# Table of Contents

## Arithmetic Operations Including Division of Fractions

### Module Overview

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividing Fractions by Fractions (6.NS.1)</td>
<td>9</td>
</tr>
<tr>
<td>Topic A: Dividing Fractions by Fractions (6.NS.1)</td>
<td>9</td>
</tr>
<tr>
<td>Lessons 1–2: Interpreting Division of a Whole Number by a Fraction—Visual Models</td>
<td>10</td>
</tr>
<tr>
<td>Lessons 3–4: Interpreting and Computing Division of a Fraction by a Fraction—More Models</td>
<td>27</td>
</tr>
<tr>
<td>Lesson 5: Creating Division Stories</td>
<td>46</td>
</tr>
<tr>
<td>Lesson 6: More Division Stories</td>
<td>56</td>
</tr>
<tr>
<td>Lesson 7: The Relationship Between Visual Fraction Models and Equations</td>
<td>64</td>
</tr>
<tr>
<td>Lesson 8: Dividing Fractions and Mixed Numbers</td>
<td>73</td>
</tr>
</tbody>
</table>

### Topic B: Multi-Digit Decimal Operations—Adding, Subtracting, and Multiplying (6.NS.3)

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 9: Sums and Differences of Decimals</td>
<td>80</td>
</tr>
<tr>
<td>Lesson 10: The Distributive Property and Product of Decimals</td>
<td>86</td>
</tr>
<tr>
<td>Lesson 11: Fraction Multiplication and the Products of Decimals</td>
<td>91</td>
</tr>
</tbody>
</table>

### Mid-Module Assessment and Rubric

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topics A through B (assessment 1 day, return 1 day, remediation or further applications 1 day)</td>
<td>96</td>
</tr>
</tbody>
</table>

### Topic C: Dividing Whole Numbers and Decimals (6.NS.2, 6.NS.3)

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 12: Estimating Digits in a Quotient</td>
<td>110</td>
</tr>
<tr>
<td>Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm</td>
<td>119</td>
</tr>
<tr>
<td>Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions</td>
<td>127</td>
</tr>
<tr>
<td>Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math</td>
<td>135</td>
</tr>
</tbody>
</table>

### Topic D: Number Theory—Thinking Logically About Multiplicative Arithmetic (6.NS.4)

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 16: Even and Odd Numbers</td>
<td>148</td>
</tr>
</tbody>
</table>

---

1 Each lesson is ONE day and ONE day is considered a 45 minute period.
Lesson 17: Divisibility Tests for 3 and 9 ................................................................. 156
Lesson 18: Least Common Multiple and Greatest Common Factor ............................. 163
Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm 178

End-of-Module Assessment and Rubric .................................................................... 186
Topics A through D (assessment 1 day, return 1 day, remediation or further applications 1 day)
Module Overview

Grade 6 • Module 2

Arithmetic Operations Including Division of Fractions

OVERVIEW

In Module 1, students used their existing understanding of multiplication and division as they began their study of ratios and rates. In Module 2, students complete their understanding of the four operations as they study division of whole numbers, division by a fraction and operations on multi-digit decimals. This expanded understanding serves to complete their study of the four operations with positive rational numbers, thereby preparing students for understanding, locating, and ordering negative rational numbers (Module 3) and algebraic expressions (Module 4).

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. They construct division stories and solve word problems involving division of fractions (6.NS.1). Through the context of word problems, students understand and use partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask, “How much is one share?” and “What part of the unit is that share?” Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. For example, students understand that \( \frac{6\text{ cm}}{2\text{ cm}} \) derives a quotient of 3 because 2 cm divides into 6 centimeters three times. They apply this method to quotients of fractions, understanding \( \frac{6}{7} \div \frac{2}{7} = \frac{6}{2}\frac{\text{sevens}}{\text{sevens}} = 3 \) because, again, two-sevenths divides into six-sevenths three times. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Using this relationship, students create equations and formulas to represent and solve problems. Later in the module, students learn to and apply the direct correlation of division of fractions to division of decimals.

Prior to division of decimals, students will revisit all decimal operations in Topic B. Students have had extensive experience of decimal operations to the hundredths and thousandths (5.NBT.7), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to mixed numbers from the last lesson in Topic A. They find that sums and differences of large mixed numbers can sometimes be more efficiently determined by first converting the number to a decimal and then applying the standard algorithms (6.NS.3). They use estimation to justify their answers.
Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. By gaining fluency in the distributive property throughout this module and the next, students will be proficient in applying the distributive property in Module 4 (6.EE.3). Estimation and place value enable students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students learn to use connections between fraction multiplication and decimal multiplication.

In Grades 4 and 5, students used concrete models, pictorial representations, and properties to divide whole numbers (4.NBT.6, 5.NBT.6). They became efficient in applying the standard algorithm for long division. They broke dividends apart into like base-ten units, applying the distributive property to find quotients place by place. In Topic C, students connect estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (6.NS.2). Students understand that when they “bring down” the next digit in the algorithm, they are essentially distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (6.NS.3). They make connections to division of fractions and rely on mental math strategies to implement the division algorithm when finding the quotients of decimals.

In Topic D, students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid’s Algorithm is a more efficient way to find the greatest common factor of larger numbers and see that Euclid’s Algorithm is based on long division.

The module comprises 21 lessons; four days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic C.

**Focus Standards**

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because 3/4 of 8/9 is 2/3. (In general, \((a/b) \div (c/d) = ad/bc\). How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
Module Overview

6.NS.3  Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.4  Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

**Foundational Standards**

**Gain familiarity with factors and multiples.**

4.OA.4  Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

**Understand the place value system.**

5.NBT.2  Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

**Perform operations with multi-digit whole numbers and with decimals to hundredths.**

5.NBT.6  Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7  Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

5.NF.4  Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a.  Interpret the product $(a/b) \times q$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*
Module Overview

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by fractions.2

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \((1/3) \div 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \(4 \div (1/5)\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \(4 \div (1/5) = 20\) because \(20 \times (1/5) = 4\).

Focus Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them. Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, “What is this problem asking me to find?” For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. They will confirm, for example, that \(10 \div 2\) can be found determining how many groups of two are in ten. They will apply that strategy to the division of fractions. Students may use pictorial representations such as area models, array models, number lines, and drawings to conceptualize and solve problems.

MP.2 Reason abstractly and quantitatively. Students make sense of quantities and their relationships in problems. They understand “how many” as it pertains to the divisor in a quotient of fractions problem. They understand and use connections between divisibility and the greatest common factor to apply the distributive property. Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students rely on estimation and properties of operations to justify the reason for their answers when manipulating decimal numbers and their operations. Students reason abstractly when applying place value and fraction sense when determining the placement of a decimal point.

MP.6 Attend to Precision. Students use precise language and place value when adding, subtracting, multiplying, and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, \(326.31\) is read as three hundred twenty-six and thirty-one hundredths. Students calculate sums, differences, products, and quotients of decimal numbers with a degree of precision appropriate to the problem context.

---

2 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement in Grade 5.
Module Overview

MP.7 Look for and make use of structure. Students find patterns and connections when multiplying and dividing multi-digit decimals. For instance, they use place value to recognize that the quotient of: $22.5 \div 0.15$, is the same as the quotient of: $2250 \div 15$. Students recognize that when expressing the sum of two whole numbers using the distributive property, for example: $36 + 48 = 12(3 + 4)$, the number 12 represents the greatest common factor of 36 and 48 and that 36 and 48 are both multiples of 12. When dividing fractions, students recognize and make use of a related multiplication problem or create a number line and use skip counting to determine the number of times the divisor is added to obtain the dividend. Students use the familiar structure of long division to find the greatest common factor in another way, specifically the Euclidean Algorithm.

MP.8 Look for and express regularity in repeated reasoning. Students determine reasonable answers to problems involving operations with decimals. Estimation skills and compatible numbers are used. For instance, when $24.385$ is divided by $3.91$, students determine that the answer will be close to the quotient of $24 \div 4$, which equals 6. Students discover, relate, and apply strategies when problem-solving, such as the use of the distributive property to solve a multiplication problem involving fractions and/or decimals (e.g., $350 \times 1.8 = 350(1 + 0.8) = 350 + 280 = 630$). When dividing fractions, students may use the following reasoning: Since $2/7 + 2/7 + 2/7 = 6/7$, then $6/7 \div 2/7 = 3$; and so I can solve fraction division problems by first getting common denominators and then solving the division problem created by the numerators. Students understand the long-division algorithm and the continual breakdown of the dividend into different place value units. Further, students use those repeated calculations and reasoning to determine the greatest common factor of two numbers using the Euclidean Algorithm.

Terminology

New or Recently Introduced Terms

- **Greatest Common Factor** (The largest quantity that factors evenly into two or more integers; the GCF of 24 and 36 is 12 because when all of the factors of 24 and 36 are listed, the largest factor they share is 12.)

- **Least Common Multiple** (The smallest quantity that is divisible by two or more given quantities without a remainder; the LCM of 4 and 6 is 12 because when the multiples of 4 and 6 are listed, the smallest or first multiple they share is 12.)

- **Multiplicative Inverses** (Two numbers whose product is 1 are multiplicative inverses of one another. For example, $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.)
Module Overview

Familiar Terms and Symbols

- Prime Number
- Composite Number
- Factors
- Multiples
- Dividend
- Divisor
- Reciprocal
- Algorithm
- Distributive Property
- Estimate

Suggested Tools and Representations

- Counters
- Fraction Tiles (example shown to the right)
- Tape Diagrams
- Area Models (example shown below)

Assessment Summary

<table>
<thead>
<tr>
<th>Assessment Type</th>
<th>Administered</th>
<th>Format</th>
<th>Standards Addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mid-Module Assessment Task</td>
<td>After Topic B</td>
<td>Constructed response with rubric</td>
<td>6.NS.1, 6.NS.3</td>
</tr>
<tr>
<td>End-of-Module Assessment Task</td>
<td>After Topic D</td>
<td>Constructed response with rubric</td>
<td>6.NS.1, 6.NS.2, 6.NS.3, 6.NS.4</td>
</tr>
</tbody>
</table>

3 These are terms and symbols students have seen previously.
In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. Students determine quotients through visual models, such as bar diagrams, tape diagrams, arrays, and number line diagrams. They construct division stories and solve word problems involving division of fractions (6.NS.1). Students understand and apply partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask themselves, “How much is one share?” and “What part of the unit is that share?” Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Later in the module, students will understand and apply the direct correlation of division of fractions to division of decimals.
Lesson 1: Interpreting Division of a Whole Number by a Fraction—Visual Models

Student Outcomes

- Students use visual models such as fraction bars, number lines, and area models to show the quotient of whole numbers and fractions. Students use the models to show the connection between those models and the multiplication of fractions.
- Students divide a fraction by a whole number.

Classwork

Opening Exercise (5 minutes)

At the beginning of class, hand each student a fraction card. Ask students to do the following:

- Draw a model of the fraction.
- Describe what the fraction means.

After about two minutes, have students share some of the models and descriptions. Emphasize the key point that a fraction shows division of the numerator by the denominator. In other words, a fraction shows a part being divided by a whole. Also remind students that fractions are numbers; therefore, they can be added, subtracted, multiplied, or divided.

To conclude the opening exercise, students can share where their fractions would be located on a number line. A number line can be drawn on a chalkboard or projected onto a board. Then students can describe how the fractions on the cards would be placed in order on the number line.

Example 1 (5 minutes)

This lesson will focus on fractions divided by whole numbers. Students learned how to divide unit fractions by whole numbers in 5th grade. Teachers can become familiar with what was taught in 5th grade on this topic by reviewing the materials used in the Grade 5, Module 4 lessons and assessments.

Scaffolding:

Each class should have a set of fraction tiles. Students who are struggling may benefit from using the fraction tiles to see the division until they are better at drawing the models.
Example 1

Maria has \( \frac{3}{4} \) lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?

*We are being asked to divide the trail mix into six equal portions. So we need to divide three-fourths by six.*

How can this question be modeled?

- Let’s take a look at how to solve this using a number line and a fraction bar.

We will start by creating a number line broken into fourths and a fraction bar broken into fourths.

![Number Line and Fraction Bar](image)

- We are going to give equal amounts of trail mix to each person. How can we show this in the model?
  - *We will divide the shaded portion so that it includes six equal-sized pieces.*

- How will we show this on the number line?
  - *There are three equal sections on the number line that also need to be divided into six equal shares.*

Next, we need to determine the unit. What did we do to each of the three sections in the fraction bar?

