/4</td>
<td>6 1/4</td>
</tr>
<tr>
<td>4</td>
<td>6 2/3</td>
</tr>
<tr>
<td>1.2</td>
<td>2</td>
</tr>
<tr>
<td>1.8</td>
<td>3</td>
</tr>
</tbody>
</table>

a. What are the unit rates for the values?

\[
\frac{5}{3} = 1 \frac{2}{3}
\]

b. Is the amount of blue paint proportional to the amount of red paint?

Yes, blue paint is proportional to red paint because there exists a constant, 2/3, such that when each amount of red paint is multiplied by the constant, the corresponding amount of blue paint is obtained.

c. Describe, in words, what the unit rate means in the context of this problem.

For every 1 2/3 quarts of blue paint, she must use 1 quart of red paint.
Lesson Summary:

A fraction whose numerator or denominator is itself a fraction is called a complex fraction.

Recall: A unit rate is a rate, which is expressed as $A/B$ units of the first quantity per 1 unit of the second quantity for two quantities $A$ and $B$.

For example: If a person walks $2\frac{1}{2}$ miles in $1\frac{3}{4}$ hours at a constant speed, then the unit rate is $\frac{\frac{5}{4}}{\frac{5}{2}} = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{2} \cdot \frac{2}{4} = \frac{2}{2}$. The person walks $2$ mph.

Closing (5 minutes)

Describe additional questions.

- Can you give an example of when you might have to use a complex fraction?
- How is the unit rate calculated? Can we calculate unit rates when both values in the ratio are fractions?
- How is finding the unit rate useful?

Exit Ticket (5 minutes)
Lesson 11: Ratios of Fractions and Their Unit Rate

Exit Ticket

Which is the better buy? Show your work and explain your reasoning.

3 ½ lb. of turkey for ten and one-half dollars  
2 ½ lb. of turkey for six and one-quarter dollars
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Which is the better buy? Show your work and explain your reasoning.

3 ⅓ lb. of turkey for ten and one-half dollars

\[ \frac{10}{2} \div \frac{3}{1} = \$3.15 \]

2 ⅔ lb. of turkey for six and one-quarter dollars

\[ \frac{6}{4} \div \frac{2}{1} = \$2.50 \]

2 ⅔ lb. is the better buy because the price per pound is cheaper.

Problem Set Sample Solutions

1. Simplify: \( 2 \frac{4}{7} \div 1 \frac{3}{6} \)

\[ 1 \frac{5}{7} \]

2. One lap around a dirt track is \( \frac{1}{3} \) mile. It takes Bryce \( \frac{1}{9} \) hour to ride one lap. What is Bryce’s unit rate around the track?

3

3. Mr. Gengel wants to make a shelf with boards that are \( 1 \frac{1}{3} \) feet long. If he has an 18 foot board, how many pieces can he cut from the big board?

13 ½ boards

4. The local bakery uses 1.75 cups of flour in each batch of cookies. The bakery used 5.25 cups of flour this morning.
   a. How many batches of cookies did the bakery make?

3 batches
   b. If there are 5 dozen cookies in each batch, how many cookies did the bakery make yesterday?

5(12) = 60 cookies per batch

60(3) = 180 cookies in 3 batches

5. Jason eats 10 ounces of candy in 5 days.
   a. How many pounds (16 ounces = 1 pound) will he eat per day?

\( \frac{1}{8} \) lb each day
   b. How long will it take Jason to eat 1 pound of candy?

8 days
Lesson 12: Ratios of Fractions and Their Unit Rates

Student Outcomes

- Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities, e.g., recipes, lengths, areas, and speed.
- Students use unit rates to solve problems and analyze unit rates in the context of the problem.

During this lesson you are remodeling a room at your house and need to figure out if you have enough money. You will work individually and with a partner to make a plan of what is needed to solve the problem. After your plan is complete then you will solve the problem determining if you have enough money or if you are short money.

Classwork

Example 1 (25 minutes) Time to Remodel

Students are given the task of determining the cost of tiling a rectangular room. The students are given the dimensions of the room, the area in square feet of one tile, and the cost of one tile.

You have decided to remodel your bathroom and put tile on the floor. The bathroom is in the shape of a rectangle and the floor measures 14 feet 8 inches long, 5 feet 6 inches wide. The tile you want to use costs $5 each, and each tile covers $\frac{4}{3}$ square feet. If you have $100 to spend, do you have enough money to complete the project?

- Make a Plan: Decide what the necessary steps are to finding the solution and complete the chart.
- If students are unfamiliar with completing a chart like this one, guide them in completing the first row.

Example 1: Time to Remodel

You have decided to remodel your bathroom and put tile on the floor. The bathroom is in the shape of a rectangle and the floor measures 14 feet 8 inches long, 5 feet 6 inches wide. The tile you want to use costs $5 each, and each tile covers $\frac{4}{3}$ square feet. If you have $100 to spend, do you have enough money to complete the project?

Make a Plan: Complete the chart to identify the necessary steps in the plan and find a solution.

<table>
<thead>
<tr>
<th>What I Know</th>
<th>What I Want to Find</th>
<th>How to Find it</th>
</tr>
</thead>
</table>
| Dimensions of the Floor | Area | 1 – Convert inches to feet as a fraction over 12  
                                    2 – Area = length x width |
| Square Foot of 1 Tile | Number of Tiles Needed | Area divided by the area of 1 tile |
| Cost of 1 Tile | Total Cost of all tiles | Multiply the total amount of tiles by the cost of one ($5). |
| Have $100 | Is this enough? | $100 – total money to spend  
                                    If the total cost is more than $100 then there is not enough money |

Scaffolding:

- Review that 12 inches = 1 foot and how to represent feet and inches as a mixed fraction.
- Review the concept of area and the formula for finding area of a rectangular shape.
- Review how to multiply mixed numbers.
- How can estimation be used to answer this problem?

MP.2
Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.

### Area

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Area (sq. feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft 6 in = 5 ½</td>
<td>( A = ) (5 ½)(14 2/3)</td>
</tr>
<tr>
<td>14 ft 8 in = 14 2/3</td>
<td>( A = ) (11/2)(44/3)</td>
</tr>
<tr>
<td></td>
<td>( A = ) 242/3 = 80 2/3 sq ft</td>
</tr>
</tbody>
</table>

**Number of tiles:**

\[ \frac{80\frac{2}{3}}{4\frac{2}{3}} = \frac{242}{14} = 17 \frac{2}{7} \]

Can’t buy part of a tile so you will need to purchase 18 tiles.

**Total Cost:** 18(5) = $90

**Enough Money?** Yes since the total is less than $100, there is enough money.

Generate discussion about completing the plan and finding the solution. If needed, pose the following questions:

- **Why was the mathematical concept of area and not perimeter or volume used?**
  - *Area was used because we were “filling” in the rectangular space of the floor. Area is 2-dimensional and we needed two dimensions, length and width of the room, to calculate the area of the floor. If we were just looking to put trim around the outside, then we would use perimeter. If we were looking to fill the room from floor to ceiling, then we would use volume.*

- **Why would 5.6 inches and 14.8 inches be incorrect representations for 5 feet 6 inches and 14 2/3 feet?**
  - *The relationship between feet and inches is 12 inches = 1 foot. To convert, you need to figure out what fractional part 6 inches is of a foot or 12 inches. If you just wrote 5.6, then you would be basing the inches out of 10 not 12. The same holds true for 14 feet 8 inches.*

- **How is the unit rate useful?**
  - *The unit rate for a tile is given as 4 2/3. We can find the total number of tiles needed by dividing the total square footage by the unit rate.*

- **Can I buy 17 2/7 tiles?**
  - *No, you have to buy whole tiles and cut what you may need.*
Lesson 12: Ratios of Fractions and Their Unit Rates

How would rounding to 17 tiles compared to 18 tiles affect the job?

- Even though the rules of rounding would say round down to 17 tiles, we would not in this problem. If we round down, then the entire floor would not be covered, and we would be a little short. If we round up to 18 tiles, the entire floor would be covered with a little extra.

Exercises (10 minutes)

1. Which car can travel further on 1 gallon of gas?
   - Blue Car: Travels \( \frac{18}{2} \) miles using 0.8 gallons of gas
   - Red Car: Travels \( \frac{17}{2} \) miles using 0.75 gallons of gas

   **Finding the Unit Rate:**
   - **Blue Car:**
     \[
     \frac{18}{2} \div \frac{5}{2} = \frac{9}{5} = 23
     \]
   - **Red Car:**
     \[
     \frac{17}{2} \div \frac{3}{4} = \frac{87}{3} = 23 \frac{1}{5}
     \]

   **Rate:**
   - \( 23 \) miles / 1 gallon
   - \( 23 \frac{1}{5} \) miles / 1 gallon

   The red car traveled 1/5 mile further on one gallon of gas.

Closing (5 minutes)

- How can unit rates with fractions be applied in the real world?

Exit Ticket (5 minutes)

Scaffolding:
Since the students are at a young age, they may not be familiar with cars, distance, miles per gallon relationships. Students may select the car with the lower unit rate because they may be confused with the better buy and lower unit prices. Further clarification may be needed to explain how a higher miles per gallon value is more favorable.
Lesson 12: Ratios of Fractions and Their Unit Rates

Exit Ticket

If \(\frac{3}{4}\) lb. of candy cost $20.50, how much would 1 lb. of candy cost?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

If \( \frac{3}{4} \) lb. of candy cost $20.50, how much would 1 lb. of candy cost?

\[
\frac{27\frac{1}{3}}{3} = 27.3\overline{3}.
\]

Students may find the unit rate by first converting 20.50 to 41/2 and dividing it by \( \frac{3}{4} \). Students may also divide 20.50 by 3 because it represents 3 parts of the total. Once finding the quotient, the student may then add this to the 20.50 to get 27.33 per pound.

Problem Set Sample Solutions

1. You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes \( 1 \frac{2}{5} \) hours to download and you downloaded for \( 5 \frac{1}{4} \) hours, how many movies did you download?

\( 3 \frac{3}{4} \)

2. The area of a blackboard is \( 1 \frac{1}{3} \) square yards. A poster’s area is \( \frac{8}{9} \) square yards. Find a unit rate and explain, in words, what the unit rate means in the context of this problem. Is there more than one unit rate that can be calculated? How do you know?

\( 1 \frac{1}{2} \) - The area of the blackboard is \( 1 \frac{1}{2} \) times the area of the poster.

Yes, there is another possible unit rate. Another possible answer: \( 2/3 \) the area of the poster is \( 2/3 \) the area of the blackboard.

3. A toy remote-control jeep is 12 \( \frac{1}{2} \) inches wide while an actual jeep is pictured to be 18 \( \frac{3}{4} \) feet wide. What is the value of the ratio of the width of the remote-control jeep to width of the actual jeep?

\( 2 \text{ in: } 3 \text{ ft; } 2 \text{ in: } 36 \text{ in; } 1 \text{ in: } 18 \text{ in} \)

4. \( \frac{1}{3} \) cup of flour is used to make 5 dinner rolls.

a. How many cups of flour are needed to make 3 dozen dinner rolls?

\( 2 \frac{2}{5} \text{ cups} \)

b. How many rolls can you make with \( \frac{5}{3} \) cups of flour?

85 rolls

c. How much flour is needed to make one dinner roll?

\( \frac{1}{15} \text{ cup} \)
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Student Outcomes

- Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions.

Classwork

Example 1 (12 minutes)

Have students work in partners to complete the chart below. NOTE: Teacher may allow students to utilize a calculator to assist in the multiplication step of converting mixed numbers to improper fractions.

Example 1

A group of 6 hikers are preparing for a one-week trip. All of the group's supplies will be carried by the hikers in backpacks. The leader decided that it would be fair for each hiker to carry a backpack that is the same fraction of his weight as all of the other hikers'. In this set-up, the heaviest hiker would carry the heaviest load. The table below shows the weight of each hiker and the weight of his/her backpack.

Complete the table. Find the missing amounts of weight by applying the same ratio as the first 2 rows.

<table>
<thead>
<tr>
<th>Hiker's Weight</th>
<th>Backpack Weight</th>
<th>Total Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>152 lb. 4 oz.</td>
<td>14 lb. 8 oz.</td>
<td>166 3/4</td>
</tr>
<tr>
<td>107 lb. 10 oz.</td>
<td>10 lb. 4 oz.</td>
<td>117 7/8</td>
</tr>
<tr>
<td>129 lb. 15 oz.</td>
<td>12 lb.</td>
<td>142 5/16</td>
</tr>
<tr>
<td>68 lb. 4 oz.</td>
<td>6 lb.</td>
<td>74 3/4</td>
</tr>
<tr>
<td>91 lb.</td>
<td>3 lb.</td>
<td>100 5/8</td>
</tr>
<tr>
<td>105</td>
<td>10 lb.</td>
<td>115</td>
</tr>
</tbody>
</table>
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

**Ratio**

\[
\frac{14}{2} = \frac{29 \times 4}{609 \times 4} = \frac{58}{609} = \frac{2}{21}
\]

**Equations**

\[
B = \text{Backpack weight} \\
H = \text{Hiker’s weight}
\]

\[
B = \frac{2}{21} H \\
B = \frac{2}{21} \left( \frac{129}{16} \right) \\
B = 12 \frac{3}{8}
\]

- What challenges did you encounter when calculating the missing values?
  - Remembering the conversions of ounces to pounds, and also dividing of fractions
- How is a third column representing the total quantity found, and how is it useful?
  - To find the third column, you need to add the total weight of both the hiker and the backpack. The third column giving the total allows one to compare the overall quantities. Also if the total and ratio are known, then you can find the all the quantities.
- When a table is given with the 3rd column, how do you fill in the missing pieces?
  - In order to find the third column, you need the first two columns or the ratio of the first two columns. If the third column is the total, then add the first two columns.
- When a table is given, and either of the first two columns is missing, how do you complete the table?
  - If one of the first two columns is missing, you need to look at the rest of the table to determine the constant rate or ratio. You can either set up proportions (if students recall this from 6th grade) or write an equation of the relationship then substitute in or write an equivalent ratio of the unknown to the constant of proportionality.
- Based on the given values and found values, is the backpack weight proportional to the hiker’s weight? How do you know?
  - The table shows the backpack weight proportional to the hiker’s weight because there exists a constant, \( \frac{2}{21} \) that when each measure of the hiker’s weight is multiplied by the constant gives the corresponding weight of the backpack.
- Would these two quantities always be proportional to each other?
  - Not necessarily, the relationship between backpack weight and hiker’s weight will not always be in the ratio \( \frac{2}{21} \), but these 6 hikers were.
- Describe how to use different approaches to finding the missing values of either quantity.
  - Writing equations, setting up proportions or writing equivalent ratios can be used.
- Describe the process of writing and using equations to find the missing values of a quantity.
  - First, find the constant of proportionality or unit rate.
  - Once that is found, then set up an equation in the form \( y = kx \), replacing \( k \) with the constant of proportionality.
  - Substitute the known value in for the variable and solve for the unknown.
When writing equations to find the missing value(s) of a quantity, are we restricted to using the variables \( x \) and \( y \)? Explain.

- No, any variable can be used. Often using a variable to represent the context of the problem makes it easier to know which variable to replace with the known value. For instance if the two quantities are hours and pay, one may use the variable \( p \) to represent pay instead of \( y \) and \( h \) to represent hours instead of \( x \).

Describe the process of using proportions to find the missing value of a quantity.

- Find the constant of proportionality or unit rate. Set up a proportion comparing the two quantities. One side of the proportion is the unit rate, and the other includes the given value and the unknown variable.

Describe the process of writing equivalent ratios to find the missing value(s) of a quantity. How is this method similar and different to writing proportions?

- Start with the unit rate or constant of proportionality. Determine what variable is known and determine what you must multiply by to obtain the known value. Multiply the remaining part of the unit rate by the same number to get the value of the unknown variable.
- Writing proportions is writing two equivalents ratios. The difference between the second and third methods is the process of solving. Students may recall to “cross multiply” from previous knowledge as in the second approach or multiplying by a common factor of the unknown.

What must be known in order to find the missing value(s) of a quantity regardless of what method is used?

- The ratio of the two quantities must be known.

If the ratio of the two quantities and the total amount are given, can you find the remaining parts of the table?

- Yes, once the ratio is determined or given, find an equivalent ratio to the given ratio that also represents the total amount.

Have students extend the table. Direct them to create a total amount and instruct them to find the two missing quantities.

**Example 2 (13 minutes)**

Example 2

When a business buys a fast food franchise, it is buying the recipes used at every restaurant with the same name. For example, all Pizzeria Specialty House Restaurants have different owners, but they must all use the same recipes for their pizza, sauce, bread, etc. You are now working at your local Pizzeria Specialty House restaurant and listed below are the amounts of meat used on one meat-lovers pizza.

\[
\begin{align*}
\frac{1}{4} & \text{ cup of sausage} \\
\frac{1}{3} & \text{ cup of pepperoni} \\
\frac{1}{6} & \text{ cup of bacon} \\
\frac{1}{8} & \text{ cup of ham}
\end{align*}
\]

Scaffolding:

May need to review solving a one-step equation requiring using the multiplicative inverse to solve.
Lesson 13

Finding Equivalent Ratios Given the Total Quantity

What is the total amount of toppings used on a meat-lovers pizza? __1__ cups

The meat must be mixed using this ratio to ensure that customers will receive the same great tasting meat-lovers pizza from every Pizzeria Specialty House Restaurant nationwide. The table below shows 3 different orders for meat-lovers pizza on Superbowl Sunday. Using the amounts and total for one pizza given above, fill in every row and column of the table so the mixture tastes the same.