- *We divided them into two pieces.*

What should we do to the remaining piece of the fraction bar?

- *Divide it into two pieces.*

How many pieces are there total?

- 8 pieces
Lesson 1: Interpreting Division of a Whole Number by a Fraction—Visual Models

- What does each piece or section represent?
  - \( \frac{1}{8} \)

Therefore, \( \frac{3}{4} \div 6 = \frac{1}{8} \). This visual model also shows that \( \frac{1}{6} \) of \( \frac{3}{4} \) is \( \frac{1}{8} \).

- This is an example of partitive division. You can tell because we were given the original amount of trail mix and how many “parts” of trail mix to make. We needed to determine the size of each part, where the size of each part is less than the original amount.

Example 2 (5 minutes)

Example 2

Let’s look at a slightly different example. Imagine that you have \( \frac{2}{5} \) of a cup of frosting that you need to share equally between three desserts. How would we write this as a division question?

\( \frac{2}{5} \div 3 \)

We can start by drawing a model of two-fifths.

How can we show that we are dividing two-fifths into three equal parts?

\( \frac{1}{6} \)
What does this part represent?

From the visual model, we can determine that \( \frac{2}{5} \div 3 = \frac{2}{15} \).

Exercises 1–5 (15 minutes)

Students will work in pairs to solve the following questions.

Exercises 1–5

For each question below, rewrite the problem as a multiplication question. Then model the answer.

1. \( \frac{1}{2} \div 6 = \frac{1}{12} \)

\[ \frac{1}{2} \div 6 = \frac{1}{12} \]

I need to divide \( \frac{1}{2} \) into 6 equal sections. Or I need to rewrite the problem as \( \frac{1}{6} \) of \( \frac{1}{2} \).

2. \( \frac{1}{3} \div 3 = \frac{1}{9} \)

\[ \frac{1}{3} \div 3 = \frac{1}{9} \]

I need to divide \( \frac{1}{3} \) into 3 equal sections. Or I need to rewrite the problem as \( \frac{1}{3} \) of \( \frac{1}{3} \).

3. \( \frac{1}{5} \div 4 = \frac{1}{20} \)

\[ \frac{1}{5} \div 4 = \frac{1}{20} \]

I need to divide \( \frac{1}{5} \) into 4 equal sections. Or I need to rewrite the problem as \( \frac{1}{4} \) of \( \frac{1}{5} \).
Lesson 1: Interpreting Division of a Whole Number by a Fraction—Visual Models

Date: 9/16/13

4. \[ \frac{3}{5} \div 4 = \frac{3}{20} \]

I need to divide \( \frac{3}{5} \) into 4 equal sections. Or I need to rewrite the problem as \( \frac{1}{4} \) of \( \frac{3}{5} \).

5. \[ \frac{2}{3} \div 4 = \frac{2}{12} \text{ or } \frac{1}{6} \]

I need to divide \( \frac{2}{3} \) into 4 equal sections. Or I need to rewrite the problem as \( \frac{1}{4} \) of \( \frac{2}{3} \).

Closing (5 minutes)

- When a fraction is divided by a whole number, how does the answer compare with the dividend (the original fraction)?
  - Students should notice that the quotient is smaller than the original fraction.

Exit Ticket (5 minutes)
Lesson 1: Interpreting Division of a Whole Number by a Fraction—Visual Models

Exit Ticket

Find the quotient using a model.

1. \( \frac{2}{3} \div 3 \)

2. \( \frac{5}{6} \div 2 \)
Exit Ticket Sample Solutions

Solve each division problem using a model.

1. \( \frac{2}{3} \div \frac{2}{3} = \frac{2}{9} \)

2. \( \frac{5}{6} \div 2 = \frac{5}{12} \)
Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model the answer.

1. \( \frac{2}{5} \div 5 \)
   
   I need to find \( \frac{1}{5} \) of \( \frac{2}{5} \). I would get \( \frac{2}{25} \).

2. \( \frac{3}{4} \div 2 \)
   
   I need to find \( \frac{1}{2} \) of \( \frac{3}{4} \). I would get \( \frac{3}{8} \). 
Fraction Cards to use at the beginning of class:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>
Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Student Outcomes

- Students use visual models such as fraction bars, number lines, and area models to show the quotient of whole numbers and fractions. Students use the models to show the connection between those models and the multiplication of fractions.
- Students understand the difference between a whole number being divided by a fraction and a fraction being divided by a whole number.

Classwork

Example 1 (15 minutes)

At the beginning of class, break students into groups. Each group will need to answer the question it has been assigned and draw a model to represent its answer. Multiple groups could have the same question.

Group 1: How many half-miles are in 12 miles? $12 \div \frac{1}{2} = 24$

Group 2: How many quarter hours are in 5 hours? $5 \div \frac{1}{4} = 20$

Group 3: How many one-third cups are in 9 cups? $9 \div \frac{1}{3} = 27$

Group 4: How many one-eighth pizzas are in 4 pizzas? $4 \div \frac{1}{8} = 32$

Group 5: How many one-fifths are in 7 wholes? $7 \div \frac{1}{5} = 35$

Models will vary but could include fraction bars, number lines, or area models (arrays).

Students will draw models on blank paper, construction paper, or chart paper. Hang up only student models, and have students travel around the room answering the following:

1. Write the division question that was answered with each model.
2. What multiplication question could this model also answer?
3. Rewrite the question given to each group as a multiplication question.

Students will be given a table to fill in as they visit each model.

When discussing the opening, ask students how these questions are different from the questions solved in Lesson 1. Students should notice that these questions are dividing whole numbers by fractions where the questions in Lesson 1 were dividing fractions by whole numbers.

Discuss how the division problem is related to the multiplication problem.
Lesson 2
Interpreting Division of a Whole Number by a Fraction—Visual Models

Example 1

Question # ______
Write it as a division question. .................................................................
Write it as a multiplication question. .........................................................

Make a rough draft of a model to represent the question:

As you travel to each model, be sure to answer the following questions:

<table>
<thead>
<tr>
<th>Original Questions</th>
<th>Write the division question that was answered in each model.</th>
<th>What multiplication question could the model also answer?</th>
<th>Write the question given to each group as a multiplication question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How many $\frac{1}{2}$ miles are in 12 miles?</td>
<td>$12 \div \frac{1}{2}$</td>
<td>$12 \times 2 = ?$</td>
<td>Answers will vary.</td>
</tr>
<tr>
<td>2. How many quarter hours are in 5 hours?</td>
<td>$5 \div \frac{1}{4}$</td>
<td>$5 \times 4 = ?$</td>
<td></td>
</tr>
<tr>
<td>3. How many $\frac{1}{3}$ cups are in 9 cups?</td>
<td>$9 \div \frac{1}{3}$</td>
<td>$9 \times 3 = ?$</td>
<td></td>
</tr>
<tr>
<td>4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?</td>
<td>$4 \div \frac{1}{8}$</td>
<td>$4 \times 8 = ?$</td>
<td></td>
</tr>
<tr>
<td>5. How many one-fifths are in 7 wholes?</td>
<td>$7 \div \frac{1}{5}$</td>
<td>$7 \times 5 = ?$</td>
<td></td>
</tr>
</tbody>
</table>

Example 2 (5 minutes)

- All of the problems in the first example show what is called measurement division. When we know the original amount and the size or measure of one part, we use measurement division to find the number of parts. You can tell when a question is asking for measurement division because it asks, “How many _____ are in _________?”
Let’s take a look at a different example:

**Example 2**
Molly used 9 cups of flour to bake bread. If this was \(\frac{3}{4}\) of the total amount of flour she started with, what was the original amount of flour?

What is different about this question from the measurement questions?

- In this example, we are not trying to figure out how many three-fourths are in 9. We know that 9 cups is a part of the entire amount of flour needed. Instead, we need to determine three-fourths of what number is 9.

- **a.** Create a model to represent what the question is asking for.

  ![Model](image)

- **b.** Explain how you would determine the answer using the model.

  *To divide 9 by \(\frac{3}{4}\), we can divide 9 by 3 to get the amount for each rectangle and then multiply by 4 because there are 4 rectangles total.*

  \[9 \div 3 = 3 \quad 3 \times 4 = 12.\] Now, I can see that there were originally 12 cups of flour.

  ![Model](image)
Exercises 1–5 (15 minutes)

Students will work in pairs or on their own to solve the following questions. First, students will write a division sentence to represent the situations. Then students will rewrite each problem as a multiplication question. Finally, they will draw a model to represent the solution.

Allow time for students to share their models. Take time to have students compare the different models that were used to solve each question. For example, allow students to see how a fraction bar and a number line can be used to model Exercise 1.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Description</th>
</tr>
</thead>
</table>
| 1 | A construction company is setting up signs on 4 miles of the road. If the company places a sign every \( \frac{1}{8} \) of a mile, how many signs will it need? 

\[ 4 \div \frac{1}{8} \Rightarrow \frac{1}{8} \text{ of what number is } 4? \]

The company will need 32 signs. |
| 2 | George bought 12 pizzas for a birthday party. If each person will eat \( \frac{3}{8} \) of a pizza, how many people can George feed with 12 pizzas? 

\[ 12 \div \frac{3}{8} \Rightarrow \frac{3}{8} \text{ of what number is } 12? \]

The pizzas will feed 32 people. |
| 3 | The Lopez family has adopted 6 miles of trail on the Erie Canal. If each family member cleans up \( \frac{3}{4} \) of a mile, how many family members will be needed to clean the adopted section? 

\[ 6 \div \frac{3}{4} \Rightarrow \frac{3}{4} \text{ of what number is } 6? \]

The Lopez family will need to bring 8 family members to clean the adopted section. |
4. Margo is freezing 8 cups of strawberries. If this is \(\frac{2}{3}\) of the total strawberries that were picked, how many cups of strawberries did Margo pick?

\[
\begin{align*}
8 \\
\frac{4}{4} & \frac{4}{4} \\
? &= 12
\end{align*}
\]

*Margo picked 12 cups of strawberries.*

5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent \(\frac{5}{8}\) of all the logs that need to be chopped, how many logs need to be chopped in all?

\[
10 \div \frac{5}{8} \rightarrow \frac{5}{8} \text{ of what number is } 10?
\]

\[
\begin{align*}
? \\
2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 \\
10
\end{align*}
\]

*Regina needs to chop 16 logs in all.*

### Closing (5 minutes)

- What are the key ideas from Lessons 1 and 2?
- Over the past two lessons, we have reviewed how to divide a whole number by a fraction and how to divide a fraction by a whole number. The next two lessons will focus on dividing fractions by fractions. Explain how you would use what we have learned about dividing with fractions in the next two lessons.

### Exit Ticket (5 minutes)
Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Exit Ticket

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

2. Rachel completed $\frac{3}{4}$ of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?
Exit Ticket Sample Solutions

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

\[
\frac{4}{\frac{1}{6}} = \frac{1}{6} \text{ of what is 4?}
\]

\[
\begin{array}{cccccc}
4 & 8 & 12 & 16 & 20 & 24
\end{array}
\]

24 people are in the group.

2. Rachel completed \(\frac{3}{4}\) of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?

\[
6 \div \frac{3}{4}
\]

\[
\frac{3}{4} \text{ of what is 6?}
\]

\[
\begin{array}{cccc}
2 & 2 & 2 & 2
\end{array}
\]

Rachel will spend 8 total hours cleaning.
Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model the answer.

1. Nicole has used 6 feet of ribbon. This represents \( \frac{3}{8} \) of the total amount of ribbon she started with. How much ribbon did Nicole have at the start?

\[
6 \div \frac{3}{8} = \frac{3}{8} \text{ of what is } 6?
\]

Nicole started with 16 feet of ribbon.

2. How many quarter hours are in 5 hours?

\[
5 \div \frac{1}{4} = \frac{1}{4} \text{ of what is } 5?
\]

There are 20 quarter hours in 5 hours.
Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes
- Students use visual models such as fraction bars and area models to show the division of fractions by fractions with common denominators.
- Students make connections to the multiplication of fractions. In addition, students understand that the division of fractions require students to ask, “How many groups of the divisor are in the dividend?” to get the quotient.

Classwork
Opening Exercise (5 minutes)
Begin class with a review of how to divide a whole number by a whole number using a model.

Opening Exercise
Draw a model to represent $\frac{12}{3}$.

There are two interpretations:

1. 12 is divided into 3 equal parts.
2. 12 is divided into 4 equal parts.
How could we reword this question?

Answers will vary. Sample Solutions:
If we divide 12 into three groups of equal size, what is the size of each group?
If we divide 12 into groups of size 3, how many groups would we have?
If I have 12 chickens and I put 3 chickens in each cage, how many cages will I need?
If I have 12 flowers and I place 3 flowers in each vase, how many vases will I need?

Example 1 (5 minutes)

Next, we will introduce an example where students are asked to divide a fraction by a fraction with the same denominator. The whole number examples in the opening are used to give students ideas to build off of when dealing with fractions.

- What is $\frac{8}{9} \div \frac{2}{9}$? Take a moment to use what you know about division to create a model to represent this division problem. Give students a chance to explore this question and draw models without giving them the answer first. After three minutes or so, ask for students to share the models that they have created and to discuss what conclusions they have made about dividing fractions with the same denominator.
  - One way to interpret the question is to say how many $\frac{2}{9}$ are in $\frac{8}{9}$. From the model, I can see that there are 4 groups of $\frac{2}{9}$ in $\frac{8}{9}$. This would give the same solution as dividing 8 by 2 to get 4.

Example 1

$\frac{8}{9} \div \frac{2}{9}$

Draw a model to show the division problem.

Here we have 4 groups of $\frac{2}{9}$. Therefore, the answer is 4.
Example 2 (5 minutes)

Another example of fractions divided by fractions will help students see the connection between the two concepts.

- What is $\frac{9}{12} \div \frac{3}{12}$? Be sure to create a model to support your answer.
  
  - One way to interpret this question is by saying how many $\frac{3}{12}$ are in $\frac{9}{12}$. In other words, I need to divide nine twelfths by three twelfths, which is the same as $9$ units $\div 3$ units, which is $3$.

Example 2

$\frac{9}{12} \div \frac{3}{12}$

Be sure to draw a model to support your answer.

Example 3 (3 minutes)

- What is $\frac{7}{9} \div \frac{3}{9}$? Be sure to create a model to support your answer.
  
  - One way to interpret this question is by saying how many $\frac{3}{9}$ are in $\frac{7}{9}$. In other words, I need to divide seven ninths by three ninths, which is the same as $7$ units $\div 3$ units, which is $2 \frac{1}{3}$.

- Start by drawing a model in order to divide the two fractions.
  
  - $\frac{7}{9} \div \frac{3}{9}$
Lesson 3
Interpreting and Computing Division of a Fraction by a Fraction—More Models

Date: 9/16/13

Example 3

\[
\frac{7}{9} \div \frac{3}{9}
\]

Be sure to create a model to support your answer.

\[
\frac{7}{9}
\]

- How many whole \(\frac{3}{9}\) will go into \(\frac{7}{9}\)?
  - 2 whole and then part of a whole

- How do we represent the remainder?
  - There is one out of the three needed pieces to make another whole. So the remainder is \(\frac{1}{3}\).
  - This means that \(\frac{7}{9} \div \frac{3}{9} = \frac{2}{3}\).
  - This is the same as \(7 \div 3\).

Exercises 1–6 (20 minutes)

Students will work in pairs or alone to solve more questions about division with like denominators.

Exercises 1–6

For the following exercises, rewrite the division problem. Then be sure to draw a model to support your answer.

1. How many fourths are in three fourths?
   
   I need to divide three fourths by one fourth, which is 3.

   \[
   \frac{3}{4} \div \frac{1}{4}
   \]

   Draw a model to support your answer.

   [Model diagram showing \(\frac{3}{4}\) divided into \(\frac{1}{4}\) parts with 3 parts highlighted]

   There are 3 one-fourths in three-fourths.
Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Date: 9/16/13

How are Example 2 and Exercise 1 similar?

Both questions have a quotient of 3.

How are the divisors and dividends related?

\[
\frac{9}{12} \text{ is equivalent to } \frac{3}{4} \text{ and } \frac{3}{12} \text{ is equivalent to } \frac{1}{4}.
\]

\[
\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad \text{and} \quad \frac{1 \times 3}{4 \times 3} = \frac{3}{12}.
\]

What conclusions can you draw from these observations?

When the dividend and divisor are scaled up or scaled down by the same amounts, the quotient stays the same.

2. \[
\frac{4}{5} \div \frac{2}{5}
\]

This is really four fifths divided by two fifths, which is 2.

3. \[
\frac{9}{4} \div \frac{3}{4}
\]

This is really nine fourths divided by three fourths, which is 3.

4. \[
\frac{7}{8} \div \frac{2}{8}
\]

This is really seven eighths divided by two eighths, which is \(\frac{7}{2}\) or \(3 \frac{1}{2}\).
Lesson 3

Interpreting and Computing Division of a Fraction by a Fraction—More Models

### 5. \( \frac{13}{10} \div \frac{2}{10} \)

This is really thirteen sixths divided by two sixths, which is \( \frac{13}{2} \) or \( 6 \frac{1}{2} \).

### 6. \( \frac{11}{9} \div \frac{3}{9} \)

This is really eleven ninths divided by three ninths, which is \( \frac{11}{3} \) or \( 3 \frac{2}{3} \).

### Closing (5 minutes)

Depending on how much time you have in the class, you could have each student write to another student an actual note that contains models and a description of the ideas discussed in class. Or, if time is short, this can be a discussion.

- Imagine that your best friend missed today’s lesson. What key ideas would you want your friends to know in order to be able to divide fractions by fractions with the same denominator?

### Lesson Summary

When dividing a fraction by a fraction with the same denominator, we can use the general rule \( \frac{a}{c} \div \frac{b}{c} = \frac{a}{b} \).

### Exit Ticket (5 minutes)

- \( \frac{0}{9} \)
- \( \frac{1}{9} \)
- \( \frac{2}{9} \)
- \( \frac{3}{9} \)
- \( \frac{4}{9} \)
- \( \frac{5}{9} \)
- \( \frac{6}{9} \)
- \( \frac{7}{9} \)
- \( \frac{8}{9} \)
- \( \frac{9}{9} \)
- \( \frac{10}{9} \)
- \( \frac{11}{9} \)
Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Exit Ticket

Draw a model to support your answer to the division questions.

1. \[ \frac{9}{4} \div \frac{3}{4} \]

2. \[ \frac{7}{3} \div \frac{2}{3} \]
Exit Ticket Sample Solutions

Draw a model to support your answer to the division questions.

1. \( \frac{9}{4} \div \frac{3}{4} \)

   This is really nine fourths ÷ three fourths = 3.

   ![Model for \( \frac{9}{4} \div \frac{3}{4} \)]

2. \( \frac{7}{3} \div \frac{2}{3} \)

   This is really asking seven thirds ÷ two thirds, which is \( \frac{7}{2} \) or \( 3 \frac{1}{2} \).

   ![Model for \( \frac{7}{3} \div \frac{2}{3} \)]
Problem Set Sample Solutions

For the following exercises, rewrite the division problem. Then be sure to draw a model to support your answer.

1. \( \frac{15}{4} \div \frac{3}{4} \)

   *Fifteen fourths ÷ three fourths = 5.*

\[
\begin{array}{cccccccccccc}
\hline
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{3}{4} \\
\hline
\end{array}
\]

2. \( \frac{8}{5} \div \frac{3}{5} \)

   *Eight fifths ÷ three fifths = \( \frac{8}{3} \) or 2 \( \frac{2}{3} \).*

\[
\begin{array}{cccccccccccc}
\hline
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{3}{5} \\
\hline
\end{array}
\]
Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes

- Students use visual models such as fraction bars and area models to divide fractions by fractions with different denominators.
- Students make connections between visual models and multiplication of fractions.

Classwork

Opening Exercise (2 minutes)

Begin class with a review of equivalent fractions. Ask each student for a new example of an equivalent fraction. Students need to share how they know that the new fraction is equivalent to the old fraction.

Example 1 (3 minutes)

For the first example, students will be asked to solve a word problem using the skills they used in Lesson 3 to divide fractions with the same denominator.

- Molly purchased 1 3/8 cups of strawberries. This can also be represented as 11/8. She eats 2/8 cups in a serving. How many servings did Molly purchase?

  - This question is really asking me how many 2/8 are in 11/8 or, in other words, to divide eleven eighths by two eighths. I can use a model to show that there are 5 1/2 servings in the 11/8 cups of strawberries.
Example 1

Molly purchased \( \frac{11}{8} \) cups of strawberries. She eats \( \frac{2}{8} \) in a serving. How many servings did Molly purchase?

Use a model to prove your answer.

\[
\begin{align*}
1 &+ 1 + 1 + 1 + 1 + \frac{1}{2} = 5 \frac{1}{2}
\end{align*}
\]

Example 2 (3 minutes)

- Now imagine that Molly’s friend Xavier purchased \( \frac{11}{8} \) cups of strawberries and that he eats \( \frac{3}{4} \) cup servings. How many servings has he purchased?

  - He has purchased \( \frac{11}{6} \) servings or 1 and \( \frac{5}{6} \) servings. (This would be answered last after a brief discussion using the questions that follow.)

- What is this question asking us to do?

  - I am being asked to divide \( \frac{11}{8} \) cups into \( \frac{3}{4} \) cup servings.

- How does the problem differ from the first example?

  - The denominators are different.

- What are some possible ways that we could divide these two fractions?

  - I could change \( \frac{3}{4} \) to \( \frac{6}{8} \). These fractions are equivalent. I scaled up from \( \frac{3}{4} \) by multiplying the top and bottom by 2.
Example 2

Now imagine that Molly’s friend Xavier purchased $\frac{11}{8}$ cups of strawberries and that he eats $\frac{3}{4}$ cup servings. How many servings has he purchased? Use a model to prove your answer.

![Model Diagram]

There are 1 and $\frac{5}{6}$ servings.

Example 3 (3 minutes)

$\frac{3}{4} \div \frac{2}{3}$

What is this question asking us to do?
- $\frac{2}{3}$ of what is $\frac{3}{4}$ or how many $\frac{2}{3}$ are in $\frac{3}{4}$

Lead students through a brief discussion about this example:
- Is your answer larger or smaller than one? Why?
  - Since $\frac{2}{3}$ is less than $\frac{3}{4}$, we will have an answer that is larger than 1.
- Why is this question more difficult to model than the questions in Lesson 3?
  - The fractions do not have common denominators.
- How can we rewrite this question to make it easier to model?
  - We can create equivalent fractions with like denominators and then model and divide.
  - We can also think of this as $\frac{9}{12} \div \frac{8}{12}$ or nine twelfths divided by eight twelfths. 9 units ÷ 8 units = $\frac{9}{8}$ or 1 $\frac{1}{8}$ units.
Example 3

Find the quotient: $\frac{3}{4} \div \frac{2}{3}$. Use a model to show your answer.

Exercises 1–5 (20 minutes)

Students will work in pairs or alone to solve more questions about division of fractions with unlike denominators.

Exercises 1–5

A model should be included in your solution.

1. $\frac{6}{2} \div \frac{3}{4}$

   We could rewrite this problem to ask $\frac{12}{4} \div \frac{3}{4} = 4$.

   $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
2. \( \frac{2}{3} \div \frac{2}{5} \)

We could rewrite this problem to ask \( \frac{10}{15} \div \frac{6}{15} = \frac{10}{6} \) or \( 1 \frac{4}{6} \).

3. \( \frac{7}{8} \div \frac{1}{2} \)

We could rewrite this as \( \frac{7}{8} \div \frac{4}{8} = \frac{7}{4} \) or \( 1 \frac{3}{4} \).

4. \( \frac{3}{5} \div \frac{1}{4} \)

This can be rewritten as \( \frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2 \frac{2}{5} \).
5. \( \frac{5}{4} ÷ \frac{1}{3} \)

We can rewrite this as \( \frac{15}{12} ÷ \frac{4}{12} = \text{fifteen twelfths ÷ four twelfths} = \frac{15}{4} = 3 \frac{3}{4} \).

Closing (10 minutes)

- When dividing fractions, is it possible to get a whole number quotient?
- When dividing fractions, is it possible to get an answer that is larger than the dividend?
- When you are asked to divide two fractions with different denominators, what is one possible way to solve?

Exit Ticket (5 minutes)
Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Exit Ticket

Draw a model to support your answer to the division questions.

1. \( \frac{9}{4} \div \frac{3}{8} \)

2. \( \frac{3}{5} \div \frac{2}{3} \)
Exit Ticket Sample Solutions

Draw a model to support your answer to the division questions.

1. \( \frac{9}{4} ÷ \frac{3}{8} \)
   
   *This can be rewritten as* \( \frac{18}{8} ÷ \frac{3}{8} = \) eighteen eighths divided by three eighths = \( \frac{18}{3} = 6 \).