<table>
<thead>
<tr>
<th></th>
<th>Order 1</th>
<th>Order 2</th>
<th>Order 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sausage (cups)</td>
<td>1</td>
<td>1½</td>
<td>2¼</td>
</tr>
<tr>
<td>Pepperoni (cups)</td>
<td>1 ⅓</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Bacon (cups)</td>
<td>2 ⅔</td>
<td>1</td>
<td>1½</td>
</tr>
<tr>
<td>Ham (cups)</td>
<td>1 ½</td>
<td>3 ⅔</td>
<td>1 ¼</td>
</tr>
<tr>
<td>Beef (cups)</td>
<td>1 ⅔</td>
<td>3 ⅔</td>
<td>1 ⅛</td>
</tr>
<tr>
<td>TOTAL (cups)</td>
<td>4</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

- What must you calculate or know to complete this table?
  - You need to know how many pizzas are being made for each order.
- How many pizzas were made for Order 1? Explain how you obtained and used your answer?
  - There were 4 pizzas ordered in this order. The amount of sausage increased from ¼ cup to 1 cup, which is 4 times as big. Knowing this, each ingredient can now be multiplied by 4, and the answer can be reduced to determine how much of each ingredient is needed for Order 1.

A bar model can be utilized as well:

The amount of sausage is represented by the green in the bar model. This represents ¼ of a cup.

If the amount of sausage becomes 1 cup, then the model should represent 1 whole (New green).

The amount of ¼’s in 1 whole is 4.
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

How many pizzas were made for Order 2? Explain how you obtained and used your answer?
- There were 6 pizzas ordered in this order. The amount of bacon increased from 1/6 to 1, which is 6 times as big. Each ingredient can then be multiplied by 6 and the answer reduced to determine how much is needed for the order.

Bar Model:

The amount of bacon, 1/6, is represented by the green portion in the model.

The amount of bacon became 1 cup, so the model should represent 1 whole (New Green.)

The number of 1/6’s in 1 whole is 6.

How many pizzas were made for Order 3? Explain how you obtained and used your answer?
- There were 9 pizzas ordered in this order. The amount of pepperoni increased from 1/3 to 3, which is 9 times as big. The other ingredients can then be multiplied by 9, and the answers reduced to determine how much of each ingredient is needed for the order.

Bar Model:

The amount of pepperoni 1/3 is represented by the green potion in the model.

The amount of pepperoni becomes 3 or 3 wholes, so we need to draw 3 whole models broken in thirds.

The amount of thirds in the total models is 9.

Is it possible to order 1 ½ or 2 ½ pizzas? If so, describe the steps to determine the amount of each ingredient necessary.
- Yes, pizzas can be sold by the halves. This may not be typical, but it is possible. Most pizza places can put the ingredients on only half of a pizza. To determine the amount of each ingredient necessary, multiply the ingredient’s original amount by the number of pizzas ordered.
Exercises (10 minutes)

Exercises

1. The table below shows 6 different-sized pans of the same recipe for macaroni and cheese. If the recipe relating the ratio of ingredients stays the same, how might it be altered to account for the different sized pans?

<table>
<thead>
<tr>
<th>Noodles (cups)</th>
<th>Cheese (cups)</th>
<th>Pan Size (number of cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{3}{4})</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>1</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>(\frac{1}{6})</td>
<td>(\frac{5}{6})</td>
</tr>
<tr>
<td>(\frac{5}{3})</td>
<td>(\frac{1}{3})</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{8})</td>
<td>(\frac{5}{8})</td>
</tr>
</tbody>
</table>

Method 1: Equations

Find the constant rate. To do this, use the row that gives both quantities, not the total. To find the unit rate: \(\frac{3}{3} = \frac{3}{4}\).

\[\frac{1}{3} = \frac{1}{4}\]

Write the equation of the relationship. \(c = \frac{1}{4} n\) where \(c = \text{cups of cheese, } n = \text{cups of noodles.}\)

\[
\begin{align*}
c & = \frac{1}{4} n \\
\frac{1}{4} & = \frac{1}{4} n \\
1 & = n
\end{align*}
\]

\[
\begin{align*}
c & = \frac{1}{6} n \\
\frac{1}{6} & = \frac{2}{3} n \\
1 & = n
\end{align*}
\]

\[
\begin{align*}
c & = \frac{1}{3} n \\
\frac{1}{3} & = \frac{1}{16} n \\
1 & = n
\end{align*}
\]

\[
\begin{align*}
c & = \frac{1}{3} n \\
\frac{1}{3} & = \frac{1}{3} n \\
1 & = n
\end{align*}
\]
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Method 2: Proportions

Find the constant rate as describe in Method 1.

Set up proportions.

\[ y = \text{cups of cheese and } x = \text{cups of noodles} \]

\[
\frac{1}{4} = \frac{1}{2} \\
\frac{x}{4} = \frac{2}{3} \\
x = 1 \\
2y = \frac{2}{3} \\
y = \frac{2}{4} \\
y = \frac{2}{3} \\
y = \frac{1}{4} \\
y = \frac{1}{6}
\]

Method 3: Writing Equivalent Ratios

Multiply both the numerator and denominator of the original fraction by a common fraction that will give the known value. For example, what is multiplied by 1 to get \( \frac{1}{4} \)? Multiply both the numerator and denominator by that fraction, likewise, what is multiplied by 4 to get \( \frac{2}{3} \)? Multiply both the numerator and denominator by that fraction.

Closing (3 minutes)

- How is the 3rd column representing the total quantity used?
- Describe how you can fill in the missing information in a table that includes the total quantity.

Lesson Summary:

To find missing quantities in a ratio table where a total is given, determine the unit rate from the ratio of two given quantities and use it to find the missing quantities in each equivalent ratio.

Exit Ticket (5 minutes)
Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Exit Ticket

The table below shows the combination of dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds. Using the information provided in the table, complete the remaining parts of the table.

<table>
<thead>
<tr>
<th>Dry Mix (pounds)</th>
<th>Water (pounds)</th>
<th>Total (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>14 1/6</td>
</tr>
<tr>
<td>4 1/2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

The table below shows the combination of dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds. Using the information given in the table, complete the remaining parts of the table.

<table>
<thead>
<tr>
<th>Dry Mix (pounds)</th>
<th>Water (pounds)</th>
<th>Total (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>8</td>
<td>68</td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>12 ⅓</td>
<td>2 ⅓</td>
<td>14 ⅓</td>
</tr>
<tr>
<td>4 ⅔</td>
<td>3 ⅕</td>
<td>5 ⅓</td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

1. Students in 6 classes, displayed below, ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table, which represents the number of slices of pizza students in each class ate.

<table>
<thead>
<tr>
<th>Slices of Cheese Pizza</th>
<th>Slices of Pepperoni Pizza</th>
<th>Total Pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>5 1/2</td>
<td>13 3/4</td>
<td>19 1/4</td>
</tr>
<tr>
<td>3 1/3</td>
<td>8 1/3</td>
<td>11 2/3</td>
</tr>
<tr>
<td>3 1/5</td>
<td>1 1/2</td>
<td>2 1/10</td>
</tr>
</tbody>
</table>
2. To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.
   a. Complete the table.

<table>
<thead>
<tr>
<th>Yellow (Y) (ml)</th>
<th>Blue (B) (ml)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ½</td>
<td>5 ½</td>
<td>8 ¼</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4 ½</td>
<td>6 ¾</td>
<td>11 ¾</td>
</tr>
<tr>
<td>6 ½</td>
<td>9 ¾</td>
<td>16 ¾</td>
</tr>
</tbody>
</table>

   b. Write an equation to represent the relationship between the amount of yellow paint and blue paint.

   \[ B = 1.5Y \]

3. a. Complete the following table.

<table>
<thead>
<tr>
<th>Distance Ran (miles)</th>
<th>Distance Biked (miles)</th>
<th>Total Amount of Exercise (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3 ½</td>
<td>7</td>
<td>10 ½</td>
</tr>
<tr>
<td>2 ¾</td>
<td>5 2/5</td>
<td>8 ½</td>
</tr>
<tr>
<td>2 1/8</td>
<td>4 ¾</td>
<td>3 3/8</td>
</tr>
<tr>
<td>1 2/3</td>
<td>3 1/3</td>
<td>5</td>
</tr>
</tbody>
</table>

   b. What is the relationship between distances biked and distances ran?

   Distance biked is twice the distance ran.

4. The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

<table>
<thead>
<tr>
<th>Milk (cups)</th>
<th>Flour (cups)</th>
<th>Total (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>9</td>
<td>16.5</td>
</tr>
<tr>
<td>8 ¼</td>
<td>10.5</td>
<td>19 ¼</td>
</tr>
<tr>
<td>12.5</td>
<td>15</td>
<td>27.5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>
Lesson 14: Multistep Ratio Problems

Student Outcomes
- Students will solve multi-step ratio problems including fractional markdowns, markups, commissions, fees, etc.

Lesson Notes
In this lesson, students will solve multi-step ratio problems including fractional markdowns, fractional commissions, fees, and discounts. Problems with similar context but applying percent concepts will be seen in Modules 2 and 4.

Classwork
Example 1 (30 minutes): Bargains

Begin this lesson by discussing advertising. Share with students that businesses will create an advertisement that will encourage consumers to come to their business in order to purchase their products. Many businesses subscribe to the idea that if a consumer thinks that he or she is saving money, then the consumer will be more motivated to purchase the product.

Have students verbalize how they would determine the sale prices with a discount rate of \( \frac{1}{4} \) off the original price of the shirt and \( \frac{1}{4} \) off the original price of the shoes.

Students should provide an idea that is similar to this: discount price = original price – rate times the original price.

Example 1: Bargains

A retail clothing store advertises the following sale: Shorts are \( \frac{1}{2} \) off the original price; pants are \( \frac{1}{3} \) off the original price, and shoes at \( \frac{1}{4} \) off the original price (called the discount rate).

a. If a pair of shoes costs $40 and is advertised at \( \frac{1}{4} \) off the original price, what is the sales price?

Method 1: Tape Diagram

\[
\begin{array}{cccc}
\$10 & \$10 & \$10 & \$10 \\
\end{array}
\]

\( \frac{1}{4} \) of the original price is \$30.

Method 2: Subtracting \( \frac{1}{4} \) of the price from the original price

\[
\begin{align*}
40 - \frac{1}{4}(40) &= 40 - 10 \\
&= 30
\end{align*}
\]

Method 3: Finding the fractional part of the price being paid by subtracting \( \frac{1}{4} \) of the price from 1 whole

\[
\begin{align*}
\frac{3}{4}(40) &= 30
\end{align*}
\]
Lesson 14
Multistep Ratio Problems

b. At Peter’s Pants Palace a pair of pants usually sells for $33.00. If Peter advertises that the store is having \( \frac{1}{3} \) off sale, what is the sale price of Peter’s pants? Use questioning to guide students to developing the methods below. The do not need to use all three methods but should have a working understanding of how and why they work in this problem.

**Method 1: Tape Diagram**

\[
\begin{array}{c|c|c|c}
$11 & $11 & $11 \\
\end{array}
\]

\[\begin{align*}
$33 \div 3 &= $11 \\
2($11) &= $22
\end{align*}\]

**Method 2: Use the given rate of discount, multiply by the price and then subtract from the original price.**

\[33 - \frac{1}{3}(33) = 33 - 11 = $22\]

\[\text{The consumer pays } \frac{2}{3} \text{ of the original price.}\]

**Method 3: Subtract the rate from 1 whole, then multiply that rate times the original price.**

\[\begin{align*}
1 - \frac{1}{3} &= \frac{2}{3} \\
\frac{2}{3}(33) &= $22.00
\end{align*}\]

**Example 2: Big Al’s Used Cars**

Have students generate an equation that would find the commission for the sales person.

**Example 2: Big Al’s Used Cars**

A used car sales person receives a commission of \( \frac{1}{12} \) of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for $21,999?

**Commission** = \[21999 \times \frac{1}{12} = $1833.25\]

**Example 3: Tax Time**

As part of a marketing ploy, some businesses mark up their prices before they advertise a sales event. Some companies use this practice as a way to entice customers into the store without sacrificing their profits.

A furniture store wants to host a sale event to improve their profit margin and to reduce their tax liability before their inventory is taxed at the end of the year.

How much profit will be business make on the sale of a couch that is marked-up by \( \frac{1}{3} \) and then sold at a \( \frac{1}{5} \) off discount if the original price is $2400?

**Mark up:**

\[2400 + 2400 \times \left(\frac{1}{3}\right) = 2400 \times \left(1 + \frac{1}{3}\right) = $3200\]

**Markdown:**

\[3200 - 3200 \times \left(\frac{1}{5}\right) = 3200 \times \left(1 - \frac{1}{5}\right) = $2560\]

**Profit** = sales price – original price = $2560 – $2400 = $160.00
Example 4: Born to Ride

A motorcycle dealer paid a certain price for a motorcycle and marked it up by \( \frac{1}{5} \) of the price he paid. Later, he sold it for $14,000 what is the original price?

Explain that a whole plus the fractional increase will give \( 1 + \frac{1}{5} = \frac{6}{5} \) of the original price.

Let \( x \) = the original price

\[
\begin{align*}
\frac{1}{5}x + \frac{1}{5}x + \frac{1}{5}x + \frac{1}{5}x + \frac{1}{5}x &= x \\
\frac{6}{5}x &= 14000 \\
\frac{5}{6}x &= (\frac{5}{6})14000 \\
x &= \frac{1}{5} \times 14000 \\
&= 11,666.67.
\end{align*}
\]

Closing (5 minutes)

- Name at least two methods used to find the solution to a fractional markdown problem.
  - Find the fractional part of the markdown and subtract it from the original price.
  - Use a tape diagram to determine how much value each part represents and then subtract the fractional part from the whole.

- Compare and contrast a commission and a discount price?
  - The commission and the discount price are both fractional parts of the whole. The difference between them is that commission is found by multiplying the commission rate times the sale, while the discount is the difference between 1 and the fractional discount multiplied by the original price.

Lesson Summary:

- Discount price = original price – rate \( \times \) original price OR (1 - rate) \( \times \) original price
- Commission = rate \( \times \) total sales amount
- Markup price = original price + rate \( \times \) original price OR (1 + rate) \( \times \) original price

Exit Ticket (5 minutes)
Lesson 14: Multistep Ratio Problems

Exit Ticket

1. A bicycle shop advertised all mountain bikes priced at a $\frac{1}{3}$ discount.
   a. What is the amount of the discount if the bicycle originally costs $327?
   b. What is the discount price of the bicycle?
   c. Explain how you found your solution to part b.

2. A hand-held digital music player was marked down by $\frac{1}{4}$ of the original price.
   a. If the sales price is $128.00, what is the original price?
   b. If the item was marked up by $\frac{1}{2}$ before it was placed on the sales floor, what was the price that the store paid for the digital player?
   c. What is the difference between the discount price and the price that the store paid for the digital player?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. A bicycle shop advertised all mountain bike’s priced at a $\frac{1}{3}$ discount.
   a. What is the amount of the discount if the bicycle originally costs $327?
      \[ \frac{1}{3}(327) = $109 \text{ discount} \]
   b. What is the discount price of the bicycle?
      \[ \frac{2}{3}(327) = $218 \text{ discount price – methods will vary} \]
   c. Explain how you found your solution to part b.
      *Answers will vary*

2. A hand-held digital music player was marked down by $\frac{1}{4}$ of the original price.
   a. If the sales price is $128.00, what is the original price?
      \[ x - \frac{1}{4}x = 128 \]
      \[ \frac{3}{4}x = 128 \]
      \[ x = 170.67 \]
   b. If the item was marked up by $\frac{1}{2}$ before it was placed on the sales floor, what was the price that the store paid for the digital player?
      \[ x + \frac{1}{2}x = 170.67 \]
      \[ \frac{3}{2}x = 170.67 \]
      \[ x = 113.78 \]
   c. What is the difference between the discount price and the price that the store paid for the digital player?
      \[ 128 - 113.78 = 14.22 \]
Problem Set Sample Solutions

1. What is \(\frac{1}{32}\) commission of sales totaling $24,000?

\[
\left(\frac{1}{32}\right) 24000 = 750
\]

2. DeMarkus says that a store overcharged him on the price of the video game he bought. He thought that the price was marked \(\frac{1}{4}\) of the original price, but it was really \(\frac{1}{4}\) of the original price. He misread the advertisement. If the original price of the game was $48, then what was the difference between the price that DeMarkus thought he should pay and the price that the store charged him?

\[
\text{\(\frac{1}{4}\) of $48 = $12 (the price DeMarkus thought he should pay); \(\frac{1}{4}\) off $48 = $36};
\]

\[
\text{Difference between prices $36 - $12 = $24}
\]

3. What is the cost of a $1200 washing machine that was on sale for a \(\frac{1}{5}\) discount?

\[
\left(1 - \frac{1}{5}\right) 1200 = $960 \text{ or } 1200 - \frac{1}{5}(1200) = $960
\]

4. If a store advertised a sale that gave customers a \(\frac{1}{4}\) discount, what is the fraction part of the original price that the customer will pay?

\[
1 - \frac{1}{4} = \frac{3}{4} \text{ of original price}
\]

5. Mark bought an electronic tablet on sale for \(\frac{3}{4}\) off its original price of $825.00. He also wanted to use a coupon for a \(\frac{1}{5}\) off the sales price. Before taxes, how much did Mark pay for the tablet?

\[
825 \left(\frac{3}{4}\right) = 618.75 \text{ then } 618.75 \left(\frac{4}{5}\right) = $495
\]

6. A car dealer paid a certain price for a car and marked it up by \(\frac{7}{5}\) of the price he paid. Later he sold it for $24,000 what is the original price?

\[
x + \frac{7}{5}x = 24000, \frac{12}{5}x = 24000, x = $10,000
\]

7. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was 60 beats per minute. If her heart rate decreased by \(\frac{2}{5}\), what was her heart rate immediately after she ran the mile?

\[
x - \frac{2}{5}x = 60, \frac{3}{5}x = 60, x = 100 \text{ beats per minute}
\]
Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

Student Outcomes

- Students use equations and graphs to represent proportional relationships arising from ratios and rates involving fractions. They interpret what points on the graph of the relationship mean in terms of the situation or context of the problem.

Classwork

Review with students the meaning of unit rate, the meaning of an ordered pair in the proportional relationship context, the meaning of (0, 0) and the meaning of (1, r) from Lesson 10. The goal here is to help students see the relationship between the unit rate and the changes in x and y.

Example 1 (10 minutes): Mother’s 10K Race

Use the table to determine the constant of proportionality and remind students how this was done in earlier lessons. Help students to understand what the constant of proportionality means in the context of this problem.