2. \( \frac{3}{5} ÷ \frac{2}{3} \)
   
   *This can be rewritten as* \( \frac{9}{15} ÷ \frac{10}{15} = \) nine fifteenths divided by ten fifteenths or 9 units ÷ 10 units.

   *So this is equal to* \( \frac{9}{10} \)
Problem Set Sample Solutions
The following problems can be used as extra practice or a homework assignment.

1. \( \frac{8}{9} \div \frac{4}{9} \)
   - Eight ninths divided by four ninths equals 2.
   - [Diagram showing fractions on a number line]

2. \( \frac{9}{10} \div \frac{4}{10} \)
   - Nine tenths divided by four tenths equals 2 1/4.
   - [Diagram showing fractions on a number line]

3. \( \frac{3}{5} \div \frac{1}{3} \)
   - Nine fifteenths divided by five fifteenths equals 4/5.
   - [Diagram showing fractions on a number line]
Lesson 4

Interpreting and Computing Division of a Fraction by a Fraction—More Models

4. \( \frac{3}{4} \div \frac{1}{5} \)

\( \frac{15}{20} \div \frac{4}{20} = \text{fifteen twentieths ÷ four twentieths} = \frac{15}{4} \)

\[
\begin{array}{cccccc}
1 & + & 1 & + & \frac{3}{4} & = 3 \frac{3}{4} \\
\frac{4}{4} & + & \frac{4}{4} & + & \frac{3}{4} & = \frac{15}{4}
\end{array}
\]
Lesson 5: Creating Division Stories

Student Outcomes

- Students demonstrate further understanding of division of fractions when they create their own word problems.
- Students choose a measurement division problem, draw a model, find the answer, choose a unit, and then set up a situation. Further, they discover that they must try several situations and units before finding which are realistic with given numbers.

Lesson Notes

During this lesson, students make sense of problems by creating them and persevere in solving them. They use concrete representations (fraction tiles or paper cutouts) and pictorial representations to show their understanding of the meaning of division and apply that understanding to the division of fractions.

There is a system for creating a division word problem. This sequence, which is noted in the second Student Outcome, is to be followed in order. Visual models showing examples of measurement division (e.g., \( \frac{1}{2} \div \frac{1}{8} \) or how many \( \frac{1}{8} \) are there in \( \frac{1}{2} \)) should be available, visible, and referenced throughout the lesson. These can be from previous lessons, student work, or from chart paper or board examples saved from Lessons 1-4 in this module. During the next lesson, partitive division problems (50 ÷ \( \frac{2}{3} \) 50 is \( \frac{2}{3} \) of what number?) will be the focus.

Before class, make a display of both types of problems (measurement and partitive).

It will be helpful to put the “5 Steps for Creating Division Word Problems” on chart paper posted around the room.

Through previous lessons, students gained experience solving problems involving division with fractions and creating the corresponding models. Students will now extend this work to choosing units and creating “division” stories. Pair up the students before the lesson, so they can collaborate on the exercises.

Classwork

Opening Exercises (5 minutes)

The classwork and problem sets from Lessons 1 through 4 include visual models made by the students. These will serve as the first three steps in writing division stories about the problems. These include fraction bars, number lines, and area models. Examples are shown here:
Opening Exercises

Fraction bar:

\[ \frac{8}{9} \div \frac{2}{9} \]

Here we have 4 groups of \(\frac{2}{9}\). Therefore, the answer is 4.

Number Line:

Molly’s friend Xavier purchased \(\frac{11}{8}\) cups of strawberries, and he eats \(\frac{3}{4}\) cup servings. How many servings has he purchased?

There are 1 and \(\frac{5}{6}\) servings.

Area Model:

\[ \frac{3}{5} \div \frac{1}{4} \]

This can be rewritten as \(\frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2 \frac{2}{5}\).
Use a few minutes to review the problem set from the last lesson. Ask students to label their answers to include fraction bars, number lines, and/or area models.

**Discussion (5 minutes)**

- When we know the original amount and the size or measure of one part, we use measurement division to find the number of parts. You can tell when a question is asking for measurement division because it asks, “How many _____ are in _________?”
- Writing division stories has five steps that are to be done in order. These steps are 1) decide on an interpretation (measurement or partitive), 2) draw a model, 3) find the answer, 4) choose a unit, and finally 5) set up a situation.
- Today we will only look at measurement division, and tomorrow we will go on to partitive division problems.
- By looking at all the work we have posted around the room, which of these steps are already done?
  - Steps 1, 2, and 3
- Today we will focus on extending our work involving division with fractions to choosing units and setting up a story situation.

**Example 1 (10 minutes)**

In this example, students reason abstractly and quantitatively. They make sense of quantities and their relationships in problems and understand “how many” as it pertains to the divisor when finding the quotients of fractions.

Encourage students to work each step in order.

- Let’s look at an example of measurement division: \( \frac{1}{2} \div \frac{1}{8} \) or how many \( \frac{1}{8} \) are there in \( \frac{1}{2} \)?
- Our first step in writing a story problem is to decide which division interpretation to use. Do we need to decide this, or is it already done for us here?

**Example 1**

\[
\begin{align*}
\frac{1}{2} & \div \frac{1}{8} \\
\text{Step 1: Decide on an interpretation. (given: measurement model)}
\end{align*}
\]

- Our second step is to sketch out a model. This should be done neatly and fairly accurately but should not take too long to do. Use a tape diagram or fraction bar to model this problem on your paper.

**Step 2: Draw a model.**

[Diagram of a tape diagram or fraction bar is not provided in the text but would typically be included in the printed version.]
Lesson 5

Creating Division Stories

Date: 9/16/13

Note: This drawing can be cut and pasted onto an interactive whiteboard document and then labeled.

The third step is to find the answer to the problem. Do this on your paper.

Step 3: Find the answer.
\[
\frac{1}{2} \div \frac{1}{8} = \frac{4}{1} = 4
\]

So the answer is 4. There are four \(\frac{1}{8}\) in \(\frac{1}{2}\).

Step 4: Choose a unit.
Step 5: Set up a situation.

Now that we have the answer, we can move on to the fourth step, choosing a unit. For measurement division, both divisor and dividend must be the same unit.

Note: Choosing the unit and using it in both the divisor and dividend consistently preserves the story situation clearly and precisely. With enough repetition, students will learn to interpret and write division story problems more clearly.

- Let’s use pounds for this example. We are asking how many \(\frac{1}{8}\) pounds are there in \(\frac{1}{2}\) pound.
- Step 5 is to set up a situation. This means writing a story problem that is interesting, realistic, short, and clear and that has all the information necessary to solve it. It may take you several attempts before you find a story that works well with the given dividend and divisor.
- One story problem that might go well with these numbers is the following: Bonnie Baker has a total of \(\frac{1}{2}\) pound of chocolate. She needs \(\frac{1}{8}\) pound of chocolate for each batch of brownies she bakes. How many batches of brownies can Bonnie bake with \(\frac{1}{2}\) pound of chocolate?
Exercise 1 (5 minutes)

Allow students to work with a partner to create the story problem. Also take time for sharing and discussion of their work.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

Possible story problems:

1. Tina uses \( \frac{1}{8} \) oz. of cinnamon each time she makes a batch of coffee cake topping. How many batches can she make if she has \( \frac{1}{2} \) oz. left in her spice jar?

2. Eugenia has \( \frac{1}{2} \) yard of ribbon. For each party decoration, she needs \( \frac{1}{8} \) yard. How many party decorations can she make?

Example 2 (5 minutes)

Let’s look at another example of measurement division: \( \frac{3}{4} \div \frac{1}{2} \) or how many \( \frac{1}{2} \) are there in \( \frac{3}{4} \)?

Our first step in writing a story problem is to decide which division interpretation to use. Do we need to decide this, or is it already done for us here?

Example 2

\( \frac{3}{4} \div \frac{1}{2} \)

Step 1: Decide on an interpretation. (given: measurement model)

Our second step is to sketch out a model. This should be done neatly and fairly accurately but should not take too long to do. Use a tape diagram to model this problem on your paper.

Step 2: Draw a diagram.
Lesson 5: Creating Division Stories

Step 3: Find the answer.

\[
\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2} = 1\frac{1}{2}
\]

So the answer is \(1\frac{1}{2}\). There are \(1\frac{1}{2}\) halves in \(\frac{3}{4}\).

Step 4: Choose a unit.

Step 5: Set up a situation.

- Step 4 is to choose a unit. Let’s choose ounces.
- Step 5 is to set up a situation. This means writing a story problem that is interesting, realistic, short, and clear and that has all the information necessary to solve it. It may take you several attempts before you find a story that works well with the given dividend and divisor.
- One story problem that might go well with these numbers is the following: Tia has \(\frac{3}{4}\) oz. of coffee left in her coffee can. She needs \(\frac{1}{2}\) oz. to make a cup of coffee. How many cups of coffee can she make?

Exercise 2 (5 minutes)

Allow students to work with a partner to create the story problem. Also take time for sharing and discussion of their work.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

Possible story problems:

1. Tiffany uses \(\frac{1}{2}\) oz. of glycerin each time she makes a batch of soap bubble mixture. How many batches can she make if she has \(\frac{3}{4}\) oz. left in her glycerin bottle?

2. Theresa has \(\frac{3}{4}\) yard of fabric. For each doll skirt she makes, she needs \(\frac{1}{2}\) yard. Does she have enough fabric to make 2 doll skirts?

Closing (5 minutes)

- How did we extend our work with division with fractions in this lesson?
  - We took an answer to a problem, added a unit, and then thought of a story problem that would fit it.
- What were your biggest challenges when writing story problems involving division with fractions?
  - Accept all answers.
Lesson Summary

The method of creating division stories has five steps, to be followed in order:
Step 1: Decide on an interpretation (measurement or partitive). Today we used only measurement division.
Step 2: Draw a model.
Step 3: Find the answer.
Step 4: Choose a unit.
Step 5: Set up a situation. This means writing a story problem that is interesting, realistic, short, and clear and that has all the information necessary to solve it. It may take you several attempts before you find a story that works well with the given dividend and divisor.

Exit Ticket (3 minutes)
Lesson 5: Creating Division Stories

Exit Ticket

Write a story problem for the following measurement division: \( \frac{3}{4} \div \frac{1}{8} = 6 \).

\[
\begin{array}{c|c|c|c}
1 & 1/4 & 1/4 & 1/4 \\
1/8 & 1/8 & 1/8 & 1/8 \\
1/8 & 1/8 & 1/8 & 1/8 \\
\end{array}
\]
Exit Ticket Sample Solution

Write a story problem for the following measurement division: \( \frac{3}{4} \div \frac{1}{8} = 6 \).

Arthur divided \( \frac{3}{4} \) of his kingdom into parcels of land, each being \( \frac{1}{8} \) of the entire kingdom. How many parcels did he make? (Or any other reasonable story problem showing \( \frac{3}{4} \div \frac{1}{8} = 6 \)).

Problem Set Sample Solutions

Please use each of the five steps of the process you learned. Label each step.

1. Write a measurement division story problem for \( 6 \div \frac{3}{4} \).

How many \( \frac{3}{4} \) are there in \( 6 \)?

Rafael had a 6 foot piece of wood. He had to cut shelves that were \( \frac{3}{4} \) foot long. How many shelves could he cut from the 6 foot board?
2. Write a measurement division story problem for \( \frac{5}{12} \div \frac{1}{6} \).

There are 12 inches in a foot. A piece of wire is 5 inches (\( \frac{5}{12} \) foot) long. Hector needs to cut pieces of wire that are 2 inches (\( \frac{1}{6} \) foot) long. How many can he cut? 2 \( \frac{1}{2} \).
Lesson 6: More Division Stories

Student Outcomes

- Students demonstrate further understanding of division of fractions when they create their own word problems.
- Students choose a partitive division problem, draw a model, find the answer, choose a unit, and then set up a situation. Further, they practice trying several situations and units before finding which are realistic with given numbers.

Lesson Notes

This lesson is a continuation of Lesson 5 and focuses on asking students to write fraction division story problems that are partitive in nature.

Classwork

Opening (5 minutes)

Use a few minutes for students to share the division stories they wrote for the previous lesson’s problem set. Clarify any misconceptions that surface regarding the process of creating story problems when using measurement division.

Discussion (5 minutes)

- Partitive division is another interpretation of division problems. What do you recall about partitive division?
  - We know that when we divide a whole number by a fraction, the quotient will be greater than the whole number we began with (the dividend). This is true regardless of whether we use a partitive approach or a measurement approach.
  - In other cases, we know what the whole is and how many groups we are making and must figure out what size the groups are.

Scaffolding:
Paper fraction tile strips can be pre-cut for students who have difficulty making accurate sketches.
Example 1 (10 minutes)

- Today we will work with Partitive Division.
- Step 1: Let’s use an example that uses partitive division: \(50 \div \frac{2}{3}\).

**Example 1**
Divide \(50 \div \frac{2}{3}\)
Step 1: Decide on an interpretation. *(given: partitive model)*

- Step 2 is to draw a model. How many equal size rectangles do we need?
  - \(3\)
- How do you know?
  - *The denominator of the fraction tells us we are using thirds. We need 3 rectangles.*

**Step 2: Draw a model.**

- The 50 accounts for how many of those 3 rectangles?
  - *Two because the numerator tells us how many thirds that the 50 represents.*
- So the 50 must be spread out evenly between two thirds. How much would be in each box?
  - \(25\)

**Step 2: Draw a model.**

- Step 3 is to find the answer. \(50 \div \frac{2}{3}\) means 50 is \(\frac{2}{3}\) of some number that is greater than 50. By looking at our tape diagram, we can see that \(25 = \frac{1}{3}\) of the number.

**Step 3: Find the answer.**

By looking at our tape diagram, \(50 \div \frac{2}{3} = 25 \cdot 3 = 75\).

Step 4: Choose a unit.
Step 5: Set up a situation.
Now that we have the answer, we can move on to the fourth step, choosing a unit. Let’s choose dollars.

Step 5 is to set up a situation. Remember that this means writing a story problem that is interesting, realistic, short, and clear and that has all the information necessary to solve it. It may take you several attempts before you find a story that works well with the given dividend and divisor.

Spending money gives a “before and after” word problem. We are looking for a situation where \( \frac{2}{3} \) of some greater dollar amount is $50.

One story problem that might go well with these numbers is the following: Adam spent $50 on a new graphing calculator. This was \( \frac{2}{3} \) of his money. How much money did he start with?

Exercise 1 (5 minutes)
Allow students to work with a partner to create the story problem. Also take time for sharing and discussion of their work.

Exercise 1
Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, dollars, but your situation must be unique. You could try another unit, such as miles, if you prefer.

Possible story problems:
1. Ronaldo has ridden 50 miles during his bicycle race and is \( \frac{2}{3} \) of the way to the finish line. How long is the race?
2. Samantha used 50 tickets (\( \frac{2}{3} \) of her total) to trade for a kewpie doll at the fair. How many tickets did she start with?

Example 2 (10 minutes)

Step 1: Let’s use an example that uses partitive division: \( 45 \div \frac{3}{8} \).

Example 2
Divide \( 45 \div \frac{3}{8} \)
Step 1: Decide on an interpretation. (given: partitive model)

Step 2 is to draw a model. How many equal size rectangles do we need?

- 8

How do you know?

- The denominator of the fraction tells us we are using eighths. We need 8 rectangles.

Step 2: Draw a model.
- The 45 accounts for how many of those 8 rectangles?
  - 3 because the numerator tells us how many eighths that the 45 represents.

- So the 45 must be spread out evenly between three eighths. How much would be in each box?
  - 15