- Discuss and model with students how to graph fractional coordinates so that the ordered pairs are as accurate as possible.

Example 1: Mother’s 10K Race

Sam’s mother has entered a 10K race. Sam and his family want to show their support for their mother, but they need to figure out where they should go along the race course. They also need to determine how long it will take her to run the race so that they will know when to meet her at the finish line. Previously, his mother ran a 5K race with a time of \( \frac{1}{2} \) hours. Assume Sam’s mother will run the same rate as the previous race in order to complete the chart.

- Discuss with your elbow partner: Can you find Sam’s mother’s average rate for the entire race based on her previous race time?
  - \( 3 \frac{3}{3} \text{ km/hr.}, \text{ or } 10 \frac{10}{3} \text{ km/hr.} \)

Scaffolding:

- A 10K race has a length of 10 kilometers (approximately 6.2 miles).
- Help students find ordered pairs from graphs that fall on coordinates that are easy to see.
- Have students use the coordinates to determine the constant of proportionality (unit rate).
Create a chart that will show how far Sam’s mother has run after each half hour from the start of the race and graph it on the grid at the right.

<table>
<thead>
<tr>
<th>hours (H)</th>
<th>Km run (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>10 (1/2) = 5 ( \times \frac{2}{3} = 1 \frac{2}{3} )</td>
</tr>
<tr>
<td>1</td>
<td>10 (1) = 10 ( \times \frac{3}{3} = 3 \frac{1}{3} )</td>
</tr>
<tr>
<td>1 1/2</td>
<td>10 (3/2) = 5</td>
</tr>
<tr>
<td>2</td>
<td>10 (2) = 20 ( \times \frac{3}{3} = 6 \frac{2}{3} )</td>
</tr>
<tr>
<td>2 1/2</td>
<td>10 (5/2) = 25 ( \times \frac{3}{3} = 8 \frac{1}{3} )</td>
</tr>
<tr>
<td>3</td>
<td>10 (3) = 10</td>
</tr>
</tbody>
</table>

a. What are some specific things you notice about this graph?
   It forms a straight line through the origin; it relates hours to km run; the straight line through the origin means that the values are proportional.

b. What is the connection between the table and the graph?
   The time in hours is on the horizontal axis, and the km run is on the vertical axis; the coordinates of the points on the line are the same as the pairs of numbers in the table.

c. What does the point (2, 6 2/3) represent?
   After 2 hours she has run 6 2/3 km

Discuss the responses with the class and draw a conclusion.

- Have students write the equation that models the data in the chart. Record the student responses so that they can see all responses.

\[ D = 3 \frac{1}{2} H, \text{ where } D = \text{ distance and } H = \text{ hours.} \ (\text{or } D = \frac{10}{3} H) \]
Example 2 (10 minutes): Organic Cooking

Students should write the equation from the data given and complete the ordered pairs in the table. Pose the questions to the students as a whole group, one question at a time:

**Example: Organic Cooking**

After taking a cooking class, you decide to try out your new cooking skills by preparing a meal for your family. You have chosen a recipe that uses an organic mushroom mix as the main ingredient. Using the graph below, complete the table of values and answer the following questions.

<table>
<thead>
<tr>
<th>Weight in pounds</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>1 ( \frac{1}{2} )</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>2 ( \frac{3}{4} )</td>
<td>18</td>
</tr>
</tbody>
</table>

1. Is this relationship proportional? How do you know from examining the graph?
   
   Yes, the relationship is proportional because the graph is a straight line that passes through the origin.

2. What is the unit rate for cost per pound?
   
   \[ k = \frac{16}{2} = 8, \text{ Unit rate } = 8 \]

3. Write an equation to model this data.
   
   \[ C = 8w \]

4. What ordered pair represents the unit rate and what does it mean?
   
   \((1, 8)\) Unit rate is 8, which means that one pound of mushrooms cost $8.00.

5. What does the ordered pair \((2, 16)\) mean in the context of this problem?
   
   \((2, 16)\) means 2 pounds of mushrooms cost $16.00.

6. If you could spend $10.00 on mushrooms, how many pounds could you buy?
   
   \[ C = 8w; \quad \frac{10}{8} = \frac{1}{4} \cdot 8w; \quad 1 \frac{1}{2} = w; \quad \text{You can buy } 1.25 \text{ pounds with } $10.00. \]

7. What would be the cost of 30 pounds of mushrooms?
   
   \[ C = 8W; \quad C = 8(30); \quad C = $240 \]

- Have students share out how they would find the cost for 3 lb. 4 oz. of mushrooms.
  - $26, students convert 3 lb. 4 oz. to 3 \( \frac{3}{4} \) lb. then multiply the weight by 8.
Discuss the usefulness of equations as models that help determine very large or very small values that are difficult or impossible to see on a graph.

Students should complete these problems in cooperative groups and then be assigned one problem per group to present in a gallery walk. As groups of students walk around the room to view the work, have them write feedback on sticky notes about presentations, clarity of explanations, etc. Students should compare their answers and have a class discussion after the walk about any solutions in which groups disagreed or found incomplete.

Closing Questions (5 minutes)

After the gallery walk, refer back to the graphs and charts that students presented.

- Are all graphs straight lines through the origin?
- Did each group write the equations that models the situation in their problem?
- Did each group find the correct constant of proportionality (unit rate) for their problem and describe its meaning in the context of the problem using appropriate units?

Lesson Summary:

Proportional relationships can be represented through the use of graphs, tables, equations, diagrams, and verbal descriptions.

In a proportional relationship arising from ratios and rates involving fractions, the graph gives a visual display of all values of the proportional relationship, especially the quantities that fall between integer values.

Exit Ticket (5 minutes)
Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

Exit Ticket

Using the graph and its title:

1. Describe the relationship that the graph depicts.

2. Identify two points on the line and explain what they mean in the context of the problem.

3. What is the unit rate?

4. What point represents the unit rate?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. Describe the relationship that the graph depicts.
   
   The graph shows that in 3 days the water was at 4 inches high. The water has risen at a constant rate. Therefore the water has risen \( \frac{1}{3} \) inches per day.

2. Identify two points on the line and explain what they mean in the context of the problem.
   
   \((6, 8)\) means that by the 6th day, the water had risen 8 inches; \((9, 12)\) means that by the 9th day the water had risen 12 inches.

3. What is the unit rate?
   
   The unit rate in inches per day is \(\frac{4}{3}\).

4. What point represents the unit rate?
   
   The point that shows the unit rate is \((1, \frac{4}{3})\).

Problem Set Sample Solutions

1. Students are responsible for providing snacks and drinks for the Junior Beta Club Induction Reception. Susan and Myra were asked to provide the punch for the 100 students and family members who will attend the event. The chart below will help Susan and Myra determine the proportion of cranberry juice to sparkling water that will be needed to make the punch. Complete the chart, graph the data, and write the equation that models this proportional relationship.

<table>
<thead>
<tr>
<th>Sparkling water (cups)</th>
<th>Cranberry juice (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4/5</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>6 2/5</td>
</tr>
<tr>
<td>12</td>
<td>9 3/5</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>80</td>
</tr>
</tbody>
</table>

   \[ \frac{4}{5} S \text{, where } C = \text{Cups of Cranberry Juice and } S = \text{Cups of Sparkling water} \]

2. Jenny is a member of a summer swim team.
   a. How many calories does she burn in one minute?
      
      Jenny burns 100 calories every 15 minutes, so she burns \(6 \frac{2}{3}\) calories each minute.
   b. Use the graph below to determine the equation that models how many calories Jenny burns within a certain number of minutes.
      
      \[ C = 6 \frac{2}{3} t \text{, where } C = \text{calories and } t = \text{time in minutes} \]
c. How long will it take her to burn off a 480 calorie smoothie that she had for breakfast?

   It will take Jenny 72 minutes of swimming to burn off the smoothie she had for breakfast.

3. Students in a World Geography Class want to determine the distances between cities in Europe. The map has a European Publisher which gives all distances in kilometers. These students want to determine the number of miles between towns so that they can compare distances with a unit of measure that they are already familiar with. The graph below shows the relationship between a given number of kilometers and the corresponding number of miles.

   a. Find the constant of proportionality or the rate of miles per kilometer for this problem and write the equation that models this relationship

   The constant of proportionality is \( \frac{1}{3.5} \text{ km/mi} \).

   The equation that models this situation is \( M = \frac{1}{3.5} K \), where \( M \) = miles and \( K \) = kilometers.

   b. What is the distance in kilometers between towns that are 5 miles apart?

   The distance between towns that are 5 miles apart is 3 \( \frac{1}{8} \) km.

   c. Describe the steps you would take to determine the distance in miles between two towns that are 200 kilometers apart?

   Solve the equation \( m = \frac{1}{3.5} (200) \) to find the number of miles for 200 km the students should multiply 200 by \( \frac{1}{3.5} \cdot 200 \left( \frac{1}{3.5} \right) = 320 \) miles.

4. During summer vacation, Lydie spent time with her grandmother picking blackberries. They decided to make blackberry jam for their family. Her grandmother said that to make jam, you must cook the berries until they become juice and then combine the juice with the other ingredients to make the jam.

   a. Use the table below to determine the constant of proportionality of cups of juice to cups of blackberries.

   One cup of juice is produced by combining 3 cups of blackberries.

   b. Write an equation that will model the relationship between the number of cups of blackberries and the number of cups of juice.

   \( j = \frac{1}{3} b \) where \( j \) = cups of juice and \( b \) = cups of blackberries

   c. How many cups of juice were made from 12 cups of berries? How many cups of berries are needed to make 8 cups of juice?

   4 cups of juice are made from 12 cups of berries.
   24 cups of berries are needed to make 8 cups of juice.
Topic D:

Ratios of Scale Drawings

**7.RP.2b, 7.G.1**

**Focus Standard:**

- **7.RP.2** Recognize and represent proportional relationships between quantities.
  - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

**Instructional Days:**

- **7**
  - **Lesson 16:** Relating Scale Drawings to Ratios and Rates (E)
  - **Lesson 17:** The Unit Rate as the Scale Factor (P)
  - **Lesson 18:** Computing Actual Lengths from a Scale Drawing (P)
  - **Lesson 19:** Computing Actual Areas from a Scale Drawing (P)
  - **Lesson 20:** An Exercise in Creating a Scale Drawing (E)
  - **Lessons 21–22:** An Exercise in Changing Scales (E)

---

In the first lesson of Topic D, students are introduced to scale drawings; they determine if the drawing is a reduction or enlargement of a two-dimensional picture. Pairs of figures are presented for students to match corresponding points. In Lesson 17, students learn the term *scale factor* and recognize it as the constant of proportionality. With a given scale factor, students make scale drawings of pictures or diagrams. In Lessons 18 and 19, students compute the actual dimensions of objects shown in pictures given the scale factor. They recognize that areas scale by the square of the scale factor that relates lengths. In the final lessons, students engage in their own scale factor projects—first, to produce a scale drawing of the top-view of a furnished room or building, and second, given one scale drawing, to produce new scale drawing using a different scale factor.

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¹ Lesson Structure Key: **P**-Problem Set Less, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson
Lesson 16: Relating Scale Drawings to Ratios and Rates

Student Outcomes

- Students understand that a scale drawing is either the reduction or the enlargement of a two-dimensional picture.
- Students compare the scale drawing picture with the original picture and determine if the scale drawing is a reduction or an enlargement.
- Students match points and figures in one picture with points and figures in the other picture.

Classwork

Intro Activity (3 minutes): Can You Guess the Image?

Use the attached Intro Activity pages to show students a series of images to see if they can guess what is pictured and then identify whether it is a reduction or enlargement of the original image. The purpose of this activity is for students to get an understanding of the terms reduction and enlargement. The scale drawings produced in grade 7 will focus on creating a scale drawing from a two-dimensional picture. Teachers could also post alternate images of choice on a projector or interactive whiteboard where only one portion is revealed. Students need to guess the object and whether it is a reduction or enlargement of the actual object.

Give students 2 minutes to try to guess each image in the student pages and share responses (see responses to the right). Then show the full size images and have them decide whether the images in the student pages are reductions or enlargements compared to what is now being shown.

Responses for attached images and points for discussion follow.

- It is picture of a subway map. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - It is a reduction since it was a scaled down picture of a map of a subway. If you compare the length from one end of a track to the other end, you can see that the cropped photo has a shorter length as compared to the original photo. Any corresponding length could be compared.

- It is a fingerprint. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - It is an enlargement since it was from a picture of a fingerprint. If you compare the length of one of the swirls to the actual fingerprint picture, you can see that the cropped photo has a longer length compared to the original fingerprint picture.
Intro Activity: Can You Guess the Image?

1. This is a reduction of a subway map.

2. This is an enlarged picture of a fingerprint.

Example 1 (3 minutes)

For each scale drawing, have students identify if it is a reduction or an enlargement to the actual object in real life or to the given original picture. Discuss:

- What are possible uses for enlarged drawings/pictures?
  - Enlarged drawings are good to observe details such as textures and parts that are hard to see to the naked eye. In art, enlargements are used in murals or portraits.

- What are the possible purposes of reduced drawings/pictures?
  - Reductions are purposeful to get a general idea of a picture/object. These scale drawings can fit in folders, books, wallets, etc.

Read over the “Key Idea” with the class and introduce the term “scale drawing”. Emphasize the importance of scale drawings being reductions or enlargements of two-dimensional drawings, not of actual objects.

Example 1

For the following problems, (a) is the actual picture and (b) is the scale drawing. Is the scale drawing an enlargement or a reduction of the actual picture?

1. a.  
   b.  
   Enlargement

2. a.  
   b.  
   Reduction
Example 2 (7 minutes)

Complete this activity together as a class. Discuss:

- Why doesn’t point V correspond with point R?
  - Although both points are on the bottom right hand corner, they are positioned on two different ends of the path. Point V only corresponds to Point W.

- What must we consider before identifying correspond points?
  - We have to make sure we are looking at the maps in the same direction. Then we can see that this is a one-to-one correspondence because they are scale drawings of each other and each point corresponds to one specific point on the other map.

Example 2

Derek’s family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.

What are the corresponding points of the scale drawings of the maps?

Point A to: [Point R]  Point V to: [Point W]  Point H to: [Point P]  Point Y to: [Point N]

Exercise 1 (10 minutes)

In this Exercise, the size of the units on the grid are enlarged then reduced to produce two different scale drawings with lengths that are proportional to one another. Guide students to notice that the number of units of length is staying the same, but the size of each unit changes from one drawing to another due to the shrinking and enlarging of the grid. This allows for students to create a scale drawing without having to complete the computation required in finding equivalent ratios (which will be done later in Topic D).
Lesson 16: Relating Scale Drawings to Ratios and Rates

Date: 8/8/13

- How will we make the enlarged robot? Will we need to adjust the number of units?
  - No, since the grid is enlarged (thus changing the size of each unit), we can draw the new robot using the same number of units for each given length.

- What is the importance of matching corresponding points and figures from the actual picture to the scale drawing (the mural piece)?
  - The scale drawing will not be proportional and the picture will be distorted.

- How can you check the accuracy of the proportions?
  - You can count the squares and check that the points match.

Exercise 1
Create scale drawings of your own modern nesting robots using the grids provided.

Example 3 (7 minutes)
Work on the problem as a class and fill in the table together. Discuss as students record important points in the “Notes” section:

- Is the second image a reduction or enlargement of the first image? How do you know?
  - It is a reduction because the second image is smaller than the first, original image.

- What do you notice about the information on the table?
  - The pairs of corresponding lengths are all proportional.

- Does a constant of proportionality exist? How do you know?
  - Yes, it does because there is a constant value to get from each length to its corresponding length.

- What is the constant of proportionality, and why is it important in scale drawings?
  - The constant of proportionality is \( \frac{1}{2} \), and it needs to exist for images to be considered scale drawings. If not, then there would be a lack of proportionality, and the images would not be a scaled up or down version of the original image.
Lesson 16: Relating Scale Drawings to Ratios and Rates

Notes:

Exercise 2 (7 minutes)

Have students work in pairs to fill out the table and answer questions. Direct students to the vertical and horizontal lengths of the legs. Reconvene as a class to discuss answers to the given questions and the following:

- Why is it difficult to determine if the second image is a reduction or an enlargement of the first image?
  - The second image is not a scale drawing of the first image, so even though it is bigger, it is not a true reduction or enlargement of the first image.

- What must one check before determining if one image is a scale drawing of another?
  - The corresponding lengths must all be proportional to each other. If only one pair is proportional and another is not, then the images cannot be identified as scale drawings of one another.

Exercise 2

Luca drew and cut out small right triangle for a mosaic piece he was creating for art class. His mother really took a liking and asked if he could create a larger one for their living room and Luca made a second template for his triangle pieces.

<table>
<thead>
<tr>
<th>Original Drawing</th>
<th>Height</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 units</td>
<td>6 units</td>
<td></td>
</tr>
<tr>
<td>Second Drawing</td>
<td>9 units</td>
<td>3 units</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lengths of the original image</th>
<th>5 units</th>
<th>3 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths of the second image</td>
<td>15 units</td>
<td>10 units</td>
</tr>
</tbody>
</table>

a. Does a constant of proportionality exist? If so, what is it? If not, explain.

*No, because the ratios of corresponding side lengths are not equal, or proportional to each other*

b. Is Luca’s enlarged mosaic a scale drawing of the first image? Explain why or why not.

*No, the enlarged mosaic image is not a scale drawing of the first image. We know this because the images do not have all side lengths proportional to each other; there is no constant of proportionality.*
Closing (3 minutes)

- What is a scale drawing?
  - It is a drawing that is a reduction or enlargement of an actual picture.
- What is an enlargement? What is a reduction?
  - A drawing that is larger in scale than its original picture. A drawing that is smaller in scale than its original picture.
- What is the importance of matching points and figures from one picture/drawing to the next?
  - The corresponding lines, points and figures need to match because otherwise the scale drawing will be distorted and not proportional throughout.
- How do scale drawings related to rates and ratios?
  - The corresponding lengths between scale drawings and original images are equivalent ratios.