**Step 2: Draw a model.**

```
45

15 15 15

?```

- Step 3 is to find the answer. $45 \div \frac{3}{8}$ means 45 is $\frac{3}{8}$ of some number that is greater than 45. By looking at our tape diagram, we can see that $15 = \frac{1}{8}$ of the number.

**Step 3: Find the answer.**

By looking at our tape diagram, $45 \div \frac{3}{8} = 15 \cdot 8 = 120$.

**Step 4: Choose a unit.**

**Step 5: Set up a situation.**

- Now that we have the answer, we can move on to the fourth step, choosing a unit. Let’s choose carnival prize tickets.
- Step 5 is to set up a situation. Remember that this means writing a story problem that is interesting, realistic, short, and clear and that has all the information necessary to solve it. It may take you several attempts before you find a story that works well with the given dividend and divisor.
- One story problem that might go well with these numbers is the following: Scott gave away 45 carnival prize tickets to his niece. This was $\frac{3}{8}$ of his tickets. How many tickets did he start with?
Exercise 2 (5 minutes)

Allow students to work with a partner to create the story problem. Also take time for sharing and discussion of their work.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. Try a different unit. Remember spending money gives a “before and after” word problem. If you use dollars, you are looking for a situation where \( \frac{3}{8} \) of some greater dollar amount is $45.

Possible story problems:

1. Daryl has been on a diet for 45 days and is \( \frac{3}{8} \) of the way to the end of the diet program. How long is the program?

2. Amy collected 45 Box Tops for Education which is \( \frac{3}{8} \) of her goal. What is the total number she is trying to collect?

Closing (2 minutes)

- How did we extend our work with division with fractions in this lesson?
  - We took an answer to a problem, added a unit, and then thought of a story problem that would fit it.
- What were your biggest challenges when writing story problems involving division with fractions?
  - Accept all answers.

Exit Ticket (2 minutes)
Lesson 6: More Division Stories

Exit Ticket

Write a word problem for the following partitive division: \( 25 \div \frac{5}{8} = 40 \).
Exit Ticket Sample Solution

Write a word problem for the following partitive division: \( 25 \div \frac{5}{8} = 40 \).

Zolanda spent \( \frac{5}{8} \) of her class period, or 25 minutes, taking notes. How long was the class period? (Or any other reasonable story problem showing \( 25 \div \frac{5}{8} = 40 \)).

Problem Set Sample Solutions

1. Write a partitive division story problem for \( 45 \div \frac{3}{5} \).

Samantha Maria spent \( \frac{3}{5} \), or $45, on a pair of earrings. How much money did she have before she spent any?
2. Write a partitive division story problem for \( \frac{100}{\frac{2}{5}} \).

There are 100 girls in the college marching band, which is \( \frac{2}{5} \) of the total. How many members are there all together in the band?
Lesson 7: The Relationship Between Visual Fraction Models and Equations

Student Outcomes

- Students formally connect models of fractions to multiplication through the use of multiplicative inverses as they are represented in models.

Lesson Notes

Using pre-cut fraction strips saves time and assures more accurate models. Fraction strips are found at the end of this document and should be reproduced and cut prior to the lesson. With each example that students work through, the concept will be reinforced: dividing by a fraction yields the same result as multiplying by the inverse of that fraction.

The terms inverse and reciprocal should be defined and displayed in the classroom, perhaps as part of a Word Wall.

The reciprocal, or inverse, of a fraction is the fraction made by interchanging the numerator and denominator.

\[
\begin{align*}
\frac{2}{3} &\quad \rightarrow \quad \frac{3}{2} \\
\frac{5}{8} &\quad \rightarrow \quad \frac{8}{5} \\
\frac{1}{4} &\quad \rightarrow \quad \frac{4}{1}
\end{align*}
\]

Classwork

Opening (2 minutes)

Introduce the definition of the term Multiplicative Inverses: Two numbers whose product is 1 are multiplicative inverses of one another. For example, \(\frac{3}{4}\) and \(\frac{4}{3}\) are multiplicative inverses of one another because \(\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1\).

Point out and ask students to write several of their own examples of multiplicative inverses. The general form of the concept \(\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1\) should also be displayed in several places throughout the classroom.

- We already know how to make tape diagrams to model division problems. Today we will use fraction tiles or fraction strips to model dividing fractions. This will help you understand this concept more completely.

During this lesson, students continue to make sense of problems and persevere in solving them. They use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, “What is this problem asking me to find?” For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem.
Example 1 (15 minutes)

Consider the following, an example that we have worked with in previous lessons:

Example 1
\[
\begin{array}{c}
\frac{3}{4} \div \frac{2}{5} \\
\frac{2}{5} \text{ of what number is } \frac{3}{4} = \frac{3}{4} \\
\end{array}
\]

Shade 2 of the 5 sections \(\frac{2}{5}\).
Label the part that is known \(\frac{3}{4}\).

Make notes below on the math sentences needed to solve the problem.

- How did we choose which fraction strip to use?
  - We are dividing by \(\frac{2}{5}\). There is a 5 in the denominator, which tells us what kind of fraction, so we chose fifths.
- How did we know to shade in two sections?
  - There is a 2 in the numerator. That tells us how many of the fifths to shade.
- How did we know to put the \(\frac{3}{4}\) in the brace that shows \(\frac{2}{5}\)?
  - That is the part we know.
- What about the bottom brace?
  - That is unknown right now and is ready to be calculated.
- Think about this problem like this:
  - 2 units = \(\frac{3}{4}\).
  - 1 unit is half of \(\frac{3}{4} = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}\).
  - 5 units = \(\frac{3}{8} \cdot 5 = \frac{15}{8}\).
  - So \(\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}\).

We have the answer to our initial question, but perhaps there is another way to solve the problem. Do you think it is possible to solve this problem without using division?
Extension: Allow time for students to inspect the problem and offer conjectures using the reciprocal of the divisor. Prompt them, if necessary, to consider another operation (multiplication). The following is a series of steps to explain how “invert and multiply” is justified.

- Using the Any-Order Property, we can say the following:
  \[
  \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{5} = \frac{3}{4} \cdot \left(\frac{1}{2} \cdot 5\right) = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}.
  \]
- So \(\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}\) and \(\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}\).
- Therefore, \(\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}\).
- This method for dividing fractions by fractions is called “invert and multiply”. Dividing by a fraction is the same as multiplying by its inverse. It is important to invert the second fraction (the divisor) and not the first fraction (the dividend).

Exercise 1 (10 minutes)

During this part of the lesson, students look for and make use of structure. They continue to find patterns and connections when dividing fractions, and they recognize and make use of a related multiplication problem to determine the number of times the divisor is added to obtain the dividend.

- Does this always work? Let’s find out using the example \(\frac{1}{4} \div \frac{2}{3}\).
- How did we choose which fraction strip to use?
  - We are dividing by \(\frac{2}{3}\). There is a 3 in the denominator, which tells us what kind of fraction, so we chose thirds.
- How did we know to shade in two sections?
  - There is a 2 in the numerator. That tells us how many of the thirds to shade.
- How did we know to put the \(\frac{1}{4}\) in the brace that shows \(\frac{2}{3}\)?
  - That is the part we know.
- What about the bottom brace?
  - That is unknown right now and is ready to be calculated.
- Think about this problem like this:
  - 2 units = \(\frac{1}{4}\).
  - 1 unit is half of \(\frac{1}{4}\). \(\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}\).
  - 3 units = \(\frac{3}{8}\).
Exercise 1
\[
\frac{1}{4} \div \frac{2}{3}
\]

Show the number sentences below.
\[
\frac{1}{4} \times \frac{2}{3} = \frac{2}{8} = \frac{8}{12} = \frac{8}{12}
\]
\[
\frac{1}{3} \div \frac{3}{2} = \frac{3}{8}
\]

Therefore, \( \frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2} \).

Exercise 2 (5 minutes)
Ask students to solve the following problem with both a tape diagram and the “invert and multiply” rule. They should compare the answers obtained from both methods and find them to be the same.

Exercise 2
\[
\frac{2}{3} \div \frac{3}{4}
\]

Show the number sentences below.
\[
\frac{2}{3} \div \frac{3}{4} = \frac{2}{9} = \frac{8}{12} = \frac{8}{12}
\]
\[
\frac{2}{3} \div \frac{3}{4} = \frac{2}{9} = \frac{8}{12} = \frac{8}{12}
\]

?
Lesson 7

The Relationship Between Visual Fraction Models and Equations

1. How did we choose which fraction strip to use?
   - We are dividing by $\frac{3}{4}$. There is a 4 in the denominator, which tells us what kind of fraction, so we chose fourths.

2. How did we know to shade in three sections?
   - There is a 3 in the numerator. That tells us how many of the fourths to shade.

3. How did we know to put the $\frac{2}{3}$ in the brace that shows $\frac{3}{4}$?
   - That is the part we know.

4. What about the bottom brace?
   - That is unknown right now and is ready to be calculated.

5. Think about this problem like this:
   - 3 units = $\frac{2}{3}$.
   - 1 unit is one third of $\frac{2}{3}$: $\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$.
   - 4 units = $\frac{2}{9} \cdot 4 = \frac{8}{9}$.

Closing (3 minutes)

- Dividing by a fraction is equivalent to multiplying by its reciprocal, or inverse. Connecting the models of division by a fraction to multiplication by its inverse strengthens your understanding.

Lesson Summary

Connecting models of fraction division to multiplication through the use of reciprocals helps in understanding the “invert and multiply” rule.

Exit Ticket (3 minutes)
Lesson 7: The Relationship Between Visual Fraction Models and Equations

Exit Ticket

1. Write the reciprocal of the following numbers:

<table>
<thead>
<tr>
<th>Number</th>
<th>$\frac{7}{10}$</th>
<th>$\frac{1}{2}$</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocal</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Rewrite this division problem as a multiplication problem: $\frac{5}{8} ÷ \frac{2}{3}$.

3. Solve the problem 2 using models.
Exit Ticket Sample Solutions

1. Write the reciprocal of the following numbers:

<table>
<thead>
<tr>
<th>Number</th>
<th>( \frac{7}{10} )</th>
<th>( \frac{1}{2} )</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocal</td>
<td>( \frac{10}{7} )</td>
<td>2</td>
<td>( \frac{1}{5} )</td>
</tr>
</tbody>
</table>

2. Rewrite this division problem as a multiplication problem: \( \frac{5}{8} ÷ \frac{2}{3} \).
   \( \frac{5}{8} ÷ \frac{2}{3} = \frac{5}{8} \cdot \frac{3}{2} \).

3. Solve the problem in #2 using models.

\[ \frac{5}{8} \]

\[ \frac{5}{16} \]

\[ \frac{5}{16} \]

\[ \frac{5}{16} \]

\[ \frac{15}{16} \]

**Answer:** \( \frac{15}{16} \)

\( \frac{5}{8} ÷ \frac{2}{3} = \frac{5}{8} \cdot \frac{3}{2} = \frac{15}{16} \)
Problem Set Sample Solutions

1. Draw a model that shows $\frac{2}{5} + \frac{1}{3}$. Find the answer as well.

   Answer: $\frac{6}{5}$

   $\frac{2}{5} + \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$

2. Draw a model that shows $\frac{3}{4} - \frac{1}{2}$. Find the answer as well.

   Answer: $\frac{6}{4}$

   $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4}$
### Lesson 7: The Relationship Between Visual Fraction Models and Equations

**Date:** 9/16/13

- **1 whole unit**

<table>
<thead>
<tr>
<th></th>
<th>1/2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td></td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td>1/4</td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td>1/5</td>
<td>1/5</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
<td></td>
<td>1/6</td>
</tr>
<tr>
<td>1/8</td>
<td>1/8</td>
<td></td>
<td>1/8</td>
</tr>
<tr>
<td>1/9</td>
<td></td>
<td>1/9</td>
<td>1/9</td>
</tr>
<tr>
<td>1/10</td>
<td></td>
<td>1/10</td>
<td></td>
</tr>
<tr>
<td>1/12</td>
<td></td>
<td>1/12</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 8: Dividing Fractions and Mixed Numbers

Student Outcomes

- Students divide fractions by mixed numbers by first converting the mixed numbers into a fraction with a value larger than one.
- Students use equations to find quotients.

Lesson Notes

There is some mandatory prep work before teaching the lesson. The memory game that is included in the lesson needs to be cut and prepared for pairs or individual students.

Classwork

Example 1 (12 minutes): Introduction to Calculating the Quotient of a Mixed Number and a Fraction

- Carli has $4 \frac{1}{2}$ walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has $\frac{5}{6}$ of a gallon left. How much paint can she use on each wall in order to have enough to paint the remaining walls?
- In order to solve the word problem, we must calculate the quotient of $\frac{5}{6} \div 4 \frac{1}{2}$.

Before dividing, discuss how the answer must be less than one because you are dividing a smaller number by a larger number. Estimation could also be used to emphasize this point: $1 \div 5 = \frac{1}{5}$.

Explain that the mixed number must be converted into a fraction with a value larger than one. You may also emphasize that converting mixed numbers to fractions with a value larger than one is important for different division strategies. Have students complete this conversion on their own and then share the process they followed.

Remind students about the formula they learned in the previous lesson and have them attempt to solve the problem. Have students show the process used to find the quotient.
Example 1: Introduction to Calculating the Quotient of a Mixed Number and a Fraction

a. Carli has \(4 \frac{1}{2}\) walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has \(\frac{5}{6}\) of a gallon left. How much paint can she use on each wall in order to have enough to paint the remaining walls?

\[
\text{Calculate the quotient: } \frac{5}{6} \div 4 \frac{1}{2}
\]

\[
\text{Convert into the fraction: } \frac{9}{2}
\]

\[
\text{Divide fractions: } \frac{5}{6} \div \frac{9}{2} = \frac{5 \times 2}{6 \times 9} = \frac{10}{54} = \frac{5}{27}
\]

Carli can use \(\frac{5}{27}\) of a gallon of paint on each of the remaining walls.

- Calculate the quotient. \(\frac{2}{5} \div 3 \frac{4}{7}\)

Students solve this problem individually as the teacher walks around checking for understanding. Students then share their answers and processes used to find the quotients. Provide time for students to ask questions.

b. Calculate the quotient.

\[
\frac{2}{5} \div 3 \frac{4}{7}
\]

\[
\text{Convert into the fraction: } \frac{25}{7}
\]

\[
\text{Divide fractions: } \frac{2}{5} \div \frac{25}{7} = \frac{2 \times 7}{5 \times 25} = \frac{14}{125}
\]

Exercise 1 (25 minutes)

Students complete the memory game individually or in small groups.

The following are some important directions for the teacher to share with students:

- The goal is to match each expression with the equivalent quotient.
- Each expression is assigned a letter. Students must show their work in that box on the student materials. Once they have solved a particular problem, they will not have to solve it again if they record their work in the correct place.
- Students are able to flip two cards over during each turn. If the expression and the quotient do not match, both cards should be flipped back over. If the expression and quotient do match, the student keeps the matches to determine a winner at the end.

If students are not clear about the directions, the teacher may choose two or three problems for students to do as a class to show how the memory game works.
### Dividing Fractions and Mixed Numbers

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.</strong></td>
<td>( \frac{3}{4} \div \frac{2}{3} )</td>
<td>( \frac{9}{80} )</td>
</tr>
<tr>
<td><strong>B.</strong></td>
<td>( \frac{1}{3} \div \frac{3}{4} )</td>
<td>( \frac{4}{57} )</td>
</tr>
<tr>