Lesson Summary:

Scale Drawing: A drawing in which all lengths between points or figures in the drawing are reduced or enlarged proportional to the lengths in the actual picture. A constant of proportionality exists between corresponding lengths of the two images.

Reduction: The lengths in the scale drawing are smaller than those in the actual object or picture.

Enlargement/Magnification: The lengths in the scale drawing are larger than those in the actual object or picture.

One-to-one Correspondence: Each point in one figure corresponds to one and only one point in the second figure.

Exit Ticket (5 minutes)
Lesson 16: Relating Scale Drawings to Ratios and Rates

Exit Ticket

Use the following figure on the graph for problems 1 and 2.

1. 
   a. If the original lengths are multiplied by 2, what are the new coordinates?

   b. Use the table to organize lengths.

<table>
<thead>
<tr>
<th>Actual Picture Lengths (in units)</th>
<th>New Picture Lengths (in units)</th>
</tr>
</thead>
</table>

   c. Is the new picture a reduction or an enlargement?

   d. What is the constant of proportionality?
2.
   a. If the original lengths are multiplied by $\frac{1}{3}$, what are the new coordinates?

   b. Use the table to organize the lengths.

<table>
<thead>
<tr>
<th>Actual Picture Lengths (in units)</th>
<th>New Picture Lengths (in units)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. Is the new picture a reduction or an enlargement?

   d. What is the constant of proportionality?
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Use the following figure on the graph for problems 1 and 2.

1. 
   a. If the original lengths are multiplied by 2, what are the new coordinates?
      
      \((0,0), (12,18), (12,0)\)

   b. Use the table to organize lengths (the vertical and horizontal legs).

      | Actual Picture Lengths (in units) | 6 units | 9 units |
      | New Picture Lengths (in units)   | 12 units | 18 units |

   c. Is the new drawing a reduction or an enlargement?
      
      Enlargement

   d. What is the constant of proportionality?
      
      2

2. 
   a. If the original lengths are multiplied by \(\frac{1}{3}\) what are the new coordinates?

      \((0,0), (2,3), (2,0)\)

   b. Use the table to organize lengths (the vertical and horizontal legs).

      | Actual Picture Lengths (in units) | 6 units | 9 units |
      | New Picture Lengths (in units)   | 2 units | 3 units |
c. Is the new drawing a reduction or an enlargement?
   
   Reduction

d. What is the constant of proportionality?

   \[
   \frac{1}{3}
   \]

Problem Set Sample Solutions

For Problems 1–3, identify if it the scale drawing is a reduction or enlargement of the actual picture.

1. **Enlargement**
   
   a. Actual Picture
   
   b. Scale Drawing

2. **Reduction**
   
   a. Actual Picture

   b. Scale Drawing
3. **Enlargement**
   a. Actual Picture
   b. Scale Drawing

4. Using the grid and the abstract picture of a face, answer the following questions:

   a. On the grid, where is the eye?
      *Intersection BG*

   b. What is located in DH?
      *Tip of the nose*

   c. In what part of the square BI is the chin located?
      *Bottom right corner*
5. Use the graph provided to decide if the rectangular cakes are scale drawings of each other.

Cake 1: (5,3), (5,5), (11,3), (11,5)
Cake 2: (1,6), (1,12), (13,12), (13,6)

How do you know?

These images are not scale drawings of each other because the short length of cake 2 is three times longer than cake 1, but the longer length of cake 2 is only twice as long as cake 1. Both should either be twice as long or three times as long to have one-to-one correspondence and to be scale drawings of each other.
Lesson 16: Intro Activity

Can you guess the image? In each problem, the first image is from the student materials and the second image is the actual picture.

1. 

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NYS COMMON CORE MATHEMATICS CURRICULUM
Lesson 16 Intro Activity

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2.
Lesson 17: The Unit Rate as the Scale Factor

Student Outcomes
- Students recognize that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distance in the original picture.
- Students recognize the scale factor to be the constant of proportionality.
- Given a picture or description of geometric figures, students make a scale drawing with a given scale factor.

Classwork

Example 1 (7 minutes): Rubin’s Icon

After reading the prompt with the class, discuss the following questions:
- What type of scale drawing is the sticker?
  - It is an enlargement or a magnification of the original sketch.
- What is the importance of proportionality for Rubin?
  - If the image is not proportional, it looks less professional. The image on the sticker will be distorted.
- How could we go about checking for proportionality of these two images? (Have students record steps onto student pages.)
  - Measure corresponding lengths and check to see if they all have the same constant of proportionality.

As a class, label points correspondingly on the original sketch and then on the sticker sketch. Use inches to measure the distance between the points and record on a table.

Example 1: Rubin’s Icon

Rubin created a simple game on his computer and shared it with his friends to play. They were instantly hooked and the popularity of his game spread so quickly that Rubin wanted to create a distinctive icon, so players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Rubin wasn’t quite sure if the stickers were proportional to his original sketch.

<table>
<thead>
<tr>
<th>Original</th>
<th>Sticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in.</td>
<td>2 in.</td>
</tr>
<tr>
<td>3 in.</td>
<td>1 1/2 in.</td>
</tr>
<tr>
<td>1 in.</td>
<td>2 in.</td>
</tr>
<tr>
<td>7 in.</td>
<td>3 3/4 in.</td>
</tr>
</tbody>
</table>
Discuss:

- What relationship do you see between the measurements?
  - The corresponding lengths are proportional.
- Is the sticker proportional to the original sketch?
  - Yes, the sticker lengths are twice as long as the original sketch.
- How do you know?
  - The unit rate is the same for the corresponding measurements, 2.
- What is this called?
  - Constant of proportionality
- Introduce the term “scale factor” and review the key idea box with students.
  - Is the new figure bigger or smaller than the original?
  - Bigger.
- What is the scale factor for the sticker? How do you know?
  - The scale factor is two because the scale factor is the same as the constant of proportionality. It is the ratio of a length in scale drawing to the corresponding actual picture length, 2 to 1. The enlargement is represented by a number greater than 1.
- Each of the corresponding lengths is how many times larger?
  - Two times
- What can you predict about an image that has a scale factor of 3?
  - The lengths will be three times as long as the original.

Scaffolding:
For students with special needs, give the closed sentence: “The _____ of any two _____ lengths in two _____ figures. The scale factor corresponds to the _______ and the _______.”

Steps to check for proportionality for scale drawing and original object/picture:
1. Measure lengths of scale drawing. Record on table.
2. Measure corresponding lengths on actual picture/drawing. Record on table.
3. Check for constant of proportionality.

Key Idea:
The scale factor can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

Scaling by factors greater than 1, enlarge the segment, and scaling by factors less than 1, reduce the segment.
Exercise 1 (5 minutes): App Icon

Give students time to measure lengths (in inches) on the app icon that corresponds to the lengths measured in Example 1 and record on tables with partners. Discuss:

- What was the relationship between the sticker and the original sketch?
  - The sticker is bigger than the original.

- What was the constant of proportionality, or scale factor, for this relationship?
  - 2

- What is the relationship between the icon and the original sketch?
  - The icon is smaller than the original sketch.

- What was the constant of proportionality, or scale factor, for this relationship?
  - $\frac{1}{2}$

- How do we determine the scale factor?
  - Measure lengths on the app icon and corresponding lengths on the original sketch and record. Find the constant of proportionality.

- What does the scale factor indicate?
  - A scale factor less than 1 indicates a reduction from the original picture, and a scale factor greater than 1 indicates a magnification from the original picture.

---

**Exercise 1: App Icon**

<table>
<thead>
<tr>
<th>Original</th>
<th>App Icon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in.</td>
<td>$\frac{1}{2}$ in.</td>
</tr>
<tr>
<td>$\frac{3}{4}$ in.</td>
<td>$\frac{3}{8}$ in.</td>
</tr>
<tr>
<td>1 in.</td>
<td>$\frac{1}{2}$ in.</td>
</tr>
<tr>
<td>$\frac{7}{8}$ in.</td>
<td>$\frac{7}{16}$ in.</td>
</tr>
</tbody>
</table>

---

**Example 2 (7 minutes)**

Begin this example by giving the scale factor 3. Demonstrate how to make a scale drawing with the scale factor. Use a table or equation to show how you computed your actual lengths. NOTE: The original image of the flag should be 1 inch by $1\frac{1}{2}$ inch. Discuss:

- Is this a reduction or an enlargement?
  - An enlargement.

- How could you determine even before the drawing?
  - A scale factor greater than one represents an enlargement.
Can you predict what the scale lengths of the scale drawing will be?
- Yes, they will be 3 times as big as the actual picture.

What steps were used to create this scale drawing?
-Measure lengths of the original drawing and record onto a table. Multiply by 3 to compute the scale drawing lengths. Record and draw.

How can you double check your work?
- Divide the scale lengths by 3 to see if they match actual lengths.

Example 2
Use a scale factor of 3 to create a scale drawing of the picture below.

Picture of the Flag of Columbia:

![Flag Picture]

A. \( \frac{1}{2} \times 3 = \frac{3}{2} \text{ in.} \)
B. \( \frac{1}{2} \times 3 = 1 \frac{1}{2} \text{ in.} \)
C. \( \frac{1}{4} \times 3 = \frac{3}{4} \text{ in.} \)
D. \( \frac{1}{4} \times 3 = \frac{3}{4} \text{ in.} \)

Exercise 2 (7 minutes)
Have students work with partners to create a scale drawing of the original picture of the flag from Example 2 but now applying a scale factor of \( \frac{1}{2} \).

- Is this a reduction or an enlargement?
  - A reduction because the scale factor is less than one

- What steps were used to create this scale drawing?
  - Compute the scale drawing lengths by multiplying by \( \frac{1}{2} \) or dividing by 2. Record. Measure new segments with a ruler and draw.
Exercise 2

Scale Factor \( \frac{1}{2} \)

Sketch and Notes:

A. \( \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} \text{ in.} \)

B. \( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ in.} \)

C. \( \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \text{ in.} \)

Example 3 (5 minutes)

Describe the following: Your family recently took a family portrait. By request, your aunt wanted you to take a picture of the portrait from your phone and send it to her. If the original portrait is 3 feet by 3 feet and the scale factor is \( \frac{1}{2} \), draw the scale drawing that would be the size of the portrait on your phone. Discuss the questions:

- What is the shape of the portrait?
  - Square.
- Will the resulting picture be a reduction or a magnification?
  - It will be a reduction because the phone picture is smaller than the original portrait. Also, the scale factor is less than one so this indications a reduction.
- One student calculated the length to be 2 inches while another student’s response was \( \frac{1}{6} \) of a foot. Which is answer is more reasonable?
  - Although both students are correct, 2 inches is more reasonable for the purpose of measuring and drawing.
- What will the scale drawing look like?
  - The scale drawing should be a square measuring 2 inches by 2 inches.
Example 3

Your family recently had a family portrait taken. Your aunt asked you to take a picture of the portrait using your cell phone and send it to her. If the original portrait is 3 feet by 3 feet and the scale factor is $\frac{1}{18}$, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

$3 \times 12 = 36$

$36 \times \frac{1}{18} = 2 \text{ in.}$

Exercise 3 (5 minutes)

Read the problem aloud and ask students to solve the problem with another student.

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

- What is the diameter of the window in the sketch of the model house?
  - 2 inches.

Exercise 3

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

$5 \times 12 = 60$

$60 \times \frac{1}{30} = 2 \text{ in.}$
Closing Questions (5 minutes)

- Where is the constant of proportionality represented in scale drawings?
  - Scale Factor

- What step(s) are used to calculate scale factors?
  - Measure the actual picture lengths and the scale drawing lengths. Write the values as a ratio of scale drawing length to actual picture length.

- What operation(s) is (are) used to create scale drawings?
  - After the lengths of the actual picture are measured and recorded, multiply each length by the scale factor to find corresponding scale drawing lengths. Measure and draw.

Exit Ticket (5 minutes)
Lesson 17: The Unit Rate as the Scale Factor

Exit Ticket

A rectangular pool in your friend’s yard is 150 ft. x 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or equation to show how you computed your scale drawing lengths.
Exit Ticket Sample Solutions

A rectangular pool in your friend’s yard is 150 ft. x 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or equation to show how you computed your scale drawing lengths.

<table>
<thead>
<tr>
<th>Actual Length</th>
<th>Scale Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 ft.</td>
<td>150 ft. multiplied by $\frac{1}{600} = \frac{1}{4}$ ft., or 3 in.</td>
</tr>
<tr>
<td>400 ft.</td>
<td>400 ft. multiplied by $\frac{1}{600} = \frac{2}{3}$ ft., or 8 in.</td>
</tr>
</tbody>
</table>

8 in.

3 in.

Problem Set Sample Solutions

1. Giovanni went to Los Angeles, California for the summer to visit his cousins. He used a map of bus routes to get from the airport to the nearest bus station from his cousin’s house. The distance from the airport to the bus station is 56 km. On his map, the distance was 4 cm. What is the scale factor?

The scale factor is $1/140,000$. Note: you must change km to cm or cm to km or both to meters to determine the scale factor.

2. Nicole is running for school president and her best friend designed her campaign poster which measured 3 feet by 2 feet. Nicole liked the poster so much she reproduced the artwork on rectangular buttons measuring 2 inches by $1 \frac{1}{3}$ inches. What is the scale factor?

The scale factor is $\frac{1}{10}$

3. Use a ruler to measure and find the scale factor.

Scale Factor: $\frac{5}{3}$

Actual Scale Drawing
4. Find the scale factor using the given scale drawings and measurements below.

Scale Factor: \( \frac{1}{2} \)

Actual Picture

![Volleyball](24\,\text{cm})

Scale Drawing

![Volleyball](6\,\text{cm})

5. Using the given scale factor, create a scale drawing from the actual pictures in centimeters:

a. Scale factor: 3

![Envelope](3\,\text{cm})

b. Scale factor: \( \frac{3}{5} \)

![Envelope](\frac{3}{5}\,\text{cm})
6. Hayden likes building radio-controlled sailboats with her father. One of the sails, shaped like a right triangle, has side lengths measuring 6 inches, 8 inches and 10 inches. To log her activity, Hayden creates and collects drawings of all the boats she and her father built together. Using the scale factor of \(\frac{1}{4}\), draw a scale drawing of sail.

**Scaffolding:**
- Extension: Students can enlarge an image they want to draw or paint by drawing a grid using a ruler over their reference picture and drawing a grid of equal ratio on their work surface. Direct students to focus on one square at a time until the image is complete. Have students compute the scale factor for the drawing.
Lesson 18: Computing Actual Lengths from a Scale Drawing

Student Outcomes

- Given a scale drawing, students compute the lengths in the actual picture using the scale. Students identify the scale factor in order to make intuitive comparisons of size then devise a strategy for efficiently finding actual lengths using the scale.

Classwork

Example 1 (14 minutes): Basketball at Recess?

The first example has students build upon the previous lesson by applying scale factor to find missing dimensions. This leads into a discussion of whether this method is the most efficient, and whether they could find another approach that would be simpler, as demonstrated in Example 2. Guide students to record responses and additional work in their student materials.

Based upon the picture, what are the actual dimensions that the half-court will be? Will the lot be big enough if its width is 25 feet and its length is 75 feet? Explain.

- How can we use the scale factor to determine the actual measurements?
  - Divide each drawing length by the scale factor to find the actual measurement. See table below.

- How can we use the scale factor to write an equation relating the scale drawing lengths to the actual lengths?
  - The scale factor is the constant of proportionality, or the $k$ in the equation $y = kx$ or $x = \frac{y}{k}$ or even $\frac{y}{x}$. It is the ratio of drawing length to actual length.

Example 1: Basketball at Recess?

Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scale Drawing: 1 inch on drawing corresponds to 15 feet of actual length.
**Lesson 18**

**Computing Actual Lengths from a Scale Drawing**

<table>
<thead>
<tr>
<th>Scale Drawing Lengths</th>
<th>1 in.</th>
<th>2 in.</th>
<th>1 2/3 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Court Lengths</td>
<td>15 ft.</td>
<td>30 ft.</td>
<td>25 ft.</td>
</tr>
</tbody>
</table>

**Scale Factor:** 1 inch corresponds to (15•12) inches, or 180 inches, so the scale factor is \( \frac{1}{180} \) Let \( k = \frac{1}{180} \) \( x \) = actual length, \( y \) = scale drawing length.

To find Actual Length:

\[ y = \frac{1}{180} x \]

\[ 2 = \frac{1}{180} x \]

\[ x = 360 \text{ inches, or 30 feet} \]

To find Actual Width:

\[ y = \frac{1}{180} x \]

\[ \frac{2}{3} = \frac{1}{180} x \]

\[ \frac{180}{1} \cdot \frac{5}{3} = x \]

\[ x = 300 \text{ inches, or 25 feet} \]

The actual court measures 25 feet by 30 feet. Yes, the lot will be big enough for the court Vincent planned. The court will take up the entire width of the lot.

**Example 2 (5 minutes)**

Teacher can guide whole class through completion of exercises below while encouraging student participation through questioning. Student should record information in their materials.

Hold discussion with students regarding the use of the word scale.

- Where have you seen this term used?
  - Bottom of a map, blueprint, etc.
- It refers to a type of ratio. 1 cm represents 20 m is an example of a ratio relationship, and the ratio 1:20 is sometimes called a scale ratio or a scale. Why can this NOT be called the scale factor?
  - The scale factor in a scaled drawing is always a scalar between distances measured in the same units.
- Do we always need to use the scale factor in order to find actual measurements from a scale drawing, or could we just use the given scale ratio (or scale)? See below.
- Take a few minutes to try to find the actual length of the garden. Give your answer in meters. When you are finished, I will ask you how you found your answer.

Allow for students to share approaches with the class. Students could calculate the scale factor and follow the steps from Example 1, or they may realize that it is not necessary to find scale factor. They may apply the scale ratio and work the problem using the ratio 1:20, perhaps setting up a proportion relationship \( y = \frac{1}{20} x \) where \( x \) represents the actual measurement and \( y \) represents the drawing length.
Lesson 18
Computing Actual Lengths from a Scale Drawing

So then, what two quantities does the constant of proportionality, $k$, relate?
- The drawing length to the actual length, when converted to the same units if scale factor is being used.
- If just the scale ratio is used, then the quantities do not need to be converted to the same units.