<td><strong>C.</strong></td>
<td>( \frac{2}{5} \div \frac{17}{8} )</td>
<td>( \frac{16}{75} )</td>
</tr>
<tr>
<td><strong>D.</strong></td>
<td>( \frac{7}{2} \div \frac{5}{6} )</td>
<td>9</td>
</tr>
<tr>
<td><strong>E.</strong></td>
<td>( \frac{4}{7} \div \frac{5}{8} )</td>
<td>( \frac{5}{7} )</td>
</tr>
<tr>
<td><strong>F.</strong></td>
<td>( \frac{5}{8} \div \frac{9}{10} )</td>
<td>( \frac{6}{4} )</td>
</tr>
<tr>
<td><strong>G.</strong></td>
<td>( \frac{1}{4} \div \frac{10}{12} )</td>
<td>( \frac{3}{131} )</td>
</tr>
<tr>
<td><strong>H.</strong></td>
<td>( \frac{5}{4} \div \frac{5}{9} )</td>
<td>( \frac{10}{7} )</td>
</tr>
<tr>
<td><strong>I.</strong></td>
<td>( \frac{3}{5} \div \frac{2}{3} )</td>
<td>( \frac{5}{5} )</td>
</tr>
<tr>
<td><strong>J.</strong></td>
<td>( \frac{3}{5} \div \frac{3}{7} )</td>
<td>( \frac{21}{110} )</td>
</tr>
<tr>
<td><strong>K.</strong></td>
<td>( \frac{10}{13} \div \frac{4}{7} )</td>
<td>( \frac{35}{117} )</td>
</tr>
<tr>
<td><strong>L.</strong></td>
<td>( \frac{2}{4} \div \frac{7}{8} )</td>
<td>( \frac{2}{7} )</td>
</tr>
</tbody>
</table>
Lesson 8
Dividing Fractions and Mixed Numbers

Exercise 1
Show your work for the memory game in the boxes provided below.

| A. | $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$ |
| B. | $\frac{1}{3} \div \frac{4}{3} = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4} \times \frac{3}{19} = \frac{3}{57}$ |
| C. | $\frac{2}{5} \div \frac{7}{8} = \frac{2}{5} \times \frac{8}{7} = \frac{16}{35}$ |
| D. | $\frac{7}{2} \div 5 = \frac{7}{2} \div \frac{5}{2} = \frac{7}{5}$ |
| E. | $4 \div \frac{5}{8} = 4 \times \frac{8}{5} = \frac{32}{5}$ |
| F. | $\frac{5}{8} \div \frac{9}{10} = \frac{5}{8} \times \frac{10}{9} = \frac{25}{18} \div \frac{5}{3} = \frac{25}{18} \times \frac{3}{5} = \frac{5}{3}$ |
| G. | $\frac{1}{4} \div 11 = \frac{1}{4} \times \frac{1}{11} = \frac{1}{44}$ |
| H. | $\frac{3}{4} \div \frac{9}{5} = \frac{3}{4} \times \frac{5}{9} = \frac{15}{36} \div \frac{5}{2} = \frac{15}{36} \times \frac{2}{5} = \frac{20}{45}$ |
| I. | $\frac{1}{5} \div 2 = \frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$ |
| J. | $\frac{3}{5} \div \frac{3}{2} = \frac{3}{5} \times \frac{2}{3} = \frac{6}{15} \div \frac{7}{31} = \frac{6}{15} \times \frac{31}{7} = \frac{186}{105}$ |
| K. | $10 \div \frac{4}{7} = 10 \times \frac{7}{4} = \frac{70}{4} \div \frac{10}{7} = \frac{70}{4} \times \frac{7}{10} = \frac{49}{2}$ |
| L. | $\frac{2}{5} \div \frac{7}{8} = \frac{2}{5} \times \frac{8}{7} = \frac{16}{35}$ |

Closing (4 minutes)

- How does the process of dividing a fraction by a mixed number compare with our previous work with division of fractions? Discuss similarities and differences.

Exit Ticket (4 minutes)
Lesson 8: Dividing Fractions and Mixed Numbers

Exit Ticket

Calculate the quotient.

1. \( \frac{3}{4} \div 5 \frac{1}{5} \)

2. \( \frac{3}{7} \div 2 \frac{1}{2} \)

3. \( \frac{5}{8} \div 6 \frac{5}{6} \)

4. \( \frac{5}{8} \div 8 \frac{3}{10} \)
Exit Ticket Sample Solutions

Calculate the quotient.

1. \[\frac{3}{4} ÷ \frac{5}{1} = \frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20} = \frac{15}{104}\]

2. \[\frac{3}{7} ÷ \frac{2}{1} = \frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}\]

3. \[\frac{5}{8} ÷ \frac{6}{1} = \frac{5}{8} \times \frac{1}{6} = \frac{5 \times 1}{8 \times 6} = \frac{5}{48}\]

4. \[\frac{5}{8} ÷ \frac{3}{1} = \frac{5}{8} \times \frac{1}{3} = \frac{5 \times 1}{8 \times 3} = \frac{5}{24}\]

Problem Set Sample Solutions

Calculate each quotient.

1. \[\frac{2}{5} ÷ \frac{3}{10} = \frac{10}{5} \times \frac{3}{1} = \frac{10 \times 3}{5 \times 1} = \frac{30}{5} = 6\]

2. \[\frac{4}{3} ÷ \frac{4}{7} = \frac{4}{3} \times \frac{7}{4} = \frac{4 \times 7}{3 \times 4} = \frac{28}{12} = \frac{7}{3}\]

3. \[\frac{3}{6} ÷ \frac{9}{10} = \frac{10}{6} \times \frac{9}{1} = \frac{10 \times 9}{6 \times 1} = \frac{90}{6} = \frac{30}{2} = 15\]

4. \[\frac{5}{8} ÷ \frac{7}{12} = \frac{12}{8} \times \frac{7}{1} = \frac{12 \times 7}{8 \times 1} = \frac{84}{8} = 10.5\]
Topic B:

Multi-Digit Decimal Operations—Adding, Subtracting, and Multiplying

6.NS.3

Focus Standard: 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Days: 3

Lesson 9: Sums and Differences of Decimals

Lesson 10: The Distributive Property and Products of Decimals

Lesson 11: Fraction Multiplication and the Products of Decimals

Prior to division of decimals, students will revisit all decimal operations in Topic B. Students have had extensive experience of decimal operations to the hundredths and thousandths (5.NBT.7), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to mixed numbers from the last lesson in Topic A. They find that sums and differences of large mixed numbers can be more efficiently determined by first converting to a decimal and then applying the standard algorithms (6.NS.3). Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. Place value enables students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students discover and use connections between fraction multiplication and decimal multiplication.
Lesson 9: Sums and Differences of Decimals

Student Outcomes

- Students relate decimals to mixed numbers and round addends, minuends, and subtrahends to whole numbers in order to predict reasonable answers.
- Students use their knowledge of adding and subtracting multi-digit numbers to find the sums and differences of decimals.
- Students understand the importance of place value and solve problems in real-world contexts.

Lesson Notes

Students gained knowledge of rounding decimals in 5th grade. Students have also acquired knowledge of all operations with fractions and decimals to the hundredths place in previous grades.

Classwork

Discussion (5 minutes)

It is important for students to understand the connection between adding and subtracting mixed numbers and adding and subtracting decimals.

- Can you describe the circumstances when it would be easier to add and subtract mixed numbers by converting them to decimals first?
  - *When fractions have large denominators that would make it difficult to find common denominators in order to add or subtract.*
  - *When a problem is solved by regrouping, it may be easier to borrow from decimals than fractions.*
- How can estimation be used to help solve addition and subtraction problems with rational numbers?
  - Using estimation can help predict reasonable answers. It is a way to check to see if an answer is reasonable or not.

Example 1 (8 minutes)

Use this example to show students how rounding addends, minuends, and subtrahends can help predict reasonable answers. Also, have students practice using correct vocabulary (addends, sum, minuends, subtrahends, and difference) when talking about different parts of the expressions.

Example 1

\[
\frac{25}{10} + \frac{376}{100} - \frac{77}{100}
\]
Have students convert the mixed numbers into decimals.

- \(25.3 + 376.77\)

Remind students how to round the addends. Then find the estimated sum.

- \(25 + 377 = 402\)

Have students line up the addends appropriately using place value and add.

- \[
\begin{align*}
25.3 & \\
+ & 376.77 \\
\hline
402.07
\end{align*}
\]

- Show students that the sum is close to the estimation. Also show how the place value is important by completing the problem without lining up the place values. If this mistake is made, the actual sum is not close to the estimated sum.

**Example 2 (8 minutes)**

This example will be used to show that changing mixed numbers into decimals may be the best choice to solve a problem.

Divide the class in half. Have students solve the same problem, with one half of the class solving the problem using fractions and the other half of the class solving using decimals. Encourage students to estimate their answers before completing the problem.

Each group should get the same value as their answer; however, the fraction group will have \(150 \frac{7}{10}\), and the decimal group will have \(150.7\).

It is important for students to see that these numbers have the same value. Students solving the problem using fractions will most likely take longer to solve the problem and make more mistakes. Point out to students that the answers represent the same value, but using decimals was easier to solve.

When discussing the problem use the necessary vocabulary. \(426 \frac{1}{5}\) is the minuend, \(275 \frac{1}{2}\) is the subtrahend, and \(150 \frac{7}{10}\) is the difference.
Exercises 1–5 (14 minutes)

Students may work in pairs or individually to complete the following problems. Encourage students to write an expression and then round the addends, minuends, and subtrahends to the nearest whole number in order to predict a reasonable answer. Also, remind students it is not always easier to change fractions to decimals before finding the sum or difference. Discuss the use of the approximation symbol when rounding decimals that repeat.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Problem Description</th>
<th>Expression</th>
<th>Estimated Answer</th>
<th>Actual Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Samantha and her friends are going on a road trip that is 245 $\frac{7}{50}$ miles long. They have already driven 128 $\frac{53}{100}$ miles. How much further do they have to drive?</td>
<td>$245 \frac{7}{50} - 128 \frac{53}{100}$</td>
<td>$245 - 129 = 116$</td>
<td>$245.14 - 128.53 = 116.61$</td>
</tr>
<tr>
<td>2.</td>
<td>Ben needs to replace two sides of his fence. One side is 367 $\frac{9}{100}$ meters long, and the other is 329 $\frac{3}{10}$ meters long. How much fence does Ben need to buy?</td>
<td>$367 \frac{9}{100} + 329 \frac{3}{10}$</td>
<td>$367 + 329 = 696$</td>
<td>$367.09 + 329.3 = 696.39$</td>
</tr>
<tr>
<td>3.</td>
<td>Mike wants to paint his new office with two different colors. If he needs $4 \frac{4}{5}$ gallons of red paint and $3 \frac{1}{10}$ gallons of brown paint, how much paint does he need in total?</td>
<td>$4 \frac{4}{5} + 3 \frac{1}{10}$</td>
<td>$5 + 3 = 8$</td>
<td>$4 \frac{8}{10} + 3 \frac{1}{10} = 7 \frac{9}{10}$</td>
</tr>
<tr>
<td>4.</td>
<td>After Arianna completed some work, she figured she still had 78 $\frac{21}{100}$ pictures to paint. If she completed another 34 $\frac{3}{25}$ pictures, how many pictures does Arianna still have to paint?</td>
<td>$78 \frac{21}{100} - 34 \frac{3}{25}$</td>
<td>$78 - 35 = 43$</td>
<td>$78.21 - 34.92 = 43.29$</td>
</tr>
</tbody>
</table>

Use a calculator to convert the fractions into decimals before calculating the sum or difference.

4. Rahzel wants to determine how much gasoline he and his wife use in a month. He calculated that he used 78 $\frac{1}{3}$ gallons of gas last month. Rahzel’s wife used 41 $\frac{3}{8}$ gallons of gas last month. How much total gas did Rahzel and his wife use last month? Round your answer to the nearest hundredth.

Expression: $78 \frac{1}{3} + 41 \frac{3}{8}$

Estimated answer: $78 + 41 = 119$

Actual answer: $78.333 + 41.375 \approx 119.71$
Closing (5 minutes)

- Have students share their answers and processes for each of the exercise problems.
- Discuss which exercises would be easiest if the addends, minuends, or subtrahends were converted to decimals.

Exit Ticket (5 minutes)
Lesson 9: Sums and Differences of Decimals

Exit Ticket

Solve each problem. Show the placement of the decimal is correct through either estimation or fraction multiplication.

1. \(382 \frac{3}{10} - 191 \frac{87}{100}\)

2. \(594 \frac{7}{25} + 89 \frac{37}{100}\)
Exit Ticket Sample Solutions

Solve each problem. Show the placement of the decimal is correct through either estimation or fraction multiplication.

1. \( \frac{382}{10} - \frac{191}{100} \)
   \[ 382.3 - 191.87 = 190.43 \]

2. \( \frac{594}{25} + \frac{89}{100} \)
   \[ 594.28 + 89.37 = 683.65 \]

Problem Set Sample Solutions

1. Find each sum or difference.
   a. \( \frac{381}{10} - \frac{214}{100} \)
      \[ 381.1 - 214.43 = 166.67 \]
   b. \( \frac{32}{4} - \frac{12}{2} \)
      \[ 32 \frac{3}{4} - 12 \frac{1}{2} = 20 \frac{1}{4} \]
   c. \( \frac{517}{50} + \frac{312}{100} \)
      \[ 517.74 + 312.03 = 829.77 \]
   d. \( \frac{632}{25} + \frac{32}{10} \)
      \[ 632.64 + 32.3 = 664.94 \]
   e. \( \frac{421}{50} - \frac{212}{10} \)
      \[ 421.06 - 212.9 = 208.16 \]

2. Use a calculator to find each sum or difference. Round your answer to the nearest hundredth.
   a. \( \frac{422}{7} - \frac{367}{9} \)
      \[ 422.428571 - 367.555556 = 54.87 \]
   b. \( \frac{23}{5} + \frac{7}{8} \)
      \[ 23.2 + 45.875 ≈ 69.08 \]
Lesson 10: The Distributive Property and the Product of Decimals

Student Outcomes

- Through the use of arrays and partial products, students strategize and apply the distributive property to find the product of decimals.

Lesson Notes

Stations are used in this lesson; therefore, some prep work needs to be completed. Prepare stations before class and have a stopwatch available.

Classwork

Opening Exercise (3 minutes)

The opening exercise should be solved using the multiplication of decimals algorithm. These problems will be revisited in Example 1 and Example 2 to show how partial products can assist in finding the product of decimals.

Opening Exercise

Calculate the product.

1. \(200 \times 32.6\)
   \(6,520\)

2. \(500 \times 22.12\)
   \(11,060\)

Example 1 (5 minutes): Introduction to Partial Products

Show students how the distributive property can assist in calculating the product of decimals. Use this example to model the process.

Example 1: Introduction to Partial Products

Use partial products and the distributive property to calculate the product.

\(200 \times 32.6\)

\(200(32) + 200(0.6) = 6,400 + 120 = 6,520\)
Lesson 10: The Distributive Property and the Product of Decimals

Separate 32.6 into an addition expression with two addends, 32 and 0.6. Emphasize the importance of the place values. The problem will now be $200 \times (32 + 0.6)$. When the distributive property is applied, the problem will be $200(32) + 200(0.6)$.

It is ideal for students to be able to solve these problems mentally using the distributive property, but we understand if additional scaffolding is needed for struggling students. Remind students they need to complete the multiplication before adding. After giving students time to solve the problem, ask for their solutions.

Show students that the answer to this example is the same as the opening exercise but that most of the calculations in this example could be completed mentally.

**Example 2 (7 minutes): Introduction to Partial Products**

Have students try to calculate the product by using partial products. When they complete the problem, encourage students to check their answers by comparing it to the product of the second problem in the opening exercise. When a majority of students complete the problem, have some students share the processes they used to find the product. Answer all student questions before moving on to exercises.

**Example 2: Introduction to Partial Products**

Use partial products and the distributive property to calculate the area of the rectangular patio shown below.

$$500 \times 22.12 = 500(22) + 500(0.12) = 11,000 + 60 = 11,060 \text{ square feet}$$

The area of the patio would be 11,060 square feet.

**Scaffolding:**
Possible extension: Have students complete more than two partial products. An example would be $500(20 + 2 + 0.1 + 0.02)$. 

---

© 2013 Common Core, Inc. Some rights reserved. commoncore.org

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.
Exercise (20 minutes)

Students complete stations individually or in pairs. Encourage students to use partial products in order to solve the problems. Students are to write the problem and their processes in the space provided in the student materials. Remind students to record each station in the correct place because not everyone will start at station one.

Exercise

Use the boxes below to show your work for each station. Make sure you are putting the solution for each station in the correct box.

Station One:
Calculate the product of 300 \times 25.4.
300(25) + 300(0.4) = 7,500 + 120 = 7,620

Station Two:
Calculate the product of 45.9 \times 100.
100(45) + 100(0.9) = 4,500 + 90 = 4,590

Station Three:
Calculate the product of 800 \times 12.3.
800(12) + 800(0.3) = 9,600 + 240 = 9,840

Station Four:
Calculate the product of 400 \times 21.8.
400(21) + 400(0.8) = 8,400 + 320 = 8,720

Station Five:
Calculate the product of 32.6 \times 200
200(32) + 200(0.6) = 6,400 + 120 = 6,520

Closing (6 minutes)

- Students share their answers to the stations and ask any unanswered questions.

Exit Ticket (4 minutes)
Lesson 10: The Distributive Property and the Product of Decimals

Exit Ticket

Complete the problem using partial products.