What method was more efficient? Explain why.
- Allow for students to respond. If we apply the scale ratio, it requires fewer steps.

Then why would we consider the scale factor?
- The scale factor gives us a sense of the comparison. In this example, the scale factor is 1/2000, so the scale drawing lengths are 1/2000th of the actual lengths. It is not always easy to see that comparison when you are basing your calculations on the scale. The scale factor helps us reason through the problem and make sense of our results.

Now, go back and find the actual width of the garden using the scale ratio.

Elicit responses from students, including an explanation of how they arrived at their answer. Record results on board for students to see and be sure students have recorded correct responses in their materials.

Example 2
The diagram shown represents a garden. The scale is 1 cm for every 20 meters of actual length. Each square in the drawing measures 1 cm by 1 cm.

<table>
<thead>
<tr>
<th></th>
<th>Scale</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawing, $y$</td>
<td>1 cm</td>
<td>8 cm</td>
<td>4 cm</td>
</tr>
<tr>
<td>Actual, $x$</td>
<td>20 m (or 2000 cm)</td>
<td>160 m (or 16000 cm)</td>
<td>80 m (or 8000 cm)</td>
</tr>
</tbody>
</table>

Method 1:
Using the given scale: 1 cm of scale drawing length corresponds to 20 m of actual length.

$k = \frac{1}{20}$  

drawing length to actual length

To find actual length:

$y = \frac{1}{20} x$  

where $x =$ actual measurement in m and $y =$ scale drawing measurement in cm

Substitute scale drawing length in place of $y$

$x = 160$

The actual length is 160 meters.

To find actual width:

Divide the actual length by 2 since its drawing width is half the length.

The actual width is 80 meters.
Method 2:

Use the scale factor:

1 cm of scale drawing length corresponds to 2000 cm of actual length.

\[ k = \frac{1}{2000} \]

drawing length to actual length (in same units)

To find Actual Length:

\[ y = \frac{1}{2000}x \]

where \( x \) = actual measurement in cm and \( y \) = drawing measurement in cm

\[ 8 = \frac{1}{2000}x \]

substitute the scale drawing length in place of \( y \)

\[ y = 16,000 \]

The actual length is 16,000 cm or 160 m.

To find Actual Width:

\[ y = \frac{1}{2000}x \]

\[ 4 = \frac{1}{2000}x \]

substitute the scale drawing width in place of \( y \)

\[ y = 8000 \]

The actual width is 8,000 cm or 80 m.

Example 3 (10 minutes)

Example 3

A graphic designer is creating an advertisement for a tablet. She needs to enlarge the picture given here so that 0.25 inches on the scale picture will correspond to 1 inch on the actual advertisement. What will be the length and width of the tablet on the advertisement?

Using a Table:

<table>
<thead>
<tr>
<th>Picture, ( y )</th>
<th>Scale</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 in.</td>
<td></td>
<td>1 ( \frac{1}{2} ) in.</td>
<td>1 ( \frac{1}{8} ) in.</td>
</tr>
<tr>
<td>Actual Advertisement, ( x )</td>
<td>1 in.</td>
<td>6 in.</td>
<td>4 ( \frac{1}{2} ) in.</td>
</tr>
</tbody>
</table>

Using an Equation:

Find the constant of proportionality, \( k \):

\[ k = \frac{0.25}{1} \]

\[ k = \frac{1}{4} \]

(scale factor since units of measure are the same; it is a reduction)

To find Actual Length:

\[ y = \frac{1}{4}x \]

where \( x \) = actual measurement and \( y \) = picture measurement

\[ 1 \frac{1}{2} = \frac{1}{4}x \]

Substitute the picture length in place of \( x \).

\[ x = 6 \]

Multiply both sides by the reciprocal.

To find Actual Width:

\[ y = \frac{1}{4}x \]

\[ 1 \frac{1}{8} = \frac{1}{4} \times \]

Substitute the picture width in place of \( y \).

\[ x = 4 \frac{1}{2} \]

Multiply both sides by the reciprocal.

The tablet will be 6 inches by 4 \( \frac{1}{2} \) inches on the actual advertisement.
Lesson 18: Computing Actual Lengths from a Scale Drawing

- Is it always necessary to write and solve an equation \( y = kx \) to find actual measurements?
  - Guide students to conclude that the actual measurement can be found by applying any of the three relationships: \( y = kx \), \( x = y/k \), or even \( k = y/x \). Encourage students to try any of these approaches in the next exercise.

Exercises (10 minutes)

Hold a brief discussion of the problem as a class and identify how to find the answer. Guide students to identify the following big ideas to address as they solve the problem:

- We need to find the relationship between the lengths in the scale drawing and the corresponding actual lengths.
- Use this relationship to calculate the width of the actual mall entrance.
- Compare this with the width of the panels.

Allow time for students to measure and complete problem (see measurement on diagram below). Encourage students to check with elbow partner to ensure that their measurements match each other.

Sample responses shown below include work for two different approaches. Students do not need to apply both and shall receive credit for using either method.

Exercises

1. Students from the high school are going to perform one of the acts from their upcoming musical at the atrium in the mall. The students want to bring some of the set with them so that the audience can get a better feel of the whole production. The backdrop that they want to bring has panels that measure 10 feet by 10 feet. The students are not sure if they will be able to fit these panels through the entrance of the mall since the panels need to be transported flat (horizontal). They obtain a copy of the mall floor plan, shown below, from the city planning office. Use this diagram to decide if the panels will fit through the entrance. Use a ruler to measure.

Scaffolding:

- Map distance of mall entrance could be noted so that students would not need to measure.
- When determining what unit to use when measuring, look at the given scale.
Answer the following questions.

a. Find the actual distance of the mall entrance and determine whether the set panels will fit.

   \textbf{Step 1: Relationship between lengths in drawing and lengths in actual}

   \textit{Scale: } \frac{1 \text{ in.}}{4 \text{ feet}}, \text{ or an equivalent ratio of } \frac{1}{36} \text{ inches to feet}

   \textit{Scale factor calculations: } \frac{1}{432} \text{ inches to inches}

   \begin{align*}
   &= \left( \frac{1}{36} \right) \cdot \frac{8}{54} \\
   &= \frac{1}{432} \text{ a reduction}
   \end{align*}

   \textbf{Step 2: Find the actual distance of entrance}

   \textit{Use the given scale: }

   \begin{align*}
   &= \frac{3}{8} \cdot \frac{36}{1} \\
   &= 13 \frac{1}{2} \text{ feet wide}
   \end{align*}

   \textit{-or-}

   \textit{Using Scale factor:}

   \begin{align*}
   &= \frac{3}{8} \cdot \frac{432}{1} \\
   &= 162 \text{ inches, or } 13 \frac{1}{2} \text{ feet wide}
   \end{align*}

   Yes, the set panels will fit (lying flat) through the mall entrance.

b. What is the scale factor? What does it tell us?

   \textbf{The scale factor is } \frac{1}{432}. \textbf{ Each length on the scale drawing is } \frac{1}{432} \textbf{ of the actual length. The actual lengths are 432 times larger than the scale drawing.}

\textbf{Closing (1 minute)}

- What does the scale factor tell us about the relationship between the actual picture and the scale drawing?
  - \textit{It gives us an understanding of how much bigger or smaller the scale drawing is compared to the actual picture.}

- How does a scale drawing differ from other drawings?
  - \textit{In a scale drawing, there exists a constant ratio of scale drawing length to actual length, whereas other drawings may not have a constant scale ratio between all corresponding lengths of the drawing and the actual.}

\textbf{Exit Ticket (5 minutes)}
Lesson 18: Computing Actual Lengths from a Scale Drawing

Exit Ticket

A drawing of a surfboard in a catalog shows its length as $8 \frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

A drawing of a surfboard in a catalog shows its length as $\frac{8}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

<table>
<thead>
<tr>
<th>Scale Drawing Length, $x$</th>
<th>equivalent scale ratio</th>
<th>surfboard</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$ inch</td>
<td>1 inch</td>
<td>$\frac{4}{9}$ inches</td>
</tr>
<tr>
<td>Actual Length, $y$</td>
<td>$\frac{3}{8}$ foot</td>
<td>$\frac{6}{8}$ ft or $\frac{3}{4}$ ft</td>
</tr>
</tbody>
</table>

$y = kx$

$y = \frac{3}{4}$

$= \frac{8}{9}$

$= \frac{76}{9}$

$= \frac{19}{3}$

The actual surfboard measures $6 \frac{1}{3}$ feet long.

Note: Students could also use an equation where $y$ represents the scale drawing and $x$ represents the actual measurement, in which case, $k$ would equal $4/3$.

Problem Set Sample Solutions

1. A toy company is redesigning their packaging for model cars. The graphic design team needs to take the old image shown below and resize it so that $\frac{1}{2}$ inch on the old packaging represents $\frac{1}{3}$ inch on the new package. Find the length of the image on the new package.

Car image length on old packaging measures 2 inches

$\frac{4}{3}$ inches; Note: It might be interesting to see how the students arrived at this answer. The scale $\frac{1}{2}$ to $\frac{1}{3}$ and the length of the original figure is 2, which is 4 halves, so in the scale drawing the length will be 4 thirds. Another approach would be to set up a proportion and solve:

\[
\frac{1}{x} = \frac{2}{3}
\]

$\frac{1}{3} = \frac{2}{x} \quad \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3} \quad x = \frac{4}{3}$
2. The city of St. Louis is creating a welcome sign on a billboard for visitors to see as they enter the city. The following picture needs to be enlarged so that \( \frac{1}{2} \) inch represents 7 feet on the actual billboard. Will it fit on a billboard that measures 14 feet in height?

Yes, the drawing measures 1 inch in height, which corresponds to 14 feet on the actual billboard.

3. Your mom is repainting your younger brother’s room. She is going to project the image shown below onto his wall so that she can paint an enlarged version as a mural. How long will the mural be if the projector uses a scale where 1 inch of the image represents \( \frac{1}{2} \) feet on the wall?

The scale drawing measures 2 inches, so the image will measure \( 2 \times 2.5 \), or 5 feet on the wall.

4. A model of a skyscraper is made so that 1 inch represents 75 feet. What is the height of the actual building if the height of the model is \( 18 \frac{3}{5} \) inches?

1,395 feet

5. The portrait company that takes little league baseball team photos is offering an option where a portrait of your baseball pose can be enlarged to be used as a wall decal (sticker). Your height in the portrait measures 3\( \frac{1}{2} \) inches. If the company uses a scale where 1 inch on the portrait represents 20 inches on the wall decal, find the height on the wall decal. Your actual height is 55 inches. If you stand next to the wall decal, will it be larger or smaller than you?

Your height on the wall decal is 70 inches. The wall decal will be larger than your actual height (when you stand next to it).

6. The sponsor of a 5K run/walk for charity wishes to create a stamp of its billboard to commemorate the event. If the sponsor uses a scale where 1 inch represents 4 feet and the billboard is a rectangle with a width of 14 feet and a length of 48 feet, what will be the shape and size of the stamp?

The stamp will be a rectangle measuring \( 3 \frac{1}{2} \) inches by 12 inches.

7. Danielle is creating a scale drawing of her room. The rectangular room measures 20 \( \frac{3}{4} \) ft. by 25 ft. If her drawing uses the scale 1 inch represents 2 feet of the actual room, will her drawing fit on an \( 8 \frac{1}{2} \) in. by 11 in. piece of paper?

No, the drawing would be \( 10 \frac{1}{4} \) inches by \( 12 \frac{1}{2} \) inches, which is bigger than the piece of paper.
8. A model of an apartment is shown below where $\frac{1}{4}$ inch represents 4 feet in the actual apartment. Use a ruler to measure the drawing and find the actual length and width of the bedroom.

Ruler measurements: $1\frac{1}{8}$ inches by $\frac{7}{16}$ inches.

The actual length would be 18 feet and the actual width would be 7 ft.
Lesson 19: Computing Actual Areas from a Scale Drawing

Student Outcomes

- Students identify the scale factor.
- Given a scale drawing, students compute the area in the actual picture.

Classwork

Examples 1–3 (13 minutes): Exploring Area Relationships

In this series of examples, students will identify the scale factor. Students can find the areas of the two figures and calculate the ratio of the areas. As students complete a few more examples, they can be guided to the understanding that the ratio of areas is the square of the scale factor.

Examples 1–3: Exploring Area Relationships

Use the diagrams below to find the scale factor and then find the area of each figure.

Example 1:

Scale factor: \( \frac{2}{1} \)

Area = \( \frac{12}{3} \) square units

Scale Drawing Area = \( \frac{48}{12} \) square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{48}{12} = 4 \)

Example 2:

Scale factor: \( \frac{1}{3} \)

Actual Area = \( \frac{54}{9} \) square units

Scale Drawing Area = \( \frac{6}{2} \) square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{6}{54} = \frac{1}{9} \)
Example 3:

Scale factor: \( \frac{4}{3} \)

Actual Area = \( 27 \) square units

Scale Drawing Area = \( 48 \) square units

Ratio of Scale Drawing Area to Actual Area: \( \frac{48}{27} = \frac{16}{9} \)

Guide students through completing the results statements on the student materials.

Results: What do you notice about the ratio of the areas in Examples 1-3? Complete the statements below.

- When the scale factor of the sides was 2, then the ratio of area was \( \frac{4}{1} \).
- When the scale factor of the sides was \( \frac{1}{3} \), then the ratio of area was \( \frac{1}{9} \).
- When the scale factor of the sides was \( \frac{4}{3} \) then the ratio of area was \( \frac{16}{9} \).

Based on these observations, what conclusion can you draw about scale factor and area?

*The ratio of area is the scale factor multiplied by itself, or squared.*

If the scale factor is \( r \), then the ratio of area is \( r^2 \) to 1.

Why do you think this is? Why do you think it is squared (opposed to cubed or something else)?

- **When you are comparing areas, you are dealing with two dimensions instead of comparing one linear measurement to another.**

How might you use this information in working with scale drawings?

- **In working with scale drawings, you could take the scale factor \( r \), calculate \( r^2 \) to determine the relationship between area of the scale drawing and the actual picture. Given a blueprint for a room, the scale drawing dimensions could be used to find scale drawing area and then applied to this new relationship to determine the actual area (the actual dimensions would not be needed).**

Suppose a rectangle has an area of 12 square m. If it is enlarged by a scale factor of three, what area would you predict the enlarged rectangle to have based on Examples 1-3? Look and think carefully!

- **If the scale factor is 3, then the ratio of scale drawing area to actual area is \( 3^2 \) to \( 1^2 \) or 9 to 1. So, if its area is 12 square meters before it is enlarged to scale, then the enlarged rectangle will have an area of \( 12 \times (\frac{9}{1}) \), or \( 12 \times 9 \), resulting in an area of 108 square meters.**
Example 4 (10 minutes): They Said Yes!

Complete problem 4 as a class asking the guiding questions below. Have students use the space in their student materials to record calculations and work.

Give students time to answer the question, possibly choosing to apply what was discovered in Examples 1–3. Allow for discussion of approaches described below and for students to decide what method they prefer.

**Example 4: They Said Yes!**

The Student Government liked your half-court basketball plan. They have asked you to calculate the actual area of the court so that they can estimate the cost of the project.

Based on the drawing below, what is the area of the planned half-court going to be?

Scale Drawing: 1 inch on drawing corresponds to 15 feet of actual length.

**Method 1:** Use the measurements we found in yesterday’s lesson to calculate the area of the half-court

Actual area = 25 feet × 30 feet
= 750 square feet

**Method 2:** Apply newly discovered Ratio of Area relationship.

This can be applied to the given scale with no unit conversions (shown on left), or to the scale factor (shown on right). Both options are included here as possible student work and would provide for a rich discussion of why they both work and what method is preferred. See questioning below.

**Using Scale:**

The ratio of Area: \( \left( \frac{1}{15} \right)^2 = \frac{1}{225} \)

Scale Drawing Area = 2 in. × 1 \( \frac{2}{3} \) in.
= \( \frac{10}{3} \) square inches

Let \( x \) = actual area and let \( y \) = scale drawing area

\[ y = kx \]

\[ \frac{10}{3} = \frac{1}{225} x \]

\[ 225 \cdot \frac{10}{3} = x \]

\[ x = 750 \text{ square feet} \]

**Using Scale Factor:**

The ratio of Area: \( \left( \frac{1}{180} \right)^2 = \frac{1}{32,400} \)

Scale Drawing Area = 2 in. × \( \frac{2}{3} \) inches
= \( \frac{10}{3} \) square inches

Let \( x \) = actual area and let \( y \) = scale drawing area

\[ y = kx \]

\[ \frac{10}{3} = \frac{1}{32,400} x \]

\[ 324,000 \cdot \frac{10}{3} = x \]

The actual area is \( \frac{324,000}{3} \) square inches,

Or \( \left( \frac{324,000}{3} \right) \div (144) \text{ feet} = 750 \text{ square feet} \)
Ask for students to share how they found their answer. Use guiding questions to find all three options as noted above.

- What method do you prefer?
- Is there a time you would choose one method over the other?

If we don’t already know the actual dimensions, it might be faster to use Method 1 (ratio of areas). If we are re-carpeting a room based upon a scale drawing, we could just take the dimensions from the scale drawing, calculate area, then apply the ratio of areas to find the actual amount of carpet we need to buy.

Guide students to complete the follow-up question in their student materials.

Does the actual area you found reflect the results we found from Examples 1–3? Explain how you know.

Yes, the scale of 1 inch to 15 feet has a scale factor of $\frac{1}{180}$, so the ratio of area should be $(\frac{1}{180})^2$, or $\frac{1}{32,400}$.

The drawing area is $(2)(1\frac{2}{3})$, or $\frac{10}{3}$ square inches.

The actual area is 25 feet by 30 feet, or 750 square feet, or 108,000 square inches.

The ratio of area is $\frac{\frac{10}{3}}{108,000}$, or $\frac{10}{324,000}$, or $\frac{1}{32,400}$.

Although it would be more efficient to apply this understanding to the scale, eliminating the need to convert units.

If we use the scale of 1/15, then the ratio of area is 1/225.

The drawing area is $(2)(1\frac{2}{3})$, or $\frac{10}{3}$ square inches.

The actual area is 25 feet by 30 feet, or 750 square feet.

The ratio of area is $\frac{\frac{10}{3}}{750}$, or $\frac{10}{2250}$, or $\frac{1}{225}$.

Exercises (15 minutes)

Allow time for students to answer independently then share results.

Exercises

1. The triangle depicted by the drawing has an actual area of 36 square units. What is the scale of the drawing? (Note: each square on grid has a length of 1 unit.)

   Scale Drawing Area = $\frac{1}{2} \cdot 6 \cdot 3$
   = 9 units$^2$

   Ratio of Scale Drawing Area to Actual Area: $\frac{9}{36} = r^2$

   Therefore scale factor, $r = \frac{3}{6}$ since $\frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36}$

   The scale factor is $\frac{1}{2}$.

   The scale is: 1 unit of drawing length represents 2 units of actual length.
For Exercise 2, allow students the time to measure the apartment using a ruler, then compare measurements with their elbow partner. Students can then continue to complete parts a-f with their elbow partner. Allow students time to share responses comparing to what is given below. Sample answers to questions are given below.

2. Use the scale drawings of two different apartments to answer the questions. Use a ruler to measure.

Suburban Apartment

City Apartment

Scale Drawing Area

- Suburban: \( \frac{1}{2} \times 2 \times \frac{1}{4} = \frac{1}{8} \) square inches
- City: \( 1 \times 1 \times \frac{1}{4} = \frac{1}{4} \) square inches

Actual Area

- Suburban: \( \frac{5}{8} \times 144 = 90 \) square feet
- City: \( \frac{3}{16} \times 256 = 80 \) square feet

The Suburban apartment has greater square footage in the closet floors.

Scaffolding:
Guide students to choose measuring units based upon how the scale is stated. For example, since 1 inch represents 12 feet, it would make sense to measure the drawing in inches.

Scaffolding:
Since the given scale is different for each drawing, it is necessary for students to compute actual areas before comparing areas in questions a, b, c.
c. Which apartment has the largest bathroom? Justify your thinking.

<table>
<thead>
<tr>
<th></th>
<th>Suburban</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Drawing Area</td>
<td>$\left( \frac{1}{2}, \frac{1}{2} \right)$</td>
<td>$\left( \frac{3}{4}, \frac{1}{2} \right)$</td>
</tr>
<tr>
<td>= $\frac{1}{2}$ square inches</td>
<td>$= \frac{3}{8}$ square inches</td>
<td></td>
</tr>
<tr>
<td>Actual Area</td>
<td>$\left( \frac{1}{2} \right) (144)$</td>
<td>$\left( \frac{3}{8} \right) (256)$</td>
</tr>
<tr>
<td>= 72 square feet</td>
<td>= 96 square feet</td>
<td></td>
</tr>
</tbody>
</table>

The city apartment has the largest bathroom.

d. A one-year lease for the suburban apartment costs $750 per month. A one-year lease for the city apartment costs $925. Which apartment offers the greater value in terms of the cost per square foot?

The suburban cost per square foot is $\frac{750}{720}$, or approximately $1.04 per square foot. The city cost per square foot is $\frac{925}{768}$, or approximately $1.20 per square foot. The suburban apartment offers greater value (cheaper cost per square foot), 1.04 versus 1.20

Closing (2 minutes)

- When given a scale drawing, how do we go about finding the area of the actual object?
  - Method 1: Compute each actual length based upon given scale and then use the actual dimensions to compute the actual area.
  - Method 2: Compute the area based upon the given scale drawing dimensions and then use the square of the scale to find actual area.

- Describe a situation where you might need to know the area of an object given a scale drawing or scale model.
  - Something where you might need to purchase materials that are priced per area, something that has a limited amount of space to fill or take up; when comparing two different plans

Lesson Summary:

Given the scale factor $r$ representing the relationship between scale drawing length and actual length, the square of this scale factor, $r^2$, represents the relationship between scale drawing area and actual area.

For example, if 1 inch on the scale drawing represents 4 inches of actual length, then the scale factor, $r$, is $\frac{1}{4}$. On this same drawing, 1 square inch of scale drawing area would represent 16 square inches of actual area since $r^2$ is $\left( \frac{1}{4} \right)^2$.

Exit Ticket (5 minutes)

Scaffolding:
Extension to Exit Ticket: Ask students to show multiple methods for finding area of the dining room.
Lesson 19: Computing Actual Areas from a Scale Drawing

Exit Ticket

1. A 1-inch length in the scale drawing below corresponds to a length of 12 feet in the actual room.

   ![Scale Drawing of Dining Room and Kitchen with dimensions: 1 ½ inches by 1 ½ inches for the dining room and ¾ inch by 1 inch for the kitchen.]

   a. Describe how the scale or the scale factor can be used to determine the area of the actual dining room.

   b. Find the actual area of the dining room.

   c. Can a rectangular table that is 7 ft. long and 4 ft. wide fit into the narrower section of the dining room? Explain your answer.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. A 1 inch length in the scale drawing below corresponds to a length of 12 feet in the actual room.

   1 ½ inches

   1 ½ inches

   ¾ in

   Kitchen

   Dining Room

   1 inch

   a. Describe how the scale or the scale factor can be used to determine the area of the actual dining room.

   Calculate the area of the scale drawing and then divide it by the square of the scale (or scale factor) to determine the actual area.

   Note: When the scale represents a reduction such as \( \frac{1}{144} \), it may be more logical to take the area and multiply it by the denominator, especially in this case where the numerator is 1.

   b. Find the actual area of the dining room.

   Scale drawing area of dining room: \( \frac{1}{2} \) by \( \frac{3}{4} \) + \( \frac{3}{4} \) by \( \frac{1}{2} \), or \( \frac{12}{8} \) square inches

   Actual area of the dining room: \( \frac{12}{8} \times \frac{1}{144} \), or \( \frac{12}{8} \times 144 \), or 216 square feet

   Or similar work completing conversions and using scale factor

   c. Can a rectangular table that is 7 ft. long and 4 ft wide fit into the narrower section of the dining room? Explain your answer.

   Narrower section of dining room measures \( \frac{3}{4} \) by \( \frac{1}{2} \) in the drawing, or 9 feet by 6 feet in the actual room. Yes, the table will fit; however, it will only allow for 1 additional foot around all sides of the table for movement or chairs.
Problem Set Sample Solutions

1. The shaded rectangle shown below is a scale drawing of a rectangle whose area is 288 square feet. What is the scale factor of the drawing? (Note: each square on grid has a length of 1 unit.)

The scale factor is \( \frac{1}{3} \).

2. A floor plan for a home is shown below where \( \frac{1}{2} \) inch corresponds to 6 feet of the actual home. Bedroom 2 belongs to 13-year old Kassie, and bedroom 3 belongs to 9-year old Alexis. Kassie claims that her younger sister, Alexis, got the bigger bedroom, is she right? Explain.

Bedroom 2 (Kassie) has an area of 135 sq. ft., and Bedroom 3 (Alexis) has an area of 144 sq. ft. Therefore, the older sister is correct. Alexis got the bigger bedroom by a difference of 9 square feet.
3. On the mall floor plan, $\frac{1}{4}$ inch represents 3 feet in the actual store.
   a. Find the actual area of Store 1 and Store 2.
      
      *Store 1 has an area of $375 \frac{3}{16}$ square feet and Store 2 has an area of $309 \frac{15}{16}$ square feet.*

   b. In the center of the atrium, there is a large circular water feature that has an area of $\left(\frac{9}{64}\right)\pi$ square inches on the drawing. Find the actual area in square feet.
      
      *The water feature has an area of $\left(\frac{9}{64}\right)\pi \times 144$, or $\left(\frac{81}{7}\right)\pi$ square feet, approximately 63.6 square feet.*

4. The greenhouse club is purchasing seed for the lawn in the school courtyard. They need to determine how much to buy. Unfortunately, the club meets after school, and students are unable to find a custodian to unlock the door. Anthony suggests they just use his school map to calculate the amount of area that will need to be covered in seed. He measures the rectangular area on the map and finds the length to be 10 inches and the width to be 6 inches. The map notes the scale of 1 inch representing 7 feet in the actual courtyard. What is the actual area in square feet?

   $60 \times 49 = 2940$ sq. ft.

5. The company installing the new in-ground pool in your back yard has provided you with the scale drawing shown below. If the drawing uses a scale of 1 inch to $1 \frac{2}{7}$ feet, calculate the total amount of two-dimensional space needed for the pool and its surrounding patio.

   *Area = 780 square feet*
Lesson 20: An Exercise in Creating a Scale Drawing

Student Outcomes

- Students create their own scale drawing of the top-view of a furnished room or building.

Today, you will be applying your knowledge from working with scale drawings to create a floor plan for your idea of the dream classroom.

Classwork: Your Dream Classroom

Preparation (Before Instructional Time): Prepare sheets of grid paper (8.5 × 11 inches), rulers and furniture catalogs for student use. Measure the perimeter of the room to give to students beforehand.

Introduction (3 minutes)

Inform students they will be working in pairs to create their dream classroom. The principal is looking for ideas to create spaces conducive to enjoyable and increased learning. Be as creative as you can be! Didn’t you always think there should be nap time? Now, you can create an area for it!

Instruction: Allow each student to work at his or her own pace. Guidelines are provided in the Student Pages.

Classroom: Your Dream Classroom

Guidelines

Take measurements: All students will work with the perimeter of the classroom as well as the doors and windows. Give students the dimensions of the room. Have students use the table provided to record measurements.

Create your dream classroom and use the furniture catalog to pick out your furniture: Students will discuss what their ideal classroom will look like with their partners and pick out furniture from the catalog. Students should record the actual measurements on the given table.

Determine scale and calculate scale drawing lengths and widths: Each pair of students will determine their own scale. The calculation of the scale drawing lengths, widths, and areas is to be included.

Scale Drawing: Using a ruler and referring back to the calculated scale length, students will draw the scale drawing including the doors, windows, and furniture.

Scaffolding:

Have some students measure the perimeter of the classroom for the class beforehand.

For struggling students: Model the measuring and recording of the perimeter of the classroom.

Extension: Have students choose flooring and record the costs. Including the furniture, students can calculate the cost of the designed room.
# NYS COMMON CORE MATHEMATICS CURRICULUM

## Lesson 20

### An Exercise in Creating a Scale Drawing

**Date:** 8/8/13

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---

## Measurements

<table>
<thead>
<tr>
<th>Classroom Perimeter</th>
<th>Windows</th>
<th>Door</th>
<th>Additional Furniture Chairs</th>
<th>Rug</th>
<th>Storage</th>
<th>Bean Bags</th>
<th>Independent Work Tables (x 4)</th>
<th>Board</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Length:</td>
<td>40 ft.</td>
<td>5 ft.</td>
<td>3 ft.</td>
<td>1 ft.</td>
<td>13 ft.</td>
<td>15 ft.</td>
<td>2 ft.</td>
<td>10 ft.</td>
</tr>
</tbody>
</table>
| Width:              | 30 ft.  | /    | /                           | 1 ft.| 10 ft.  | 2.5 ft.   | 2 ft.                          | 3 ft.  | /
| Scale Drawing:      | 4 in.   | 60   | $\frac{36}{120}$ in.        | 120 | $\frac{160}{120}$ in. | $\frac{24}{120}$ in. | $\frac{120}{120}$ in. | $\frac{72}{120}$ in. |
| Width:              | 3 in.   | /    | /                           | $\frac{1}{10}$ in. | $\frac{120}{120}$ in. | $\frac{1}{5}$ in. | $\frac{36}{120}$ in. | /

Scale: \(\frac{1}{120}\)

### Initial Sketch

Use this space to sketch the classroom perimeter. Draw your ideas and play with the placement of the furniture.
Lesson 20: An Exercise in Creating a Scale Drawing

Date: 8/8/13

<table>
<thead>
<tr>
<th>Actual Area</th>
<th>Scale Drawing Area</th>
<th>Classroom</th>
<th>Chairs</th>
<th>Rug</th>
<th>Storage</th>
<th>Bean Bags</th>
<th>Independent Work Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 × 30</td>
<td>4 × 3</td>
<td>1 × 1</td>
<td>1 × 1</td>
<td>13 × 10</td>
<td>15 × 2.5</td>
<td>2 × 2</td>
<td>10 × 3</td>
</tr>
<tr>
<td>= 1200 ft²</td>
<td>= 12 in.²</td>
<td>= 1 ft²</td>
<td>= 1 ft²</td>
<td>= 130 ft²</td>
<td>= 37.5 ft²</td>
<td>= 4 ft²</td>
<td>= 30 ft²</td>
</tr>
<tr>
<td>1 × 10</td>
<td>1/10</td>
<td>1 × 1/3</td>
<td>1/3 in.²</td>
<td>1/2 × 4</td>
<td>1/3 × 4</td>
<td>1/25 in²</td>
<td>1/30 in.²</td>
</tr>
</tbody>
</table>
Lesson 20

Closing (3 minutes)

- Why are scale drawings used in construction and design projects?
  - Scale drawings can be used to rearrange furniture, find appropriate sizes for new items, and reconfigure room size and building size without having to refer back to the actual room or building being worked on.
- How can we double check our area calculations?
  - We can check to see if our calculations for area are equal to the number of boxes for each object on the graph paper.
- What were the biggest challenges you faced when creating your floor plan? How did you overcome these challenges?
  - Arranging the furniture and realizing the pieces we chose were too big for the space. Choose another item based on the measurements.

Lesson Summary:

Scale Drawing Process:

1. Measure lengths and widths carefully with a ruler or tape measure. Record in an organized table.
2. Calculate the scale drawing lengths, widths, and areas using what was learned in previous lessons.
3. Calculate the actual areas.
4. Begin by drawing the perimeter, windows and doorways.
5. Continue to draw the pieces of furniture making note of placement of objects (distance from nearest wall).
6. Check for reasonableness of measurements and calculations.

Exit Ticket (5 minutes)
Lesson 20: An Exercise in Creating a Scale Drawing

Exit Ticket

1. Your sister has just moved into a loft style apartment in Manhattan and has asked you to be her designer. Indicate the placement of the following objects on the floorplan using the appropriate scale: queen-size bed (60 in. by 80 in.), sofa (36 in. by 64 in.), and dining table (48 in. by 48 in.). In the following scale drawing, 1 cm represents 2 ft. Each square on the grid is 1 cm².

![Scale Drawing](image)

2. Choose one object and explain the procedure to find the scale lengths.
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. Your sister has just moved into a loft style apartment in Manhattan and has asked you to be her designer. Indicate the placement of the following objects on the floorplan using the appropriate scale: queen-size bed (60 in. by 80 in.), sofa (36 in. by 64 in.), and dining table (48 in. by 48 in.). In the following scale drawing, 1 cm represents 2 ft. Each square on the grid is 1 cm².

   Queen Bed: \[ 60 \div 12 = 5 \div 2 = 2 \frac{1}{2} \text{ cm} \]
   \[ 80 \div 12 = 6 \frac{2}{3} \div 2 = 3 \frac{1}{3} \text{ cm} \]

   Sofa: \[ 36 \div 12 = 3 \div 2 = 1 \frac{1}{2} \text{ cm} \]
   \[ 64 \div 12 = 5 \frac{1}{3} \div 2 = 2 \frac{2}{3} \text{ cm} \]

   Dining Table: \[ 48 \div 12 = 4 \div 2 = 2 \text{ cm} \]

2. Choose one object and explain the procedure to find the scale lengths.

   Take the actual measurements in inches and divide by 12 inches to express the value in feet. Then divide the actual length in feet by two since two feet represent 1 cm. The resulting quotient is the scale length.

Problem Set Sample Solutions

Interior Designer:

You won a spot on a famous interior designing TV show! The designers will work with you and your existing furniture to redesign a room of your choice. Your job is to create a top-view scale drawing of your room and the furniture within it.

- With the scale factor being \( \frac{1}{24} \), create a scale drawing of your room or other favorite room in your home on a sheet of 8.5 x 11 inch graph paper.
- Include the perimeter of the room, windows, doorways, and three or more furniture pieces (such as tables, desks, dressers, chairs, bed, sofa, ottoman, etc.).
- Use the table to record lengths and include calculations of areas.
- Make your furniture “moveable” by duplicating your scale drawing and cutting out the furniture.
- Create a “before” and “after” to help you decide how to rearrange your furniture. Take a photo of your “before.”
- What changed in your furniture plans?
- Why do you like the “after” better than the “before”?

Answers will vary.
Lesson 20: An Exercise in Creating a Scale Drawing
Date: 8/8/13
Lesson 20
An Exercise in Creating a Scale Drawing
Date: 8/8/13
Lesson 20: An Exercise in Creating a Scale Drawing

<table>
<thead>
<tr>
<th>Entire Room</th>
<th>Windows</th>
<th>Doors</th>
<th>Desk/Tables</th>
<th>Seating</th>
<th>Storage</th>
<th>Bed</th>
<th>Shelf</th>
<th>Side Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual : Length</td>
<td>10 ft.</td>
<td>5 ft.</td>
<td>3 ft.</td>
<td>5 ft.</td>
<td>1 ft.</td>
<td>3 ft.</td>
<td>6 ft.</td>
<td>1 ft</td>
</tr>
<tr>
<td>Width</td>
<td>13 ft.</td>
<td>/</td>
<td>2 1/2 ft.</td>
<td>2 5/12 ft.</td>
<td>1 ft.</td>
<td>2 ft.</td>
<td>2 1/4 ft.</td>
<td>1 ft.</td>
</tr>
<tr>
<td>Scale Drawing : Length</td>
<td>5 in.</td>
<td>2 1/2 in.</td>
<td>1 1/2 in.</td>
<td>2 1/2 in.</td>
<td>1/2 in.</td>
<td>1 1/2 in.</td>
<td>3 in.</td>
<td>2 5/8 in.</td>
</tr>
<tr>
<td>Width</td>
<td>6 1/2 in.</td>
<td>/</td>
<td>1 1/4 in.</td>
<td>~1 1/4 in.</td>
<td>1/2 in.</td>
<td>1 in.</td>
<td>1 1/8 in.</td>
<td>1/2 in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Entire Room Length</th>
<th>Desk/Tables</th>
<th>Seating</th>
<th>Storage</th>
<th>Bed</th>
<th>Shelf</th>
<th>Side Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Area</td>
<td>10 × 13 = 130 ft.²</td>
<td>5 × 2 5/12 = 25/12 = 20/12 = 10/6 = 5/3 ft²</td>
<td>1 × 1 = 1 ft²</td>
<td>3 × 2 = 6 ft²</td>
<td>6 × 1 4/9 = 27/9 = 3 ft²</td>
<td>1 1/2 × 1 1/2 = 5/4 ft²</td>
</tr>
<tr>
<td>Scale Drawing Area</td>
<td>5 × 6 1/2 = 31 1/2 ft²</td>
<td>2 1/2 × 1 1/2 = 5/2 × 3/2 = 15/4 = 3 3/4 ft²</td>
<td>1 1/2 × 1 1/2 = 1 1/2 in²</td>
<td>3 × 1 1/2 = 3 × 3/2 = 9/2 = 4 1/2 in²</td>
<td>1 1/8 × 1 1/2 = 3/16 in²</td>
<td>3 3/4 = 4 1/4 in²</td>
</tr>
</tbody>
</table>
Lesson 21: An Exercise in Changing Scales

Student Outcomes

- Given a scale drawing, students produce a scale drawing of a different scale.
- Students recognize that the scale drawing of a different scale is a scale drawing of the original scale drawing.
- For the scale drawing of a different scale, students compute the scale factor for the original scale drawing.

How does your scale drawing change when a new scale factor is presented?

Classwork

Example 1 (20 minutes): A New Scale Factor

Example 1: A New Scale Factor

The school plans to publish your work on the dream classroom in the next newsletter. Unfortunately, in order to fit our drawing on the page, it must be \( \frac{1}{4} \) its current length to be published in the magazine. Create a new drawing (SD2) in which all of the lengths are \( \frac{1}{4} \) those in the original scale drawing (SD1) from lesson 20.

Example is included for students unable to create SD1 from lesson 20 at the end. Pose the following questions:

- Would the new scale create a larger or smaller scale drawing as compared to the original drawing?
  - \( \text{It would be smaller because } \frac{1}{4} \text{ is smaller than one.} \)

- How would you use the scale factor between SD1 to SD2 to calculate the new scale drawing lengths without having to get the actual measurement first?
  - \( \text{Take the original scale drawing lengths and multiply by this by } \frac{1}{4} \text{ to find the new scale lengths.} \)

Once the students have finished creating SD2, ask students to prove to the architect that SD2 is actually a scale drawing of the original room.
Lesson 21

An Exercise in Changing Scales

Date: 8/8/13

- How can we go about proving that the new scale drawing (SD2) is actually a scale drawing of the original room?
  - The scale lengths of SD2 have to be proportional to the actual lengths. We need to find the constant of proportionality, the scale factor.

- How do we find the new scale factor?
  - Divide one of the new scale lengths by its corresponding actual length.

- If the actual measurement was not known, how could we find it?
  - Calculate the actual length by using the scale factor on the original drawing. Multiply the scale length of the original drawing by the original scale factor.

Exercise (20 minutes)

Write different scale factors on cards, which the students will choose: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{2}$, 2, 3, 4. They will then create a new scale drawing and calculate the scale factor between their drawing and the original trapezoid in the student material.

After completing a – c independently, have all of the students who were working with enlargements move to the right side of the room and those with reductions to the left. Have students first discuss in smaller groups on their side of the room and then come together as a class to discuss:

- Compare your answers to part a. What can you conclude?
  - All of the enlargements had a scale factor that was greater than 1. The reductions have a scale factor between zero and 1.

- What methods did you use to answer part c?
  - The scale factor between SD2 (student drawn trapezoid) and the original figure can be determined by multiplying the scale factor of SD1 (scale drawing given in the materials) to the original figure by the scale factor of SD2 to SD1.

Exercise

The picture shows an enlargement or reduction of a scale drawing of a trapezoid.

Using the scale factor written on the card you chose, draw your new scale drawing with correct calculated measurements.
Lesson 21: An Exercise in Changing Scales

Class Date: 8/8/13

Changing Scale Factors:

- To produce a scale drawing at a different scale, you must determine the new scale factor. The new scale factor is found by dividing the different (new drawing) scale factor by the original scale factor.
- To find each new length, you can multiply each length in the original scale drawing by this new scale factor.

Steps:

- Find each scale factor.
- Divide new scale factor by original scale factor.
- Divide the given length by the new scale factor (the quotient from the prior step).

a. What is the scale factor between the original scale drawing and the one you drew?

\[
\frac{1}{3}
\]

b. The longest base length of the actual trapezoid is 10 cm. What is the scale factor between the original scale drawing and the actual trapezoid?

\[
\frac{7}{10}
\]

c. What is the scale factor between the new scale drawing you drew and the actual trapezoid?

\[
\frac{\frac{1}{3}}{10} = \frac{7}{10} \times \frac{1}{10} = \frac{7}{100}
\]

Closing (5 minutes)

- Why might you want to produce a scale drawing of a different scale?

  - To produce multiple formats of a drawing (e.g. different sized papers for a blueprint)

- How do you produce another scale drawing given the original scale drawing and a different scale?

  - Take the lengths of the original scale drawing and multiply by the different scale. Measure and draw out the new scale drawing.

- How can you tell if a new scale drawing is a scale drawing of the original figure?

  - If the new scale drawing (SD2) is a scale drawing of SD1, then it is a scale drawing of the original figure with a different scale.

- How can the scale factor of the new drawing to the original figure be determined?

  - Take the scale length of the new scale drawing and divide it by the actual length of the original figure.
Lesson Summary:

Variations of Scale Drawings with different scale factors are scale drawings of an original scale drawing. From a scale drawing at a different scale, the scale factor for the original scale drawing can be computed without information of the actual object, figure or picture.

- For example...if scale drawing one has a scale factor of $\frac{1}{24}$ and scale drawing two has a scale factor of $\frac{1}{72}$, then the scale factor relating scale drawing two to scale drawing one is:

$$\frac{\frac{1}{72}}{\frac{1}{24}} = \frac{72}{24} = \frac{1}{3}$$

Scale drawing two has lengths that are 1/3 the size of the lengths of scale drawing one.
Problem Set Sample Solutions

1. Jake reads the following problem: If the original scale factor for a scale drawing of a square swimming pool was $\frac{1}{90}$, and length of the original drawing measured to be 8 inches, what is the length on the new scale drawing if the scale factor of the new scale drawing length to actual length is $\frac{1}{144}$?

   He works out the problem like so:
   
   $8 \div \frac{1}{90} = 720$ inches.
   
   $720 \times \frac{1}{144} = 5$ inches.

   Is he correct? Explain why or why not?

   Jake is correct. He took the original scale drawing length and divided by the original scale drawing to get the actual length, 720 inches. To get the new scale drawing length he takes the actual length, 720, and multiplies by the new scale factor, $\frac{1}{144}$ to get 5 inches.

2. What is the scale factor of the new scale drawing to the original scale drawing (SD2 to SD1)?

   $\frac{5}{8}$

3. If the length of the pool measures 10 cm on the new scale drawing:
   
   a. What is the actual length of the pool in meters?
      
      $14.40$ m
   
   b. What is the surface area of the actual pool?
      
      $207.36$ m$^2$
   
   c. If the pool has a constant depth of 4 feet, what is the volume of the pool?
      
      $252.81$ m$^3$
   
   d. If 1 cubic meter of water is equal to 264.2 gallons, how much water will the pool contain when completely filled?
      
      $66,792.40$ gal.

4. Complete a new scale drawing of your dream room from Lesson 20's problem set by either reducing by $\frac{1}{4}$ or enlarging it by 4.

   Scale Drawings will vary.

   Original Scale Drawing Length: $6 \frac{1}{2}$ in

   New Scale Drawing Length: $5$ in. $6 \frac{1}{2} \times x = 5$

   $\frac{13}{2} \times x = 5$

   $x = \frac{10}{13}$ in. But because there are 16 units in one inch, $16 \times \frac{10}{13} = 12.3$, which is $\approx \frac{12.3}{16}$.
Lesson 21: An Exercise in Changing Scales

**Date:** 8/8/13

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**NYS COMMON CORE MATHEMATICS CURRICULUM** 7•1

### Equivalent Fraction Computations

<table>
<thead>
<tr>
<th>1/13</th>
<th>1</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3/13</td>
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<td>48</td>
</tr>
<tr>
<td>4/13</td>
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<td>11</td>
<td>176</td>
</tr>
<tr>
<td>12/13</td>
<td>12</td>
<td>192</td>
</tr>
</tbody>
</table>

16 x 1/13 = 16/13 = 1.2
16 x 2/13 = 32/13 = 2.5
16 x 3/13 = 48/13 = 3.7
16 x 4/13 = 64/13 = 4.9
16 x 5/13 = 80/13 = 6.2
16 x 6/13 = 96/13 = 7.4
16 x 7/13 = 112/13 = 8.6
16 x 8/13 = 128/13 = 9.8
16 x 9/13 = 144/13 = 11.0
16 x 10/13 = 160/13 = 12.3
16 x 11/13 = 176/13 = 13.5
16 x 12/13 = 192/13 = 14.8
Lesson 21: An Exercise in Changing Scales

SD1 Example for students who were unable to create their own from Lesson 20
Lesson 22: An Exercise in Changing Scales

Student Outcomes

- Given a scale drawing, students produce a scale drawing of a different scale.
- Students recognize that the scale drawing of a different scale is a scale drawing of the original scale drawing.
- For the scale drawing of a different scale, students compute the scale factor for the original scale drawing.

Classwork

Reflection on Scale Drawings (15 minutes): Ask students to take out the original scale drawing and new scale drawing of their dream rooms they completed as part of Lesson 20 and 21 problem sets. Have students discuss their answers with a partner. Discuss as a class:

- How are the two drawings alike?
- How are the two drawings different?
- What is the scale factor of the new scale drawing to the original scale drawing?

Direct students to fill-in-the-blanks with the two different scale factors. Allow pairs of students to discuss the posed question, “What is the relationship?” for 3 minutes and share responses for 4 minutes. Summarize the Key Idea with students.

Classwork

Using the new scale drawing of your dream room, list the similarities and differences between this drawing and the original drawing completed for Lesson 20.

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Same room shape</td>
<td>-one is bigger than the other</td>
</tr>
<tr>
<td>-Placement of furniture</td>
<td>-different scale factors</td>
</tr>
<tr>
<td>-Space between furniture</td>
<td></td>
</tr>
<tr>
<td>-Drawing of the original room</td>
<td></td>
</tr>
<tr>
<td>-Proportional</td>
<td></td>
</tr>
</tbody>
</table>

Original Scale Factor: \( \frac{1}{120} \)  
New Scale Factor: \( \frac{1}{30} \)

What is the relationship between these scale factors? \( \frac{1}{4} \)

Key Idea:

Two different scale drawings of the same top-view of a room are also scale drawings of each other. In other words, a scale drawing at a different scale can also be considered a scale drawing of the original scale drawing.
Example 1 (10 minutes): Building a Bench

Students are given the following information: the scale factor of Taylor’s scale drawing to the actual bench is \( \frac{1}{12} \), Taylor’s scale drawing and the measurements of the corresponding lengths (2 in. and 6 in. as shown). Ask the students the following questions:

- What information is important in the diagram?
  - The scale factor of Taylor’s reproduction.

- What information can be accessed from the given scale factor?
  - The actual length of the bench can be computed from the scale length of Taylor’s drawing.

- What are the processes used to find the original scale factor to the actual bench?
  - Take the length of the new scale drawing, 6 inches, and divide by the scale factor, \( \frac{1}{12} \), to get the actual length of the bench, 72 inches. The original scale factor, \( \frac{1}{36} \), can be computed by dividing the original scale length, 2 inches, by the actual length, 72 inches.

- What is the relationship of Taylor’s drawing to the original drawing?
  - Taylor’s drawing is 3 times as big as her father’s drawing. The lengths corresponding to the actual length, which is 72 inches, are 6 inches from Taylor’s drawing and 2 inches from the original drawing. \( \frac{6}{2} = 3 \), so the scale factor is 3.

Example 1: Building a Bench

To surprise her mother, Taylor helped her father build a bench for the front porch. Taylor’s father had the instructions with drawings, but Taylor wanted to have her own copy. She enlarged her copy to make it easier to read. Using the following diagram, fill in the missing information.

The pictures below show the diagram of the bench shown on the original instructions and the diagram of the bench shown on Taylor’s enlarged copy of the instruction.

![Diagram of bench with scale factors]
Exercise 1 (5 minutes)

Allow students to work problem with partners for 3 minutes. Discuss for 2 minutes:

- How did you find the original scale factor?
  - Divide the Carmen’s map distance, 4 cm, by the scale factor $\frac{1}{563,270}$ to get the actual distance, 2,253,080 cm. Take the distance from Jackie’s map, 26 cm, and divide by the actual distance to get the original scale factor $\frac{1}{86,657}$.

- What are the steps to find the scale of new to original scale drawing?
  - Divide the new scale distance, 4 cm, to the corresponding original scale distance, 26 cm, to get $\frac{2}{13}$.

- What is the actual distance in miles?
  - $2,253,080$ cm divided by $2.54$ cm gives $887,039.37$ inches. Divide $887,039.37$ by $12$ to get $73,919.95$ feet. Then divide $73,919.95$ by $5280$ to get around $14$ miles.

- Would it make more sense to answer in centimeters or miles?
  - Although both are valid units, miles would be a more useful unit to describe the distance driven in a car.

Exercise 1

Carmen and Jackie were driving separately to a concert. Jackie printed a map of the directions on a piece of paper before the drive, and Carmen took a picture of Jackie’s map on her phone. Carmen’s map had a scale factor of $\frac{1}{563,270}$. Using the pictures, what is the scale of Carmen’s map to Jackie’s map? What was the scale factor of Jackie’s printed map to the actual distance?

Jackie’s Map

\[
\begin{align*}
\text{Scale Factor of SD2 to SD1:} & \quad \frac{4}{26} = \frac{2}{13} \\
\text{Scale Factor of SD1 to actual distance:} & \quad \frac{1}{563,270} \times \frac{13}{2} = \frac{13}{112,654,000}
\end{align*}
\]

Carmen’s Map:

\[
\begin{align*}
\text{Scale Factor of SD1 to SD2:} & \quad \frac{13}{2} \times \frac{26}{4} = \frac{13}{112,654,000}
\end{align*}
\]
Exercise 2 (10 minutes)

Allow students to work in pairs to find the solutions. Ask:

- What is another way to find the scale factor of the toy set to the actual boxcar?
  - Take the length of the toy set and divide it by the actual length.
- What is the purpose of question c?
  - To take notice of the relationships between all the scale factors.

Exercise 2

Ronald received a special toy train set for his birthday. In the picture of the train on the package, the box car has the following dimensions: length is $\frac{4}{5}$ inches; width is $\frac{1}{16}$ inches; height is $\frac{1}{16}$ inches. The toy box car that Ronald received has dimensions $l$ is 17.25 inches; $w$ is 4.5 inches; $h$ is 6.5 inches. If the actual boxcar is 50 feet long:

a. Find the scale factor of the picture on the package to the toy set.

$$\frac{\frac{4}{5}}{17} = \frac{\frac{5}{16}}{17} = \frac{69}{16} = \frac{4}{1} = 4$$

b. Find the scale factor of the picture on the package to the actual boxcar.

$$\frac{\frac{5}{16}}{50 \times 12} = \frac{\frac{5}{16}}{600} = \frac{69}{16} \times \frac{1}{600} = \frac{23}{3200}$$

c. Use these two scale factors to find the scale factor between the toy set and the actual boxcar.

$$\frac{\frac{4}{16}}{600} = \frac{\frac{5}{16}}{17} = \frac{23}{3200} \times \frac{1}{4} = \frac{23}{800} \times 4 = \frac{23}{800}$$

d. What are the width and height of the actual boxcar?

$$W: \frac{1}{2} \div \frac{23}{800} = \frac{9}{2} \times \frac{800}{23} = 156 \frac{12}{23} \text{ in.}$$

$$H: \frac{6}{2} \div \frac{23}{800} = \frac{13}{2} \times \frac{800}{23} = 226 \frac{2}{23} \text{ in.}$$

Closing (5 minutes)

- What is the relationship between the scale drawing of a different scale to the original scale drawing?
  - The scale drawing at a different scale is scale drawing of the original scale. If the scale factor of one of the drawings is known, the other scale factor can be computed.
- Describe the process of computing the scale factor for the original scale drawing from the scale drawing at a different scale.
  - Find corresponding known lengths and compute the actual length from the given scale factor using the new scale drawing. To find the scale factor for the original drawing, write a ratio to compare a drawing length from original drawing to its corresponding actual length from the second scale drawing.
Lesson Summary:

The scale drawing at a different scale is a scale drawing of the original scale drawing.

To find the scale factor for the original drawing, write a ratio to compare a drawing length from original drawing to its corresponding actual length from the second scale drawing.

Refer to the example below where we compare drawing length from Original Scale drawing to its corresponding actual length from the New Scale drawing:

6 inches / 12 feet, or 0.5 feet / 12 feet converting to the same units

This gives an equivalent ratio of $\frac{1}{24}$ for the scale factor of the original drawing.

Original Scale drawing:

\[ \text{[unknown SF]} \]

Length is 6 inches on drawing

New Scale drawing [different scale]:

\[ \text{[}\]

Length is 2 inches on drawing, or 12 feet actual length using given scale

1 inch represents 6 feet

Exit Ticket (5 minute)
Lesson 22: An Exercise in Changing Scales

Exit Ticket

The school is building a new wheelchair ramp for one of the remodeled bathrooms. The original drawing was created by the contractor, but the principal drew another scale drawing to see the size of the ramp relative to the walkways surrounding it. Find the missing values on the table.

<table>
<thead>
<tr>
<th>Original Scale Drawing</th>
<th>Principal’s Scale Drawing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 in.</td>
<td>New Scale Factor of SD2 to the actual ramp: $\frac{1}{700}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actual Ramp</th>
<th>Original Scale Drawing</th>
<th>Principal’s Scale Drawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Ramp</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Original Scale Drawing</td>
<td>1</td>
<td>175</td>
</tr>
<tr>
<td>Principals’ Scale Drawing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. The school is building a new wheelchair ramp for one of the remodeled bathrooms. The original drawing was created by the contractor, but the principal drew another scale drawing to see the size of the ramp relative to the walkways surrounding it. Find the missing values on the table.

   Original Scale Drawing
   Principal’s Scale Drawing:
   New Scale Factor of SD2 to the actual ramp: \( \frac{1}{700} \)

   12 in. 3 in.

   Scale Factor Table

<table>
<thead>
<tr>
<th>Actual Ramp</th>
<th>Original Scale Drawing</th>
<th>Principals’ Scale Drawing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Ramp</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Original Scale Drawing</td>
<td>( \frac{1}{175} )</td>
<td>1</td>
</tr>
<tr>
<td>Principals’ Scale Drawing</td>
<td>( \frac{1}{700} )</td>
<td>( \frac{1}{4} )</td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

1. For the scale drawing, the actual lengths are labeled onto the scale drawing. Measure the lengths of the scale drawing and draw a new scale drawing with a scale factor (SD2 to SD1) of \( \frac{1}{2} \).

   2 ft. 10 feet
   4 feet

   5 cm
   1 cm
   2 cm
2. Use the measurements on the diagrams below to identify whether each would be scale drawings of a garden. The garden contains a rectangular portion measuring 24 ft. by 6 ft. and two circular fountains each with a diameter of 5 ft.

\[ \begin{array}{c}
\text{a.} & \text{10 in.} \\
\text{b.} & 2.5 \text{ ft.} \\
\text{c.} & 3 \frac{1}{3} \text{ cm} \\
\text{d.} & 1 \frac{2}{3} \text{ cm}
\end{array} \]

b and c

3. Compute the scale factor of the new scale drawing (SD2) to original scale drawing (SD1) using information from the given scale drawing.

a. Original Scale Factor: 6/35
   New Scale Factor: 1/280
   Scale Factor: \( \frac{1}{48} \)

b. Original Scale Factor: 1/12
   New Scale Factor: 3
   Scale Factor: \( \frac{36}{1} \)

c. Original Scale Factor: 20
   New Scale Factor: 25
   Scale Factor: \( \frac{5}{4} \)
1. It is a Saturday morning and Jeremy has discovered he has a leak coming from the water heater in his attic. Since plumbers charge extra to come out on weekends, Jeremy is planning to use buckets to catch the dripping water. He places a bucket under the drip and steps outside to walk the dog. In half an hour the bucket is 1/5 of the way full.

   a. What is the rate at which the water is leaking?

   b. Write an equation that represents the relationship between the number of buckets filled, \( y \), in \( x \) hours.

   c. What is the longest that Jeremy can be away from the house before the bucket will overflow?
2. Farmers often plant crops in circular areas because one of the most efficient watering systems for crops provides water in a circular area. Passengers in airplanes often notice the distinct circular patterns as they fly over land used for farming. A photographer takes an aerial photo of a field on which a circular crop area has been planted. He prints the photo out and notes that 2 centimeters of length in the photo corresponds to 100 meters in actual length.

   a. What is the scale factor of the photo?

   b. If the dimensions of the entire photo are 25 cm by 20 cm, what are the actual dimensions of the rectangular land area in meters captured by the photo?

   c. If the area of the circular area on the photo is $64\pi$ cm$^2$, what is the actual area of the circular crop area in square meters?
3. A store is having a sale to celebrate President’s Day. Every item in the store is advertised as one fifth off the original price. If an item is marked with a sale price of $140, what was its original price? Show your work.

4. Over the break, your uncle and aunt ask you to help them cement the foundation of their newly purchased land and give you a top-view blueprint of the area and proposed layout. A small legend on the corner states that 4 inches of the length corresponds to an actual length of 52 feet.

   a. What is the scale factor?
b. If the dimensions of the foundation on the blueprint are 11 inches by 13 inches, what are the actual dimensions in feet?

c. You're asked to go buy bags of dry cement and know that one bag covers 350 square feet. How many bags do you need to buy to finish this project?

d. After the first 15 minutes of laying down the cement, you had used 1/5 of the bag. What is the rate you are laying cement in bags per hour? What is the unit rate?
e. Write an equation that represents the relationship between the number of bags used, $y$, in $x$ hours.

f. Your uncle is able to work faster than you. He uses 3 bags for every 2 bags you use. Is the relationship proportional? Explain your reasoning using a graph on a coordinate plane.

g. What does $(0,0)$ represent in terms of the situation being described by the graph created in part (f)?
h. Using a graph, show how many bags you would have used if your uncle used 18 bags.
<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</th>
<th>STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</th>
<th>STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</th>
<th>STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1   a  7.RP.1</td>
<td>Student answered rate incorrectly and showed no or very limited calculations.</td>
<td>Student set the problem up incorrectly resulting in an incorrect rate.</td>
<td>Student set the problem up correctly but made minor mistakes in the calculation.</td>
<td>Student correctly stated the rate as ( \frac{2}{5} ) buckets per hour with correct problem set up and calculations.</td>
</tr>
<tr>
<td>b  7.RP.1  7.RP.2c  7.EE.4a</td>
<td>Student was unable to write an equation OR wrote an equation that was not in the form ( y = kx ) or even ( x = ky ) for any value ( k ).</td>
<td>Student wrote an incorrect equation, such as ( y = \frac{5}{2}x ) or ( x = \frac{2}{5}y ), AND/OR used an incorrect value of unit rate from part (a) to write an their equation in the form ( y = kx ).</td>
<td>Student created an equation using the constant of proportionality, but wrote the equation in the form ( x = \frac{5}{2}y ) or some other equivalent equation.</td>
<td>Student correctly answered ( y = \frac{2}{5}x ).</td>
</tr>
<tr>
<td>c  7.RP.1  7.RP.2c  7.EE.4a</td>
<td>Student answer is incorrect. Little or no evidence of reasoning is given.</td>
<td>Student answer is incorrect, but shows some evidence of reasoning and usage of an equation for the proportional relationship (though the equation itself may be incorrect).</td>
<td>Student correctly answers 2.5 hours but with minor errors in the use of and calculations based on the equation ( y = \frac{5}{2}x ).</td>
<td>Student correctly answers 2.5 hours with correct use of AND calculations based on the equation ( y = \frac{2}{5}x ).</td>
</tr>
<tr>
<td>2   a  7.G.1</td>
<td>Student is unable to answer OR the answer gives no evidence of understanding the fundamental concept of scale factor as a ratio comparison of corresponding lengths between the image and the actual object.</td>
<td>Student incorrectly answers the scale factor to be 2:100, 1:50, OR 1/50. The answer expresses scale factor as a comparison of corresponding lengths, but does not show evidence of choosing the same measurement unit.</td>
<td>Student correctly answers the scale factor to be 1:5000 OR 1/5000, but has a minor error in calculations or notation. For example, student writes 1/5000 cm.</td>
<td>Student correctly answers the scale factor to be 1:5000 OR 1/5000 with correct calculations and notation.</td>
</tr>
<tr>
<td>Table row</td>
<td>7.G.1</td>
<td>7.G.1</td>
<td>7.G.1</td>
<td>7.G.1</td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>Student answers incorrectly and gives little or no evidence of understanding scale factor.</td>
<td>Student shows some evidence of reasoning, but makes one or more calculation errors thereby providing an incorrect answer.</td>
<td>Student correctly answers the actual dimensions as 1,250 m x 1,000 m, but does not show work to support their answer.</td>
<td>Student correctly answers the actual dimensions as 1,250 m x 1,000 m with correct calculations.</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>Student answers incorrectly and gives little or no evidence of understanding scale factor.</td>
<td>Student shows some evidence of reasoning, but makes one or more calculation errors thereby providing an incorrect answer.</td>
<td>Student correctly answers the actual area as 160,000π m², but does not show work to support their answer.</td>
<td>Student correctly answers the actual area as 160,000π m² with correct calculations.</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>7.RP.3</td>
<td>Student answer is missing or incorrect. Student shows little or no evidence of reasoning.</td>
<td>Student answers the original price incorrectly, but provides some evidence of reasoning.</td>
<td>Student shows solid evidence of reasoning, but makes minor errors in calculations or representations. The answer may or may not be accurate.</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>7.RP.1</td>
<td>7.RP.2 7.RP.3</td>
<td>7.RP.1 7.RP.2b</td>
<td>7.RP.1 7.RP.2b</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>Student answered rate incorrectly and showed no or very limited calculations.</td>
<td>Student incorrectly answers the scale factor to be 4/52 OR another incorrect response. Limited calculations are shown.</td>
<td>Student correctly states the rate as 4/5 bags per hour AND identified the unit rate as 4/5 with correct problem setup and calculations.</td>
<td>Student correctly states the rate as 4/5 bags per hour AND identified the unit rate as 4/5 with correct problem setup and calculations.</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>Student answers both of the actual dimensions incorrectly. No or little evidence of understanding scale factor is shown.</td>
<td>Student correctly answers at least one of the dimensions correctly with errors in calculations.</td>
<td>Student correctly answers the actual dimensions as 143 feet x 169 feet with one or two minor errors in calculations.</td>
<td>Student correctly answers the actual dimensions as 143 feet x 169 feet with correct calculations.</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>Student answers incorrectly with no or little evidence of understanding scale factor shown.</td>
<td>Student answers incorrectly, but showed some understanding of scale factor in calculations.</td>
<td>Student incorrectly answers 69 bags OR correctly answers 70 bags with one or two minor errors in calculations.</td>
<td>Student correctly answers 70 bags with correct calculations.</td>
</tr>
<tr>
<td><strong>d</strong></td>
<td>Student answered rate incorrectly and showed no or very limited calculations.</td>
<td>Student set the problem up incorrectly resulting in an incorrect rate.</td>
<td>Student set the problem up correctly, but made minor mistakes in the calculation.</td>
<td>Student correctly stated the rate as 4/5 bags per hour AND identified the unit rate as 4/5 with correct problem setup and calculations.</td>
</tr>
</tbody>
</table>
### e 7.RP.2c 7.EE.4a
Student was unable to write an equation or wrote an equation that was not in the form \( y = kx \) or even \( x = ky \) for any value \( k \).

Student wrote an incorrect equation, such as \( y = \frac{5}{2}x \), or \( x = \frac{4}{5}y \), AND/OR used an incorrect value of unit rate from part (d) to write an their equation in the form \( y = kx \).

Student created an equation using the constant of proportionality, but wrote the equation in the form \( x = \frac{5}{4}y \) or some other equivalent equation.

Student correctly answered \( y = \frac{2}{5}x \).

### f 7.RP.2
Student may or may not have answered that the relationship was proportional. Student was unable to provide a complete graph. Student was unable to relate the proportional relationship to the graph.

Student may or may not have answered that the relationship was proportional. Student provided a graph with mistakes (i.e., unlabeled axis, incorrect points). Student provided a limited expression of reasoning.

Student correctly answered that the relationship was proportional. Student labeled the axis AND plotted points with minor error. Student explanation was slightly incomplete.

Student correctly answered that the relationship was proportional. Student correctly labeled the axis AND plotted the graph on the coordinate plane. Student reasoned that the proportional relationship was due to the graph being straight and going through the origin.

### g 7.RP.2d
Student was unable to describe the situation correctly.

Student was able to explain that the zero was the amount of bags used by either him/her or the uncle, but unable to describe the relationship.

Student describes the relationship correctly, but with minor error.

Student correctly explains that \((0,0)\) represents when she/he used zero bags, the uncle doesn't use any bags.

### h 7.RP.2
Student answers incorrectly and shows no or little understanding of analyzing graphs.

Student answers incorrectly, but shows some understanding of analyzing graphs.

Student correctly answers 12 bags, but does not identify the point on the graph clearly.

Student correctly answers 12 bags by identifying the point on the graph.
1. It is a Saturday morning and Jeremy has discovered he has a leak coming from the water heater in his attic. Since plumbers charge extra to come out on weekends, Jeremy is planning to use buckets to catch the dripping water. He places a bucket under the drip and steps outside to walk the dog. In half an hour the bucket is 1/5 of the way full.

   a. What is the rate at which the water is leaking?

   \[
   \text{rate:} \quad \frac{\frac{1}{5}}{\frac{1}{2}} \text{ bucket/ hr} = \frac{1}{5} \div \frac{1}{2} \text{ buckets/hr} = \frac{2}{5} \text{ buckets/hr}
   \]

   b. Write an equation that represents the relationship between the number of buckets filled, \( y \), in \( x \) hours.

   \[
   y = \frac{2}{5} x
   \]

   c. What is the longest that Jeremy can be away from the house before the bucket will overflow?

   \[
   \frac{2}{5} x \leq 1 \\
   x \leq \frac{5}{2} \text{ hours}
   \]

   Jeremy can be away for not more than \( 2\frac{1}{2} \) hours.
2. Farmers often plant crops in circular areas because one of the most efficient watering systems for crops provides water in a circular area. Passengers in airplanes often notice the distinct circular patterns as they fly over land used for farming. A photographer takes an aerial photo of a field on which a circular crop area has been planted. He prints the photo out and notes that 2 centimeters of length in the photo corresponds to 100 meters in actual length.

![Circular area diagram]

a. What is the scale factor of the photo?

\[
\begin{align*}
2\text{cm to } 100\text{ m} & \quad 1:5000 \\
1\text{ cm to } 50\text{ m} & \quad \text{or } 1:5000 \\
1\text{ cm to } 5000\text{ cm} & \quad \\
\end{align*}
\]

b. If the dimensions of the entire photo are 25 cm by 20 cm, what are the actual dimensions of the rectangular land area in meters captured by the photo?

\[
\begin{align*}
25\text{ cm} \times 50\text{ m} & = 1250\text{ m}^2 \\
\text{by } 20\text{ cm} \times 50\text{ m} & = 1000\text{ m} \\
1250\text{ m} \times 1000\text{ m} & = 1250000\text{ m}^2 \\
\end{align*}
\]

c. If the area of the circular area on the photo is \(64\pi\) cm\(^2\), what is the actual area of the circular crop area in square meters?

\[
\begin{align*}
\text{Scale Factor} = \frac{1}{5000} & \quad \text{therefore Area is } 5,000^2 \text{ times larger} \\
100\text{ cm} = 1\text{ m} & \quad 10,000\text{ cm}^2 = 1\text{ m}^2 \\
\text{Area of Photo} = 64\pi\text{ cm}^2 & = 0.0064\pi\text{ m}^2 \\
x 5000^2 & = 160,000\pi\text{ m}^2 \\
\text{Area of Actual} & = \\
\end{align*}
\]
3. A store is having a sale to celebrate President’s Day. Every item in the store is advertised as one fifth off the original price. If an item is marked with a sale price of $140, what was its original price? Show your work.

\[
\frac{35}{35} \frac{35}{35} \frac{35}{35} \frac{35}{35} \quad \text{original price} = \frac{140}{5} = 28 \text{ original price} = \frac{175}{5} = 35
\]

4. Over the break, your uncle and aunt ask you to help them cement the foundation of their newly purchased land and give you a top-view blueprint of the area and proposed layout. A small legend on the corner states that 4 inches of the length corresponds to an actual length of 52 feet.

\[\text{Property Line} \quad \text{Deck} \]

\[\text{Foundation Line} \quad \text{Driveway} \]

a. What is the scale factor?

\[
4 \text{ in.} = 52 \text{ ft.} \\
1 \text{ in.} = 13 \text{ ft.} \\
1 \text{ in.} = 156 \text{ in.}
\]

\[\text{The scale factor is } \frac{1}{156}\]
b. If the dimensions of the foundation on the blueprint are 11 inches by 13 inches, what are the actual dimensions in feet?

\[
\begin{align*}
11 \text{ in} \times \frac{13 \text{ ft}}{\text{in}} &= 143 \text{ ft} \\
13 \text{ in} \times \frac{13 \text{ ft}}{\text{in}} &= 169 \text{ ft} \\
143 \text{ ft} & \text{ by } 169 \text{ ft}
\end{align*}
\]

c. You’re asked to go buy bags of dry cement and know that one bag covers 350 square feet. How many bags do you need to buy to finish this project?

\[
\begin{array}{c|c|c|c|c}
\text{Area} & 143 & 169 \\
1287 & \times 5 & \text{bags} & 350 \text{ ft}^2 \\
8580 & & & 3500 \text{ ft}^2 \\
14300 & & & 17500 \\
24167 & & & 21100 \text{ ft} \\
\end{array}
\]

\[
\text{70 bags}
\]

d. After the first 15 minutes of laying down the cement, you had used 1/5 of the bag. What is the rate you are laying cement in bags per hour? What is the unit rate?

\[
\frac{\frac{1}{5} \text{ bag}}{\frac{1}{4} \text{ hour}} = \frac{1}{5} \times \frac{4}{1} \text{ bags/hr} = \frac{4}{5} \text{ bags/hr}
\]

\[
\text{Unit rate} = \frac{4}{5}
\]
e. Write an equation that represents the relationship between the number of bags used, \( y \), in \( x \) hours.

\[ y = \frac{4}{5}x \]

f. Your uncle is able to work faster than you. He uses 3 bags for every 2 bags you use. Is the relationship proportional? Explain your reasoning using a graph on a coordinate plane.

[Graph showing a straight line through the point \((0,0)\).]

yes, the relationship is proportional, the graph is a straight line through the point \((0,0)\).

g. What does \((0,0)\) represent in terms of the situation being described by the graph created in part (f)?

If my uncle uses 0 bags, I also use 0 bags.
h. Using a graph, show how many bags you would have used if your uncle used 18 bags.