500 × 12.7
Exit Ticket Sample Solutions

Complete the problem using partial products.

\[ 500 \times 12.7 \]

\[ 500 \times 12.7 = 500(12) + 500(0.7) = 6,000 + 350 = 6,350 \]

Problem Set Sample Solutions

Calculate the product using partial products.

1. \( 400 \times 45.2 \)
   \[ 400(45) + 400(0.2) = 18,000 + 80 = 18,080 \]

2. \( 14.9 \times 100 \)
   \[ 100(14) + 100(0.9) = 1,400 + 90 = 1,490 \]

3. \( 200 \times 38.4 \)
   \[ 200(38) + 200(0.4) = 7,600 + 80 = 7,680 \]

4. \( 900 \times 20.7 \)
   \[ 900(20) + 900(0.7) = 18,000 + 630 = 18,630 \]

5. \( 76.2 \times 200 \)
   \[ 200(76) + 200(0.2) = 15,200 + 40 = 15,240 \]
Lesson 11: Fraction Multiplication and the Products of Decimals

Student Outcomes

- Students use estimation and place value to determine the placement of the decimal point in products and to determine that the size of the product is relative to each factor.
- Students discover and use connections between fraction multiplication and decimal multiplication.
- Students recognize that the sum of the number of decimal digits in the factors yields the decimal digits in the product.

Lesson Notes

To complete this lesson, students will need large poster paper and markers in order to present to the class.

Classwork

Exploratory Challenge (20 minutes)

Students work in small groups to complete the two given problems. After finding each product, group members will have to use previous knowledge to convince their classmates that the product has the decimal in the correct location.

- Students will solve their problems on poster paper using the markers provided.
- Students will include on the poster paper all work that supports their solutions and the placement of the decimal in the answer. You may need to prompt students about their previous work with rounding and multiplication of mixed numbers.
- All groups, even those whose solutions and/or supporting work contain errors, will present their solutions and explain their supporting work. Having the decimal in the wrong place will allow for a discussion on why the decimal placement is incorrect. Since all groups are presenting, allow each group to present only one method of proving where the decimal should be placed.

Exploratory Challenge

You will not only solve each problem, but your groups will also need to prove to the class that the decimal in the product is located in the correct place. As a group, you will be expected to present your informal proof to the class.

1. Calculate the product.  
   \[ 34.62 \times 12.8 = 443.136 \]
   
   Some possible proofs:
   
   Using estimation:  \[ 35 \times 13 = 455. \] If the decimal was located in a different place, the product would not be close to 455.
   
   Using fractions:  \[ \frac{62}{100} \times \frac{8}{10} = \frac{3.462}{100} \times \frac{128}{10} = \frac{443.136}{1000}. \] Because the denominator is 1,000, when writing the fraction as a decimal, the last digit should be in the thousandths place. Therefore, the answer would be 443.136.
2. Xavier earns $11.50 per hour working at the nearby grocery store. Last week, Xavier worked for 13.5 hours. How much money did Xavier earn last week? Remember to round to the nearest penny.

11.5 × 13.5 = 155.25

Some possible proofs:

Using estimation: 12 × 14 = 168. If the decimal was located in a different place, the product would not be close to 168.

Using fractions: \( \frac{115}{10} \times \frac{135}{10} = \frac{15525}{100} \). Because the denominator is 100, when writing the fraction as a decimal, the last digit should be in the hundredths place. Therefore, the answer would be $155.25.

Discussion (5 minutes)

- Do you see a connection between the number of decimal digits in the factors and the product?
  - In the first problem, there are two decimal digits in the first factor and one decimal digit in the second factor, which is a total of three decimal digits. The product has three decimal digits.
  - In the second problem, both factors have one decimal digit for a total of two decimal digits in the factors. The product also has two decimal digits.

Show students that this is another way to determine if their decimal is in the correct place. If this point was brought up by students in their presentations, the discussion can reiterate this method to find the correct placement of the decimal. Remind students to place the decimal before eliminating any unnecessary zeros from the answer.

At the end of the discussion, have students record notes on decimal placement in the student materials.
Lessons 11
NYS COMMON CORE MATHEMATICS CURRICULUM

Lesson 11  6.2

Exercises 1–4 (10 minutes)

Students work individually to solve the four practice problems. Emphasize the importance of decimal placement to hold place value.

Exercises 1–4
1. Calculate the product. \(324.56 \times 54.82\)
   \[324.56 \times 54.82 = 17,792.3792\]

2. Kevin spends $11.25 on lunch every week during the school year. If there are 35.5 weeks during the school year, how much does Kevin spend on lunch over the entire school year? Remember to round to the nearest penny.
   \[11.25 \times 35.5 = 399.375 \approx 399.38\] Kevin would spend $399.38 on lunch over the entire school year.

3. Gunnar’s car gets 22.4 miles per gallon, and his gas tank can hold 17.82 gallons of gas. How many miles can Gunnar travel if he uses all of the gas in the gas tank?
   \[22.4 \times 17.82 = 399.168\] Gunnar can drive 399.168 miles on an entire tank of gas.

4. The principal of East High School wants to buy a new cover for the sand pit used in the long jump competition. He measured the sand pit and found that the length is 29.2 feet and the width is 9.8 feet. What will the area of the new cover be?
   \[29.2 \times 9.8 = 286.16\] The cover should have an area of 286.16 square feet.

Closing (5 minutes)

- How can we use information about the factors to determine the largest place value of the product and the number of decimal digits in the product?

Exit Ticket (5 minutes)
Lesson 11: Fraction Multiplication and the Product of Decimals

Exit Ticket

Use estimation or fraction multiplication to determine if your answer is reasonable.

1. Calculate the product.  
   \[78.93 \times 32.45\]

2. Paint costs $29.95 for a gallon of paint. Nikki needs 12.25 gallons to complete a painting project. How much will Nikki spend on paint? Remember to round to the nearest penny.
Exit Ticket Sample Solutions

1. Calculate the product. \(78.93 \times 32.45\)
   
   \[2,561.2785\]

2. Paint costs $29.95 for a gallon of paint. Nikki needs 12.25 gallons to complete a painting project. How much will Nikki spend on paint? Remember to round to the nearest penny.
   
   \[\text{Nikki would spend $366.89 on paint to complete her project.}\]

Problem Set Sample Solutions

Solve each problem. Remember to round to the nearest penny when necessary.

1. Calculate the product: \(45.67 \times 32.58\)
   
   \[1,487.9286\]

2. Deprina buys a large cup of coffee for $4.70 on her way to work every day. If there are 24 works days in the month, how much does Deprina spend on coffee throughout the entire month?
   
   \[4.70 \times 24 = 112.80 \text{ Deprina would spend $112.80 a month on coffee.}\]

3. Krego earns $2,456.75 every month. He also earns an extra $4.75 every time he sells a new gym membership. Last month, Krego sold 32 new gym memberships. How much money did Krego earn last month?
   
   \[2,456.75 + (4.75 \times 32) = 2,608.75 \text{ Krego earned $2,608.75 last month.}\]

4. Kendra just bought a new house and needs to buy new sod for her backyard. If the dimensions of her yard are 24.6 feet by 14.8 feet, what is the area of her yard?
   
   \[24.6 \times 14.8 = 364.08 \text{ The area of Kendra's yard is 364.08 square feet.}\]
1. Yasmine is having a birthday party with snacks and activities for her guests. At one table, five people are sharing three-quarters of a pizza. What equal-sized portion of the pizza will each of the five people receive?

   a. Use a model (e.g., picture, number line, or manipulative materials) to represent the quotient.

   b. Write a number sentence to represent the situation. Explain your reasoning.

   c. If three-quarters of the pizza provided 12 pieces to the table, how many pieces were in the pizza when it was full? Support your answer with models.
2. Yasmine needs to create invitations for the party. She has $\frac{3}{4}$ of an hour to make the invitations. It takes her $\frac{1}{12}$ of an hour to make each card. How many invitations can Yasmine create?

a. Use a number line to represent the quotient.

b. Draw a model to represent the quotient.

c. Compute the quotient without models. Show your work.
3. Yasmine is serving ice cream with the birthday cake at her party. She has purchased $19 \frac{1}{2}$ pints of ice cream. She will serve $\frac{3}{4}$ of a pint to each guest.

   a. How many guests can be served ice cream?

   b. Will there be any ice cream left? Justify your answer.
4. L.B. Johnson Middle School held a track and field event during the school year. Miguel took part in a four-person shot put team. Shot put is a track and field event where athletes throw (or “put”) a heavy sphere, called a “shot,” as far as possible. To determine a team score, the distances of all team members are added. The team with the greatest score wins first place. The current winning team’s final score at the shot put is 52.08 ft. Miguel’s teammates threw the shot put the following distances: 12.26 ft., 12.82 ft., and 13.75 ft. Exactly how many feet will Miguel need to throw the shot put in order to tie the current first place score? Show your work.

5. The sand pit for the long jump has a width of 2.75 meters and a length of 9.54 meters. Just in case it rains, the principal wants to cover the sand pit with a piece of plastic the night before the event. How many square meters of plastic will the principal need to cover the sand pit?
6. The chess club is selling drinks during the track and field event. The club purchased water, juice boxes, and pouches of lemonade for the event. They spent $138.52 on juice boxes and $75.00 on lemonade. The club purchased three cases of water. Each case of water cost $6.80. What was the total cost of the drinks?
## A Progression Toward Mastery

<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
<th>STEP 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 a</strong> 6.NS.1</td>
<td>Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</td>
<td>Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
</tr>
<tr>
<td>Student response is incorrect and is not supported by a visual model, OR the student did not answer the question.</td>
<td>Student response is incorrect but some evidence of reasoning is presented with a flawed visual model.</td>
<td>Student visual model is correct; however, the quotient of $\frac{3}{20}$ is not determined. OR The student response is correct and the answer is supported with a visual model, but the model is inaccurate.</td>
<td>Student response is correct. The visual model is appropriate AND supports the quotient of $\frac{3}{20}$. The student may have chosen to support their quotient with the use of more than one visual model.</td>
<td></td>
</tr>
<tr>
<td><strong>1 b</strong> 6.NS.1</td>
<td>Student response is incorrect, OR the student did not answer the question.</td>
<td>Student response is incorrect, but a portion of the equation has reasoning. For example, the student may have figured to divide by five, but did not multiply by $\frac{1}{5}$ to determine the quotient.</td>
<td>Student response is incorrect; however, the equation shows reasoning. The equation supports dividing by 5 and makes connection to multiplying by $\frac{1}{5}$ to determine the quotient of $\frac{3}{20}$, but computation is incorrect.</td>
<td>Student response of $\frac{3}{20}$ is correct. The equation depicts the situation and makes connections between division and multiplication. All calculations are correct.</td>
</tr>
<tr>
<td>Student response of 16 pieces is correct, but is not supported with visual models OR student response is incorrect and is not supported with visual models.</td>
<td>Student response of $\frac{3}{20}$ is correct. Student arrived at the answer using an equation, but did not support reasoning with a model, OR student calculation is incorrect, but visual models support reasoning.</td>
<td>Student response of $\frac{3}{20}$ is correct. Student supported the solution with appropriate visual models, AND the student determined the amount of each portion in order to determine the full amount.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1 c</strong> 6.NS.1</td>
<td>Student response of 16 pieces is correct, but is not supported with visual models OR student response is incorrect and is not supported with visual models.</td>
<td>Student response of $\frac{3}{20}$ is correct. Student arrived at the answer using an equation, but did not support reasoning with a model, OR student calculation is incorrect, but visual models support reasoning.</td>
<td>Student response of $\frac{3}{20}$ is correct. Student supported the solution with appropriate visual models, AND the student determined the amount of each portion in order to determine the full amount.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td><strong>a</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is incorrect or missing, OR the student found the product of ( \frac{3}{4} \times \frac{1}{12} ) to reach the response of ( \frac{1}{16} ). A number line diagram does not support the student response.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>b</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is incorrect, but depicts some reasoning in an incomplete number line diagram, OR student response of 9 invitations is correct with no support with a number line diagram.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>c</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response of 9 invitations is correct. Reasoning is evident through the use of a number line diagram, but the response is in terms of time and not amount of cards, such as ( \frac{9}{12} ) or ( \frac{3}{4} ) of an hour, OR student response is correct through the use of calculation and a misinterpretation of the number line diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td><strong>a</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response of 9 invitations is correct. Reasoning is evident through the depiction of an accurately designed number line diagram.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>b</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is incorrect or missing, OR the student computed the product of ( \frac{2}{4} \times \frac{1}{2} ) to reach the response of ( \frac{3}{8} ). No visual representation supports the student response.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>c</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is incorrect, but depicts some reasoning in an incomplete visual model. OR Student response is correct, but reasoning is unclear through the misuse of a visual model.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>d</strong></td>
<td><strong>6.NS.1</strong></td>
<td>The response is correct, but includes no computation to support reasoning.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>e</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is correct. Student computed the quotient as 9 invitations but showed minimal computation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>f</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response of 9 invitations is correct. Student demonstrated evidence of reasoning through concise application of an equation with accurate calculations.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>g</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is correct. Reasoning is evident through calculation of the quotient of 26 people is determined using apparent understanding of factors.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>h</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is correct. Reasoning is evident through correct mixed number conversion. The quotient of 26 people is determined using apparent understanding of factors.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>i</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response determined that there will be no leftover ice cream is correct, but is</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>j</strong></td>
<td><strong>6.NS.1</strong></td>
<td>Student response is correct. Student explanation and reasoning include the</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>6.NS.3</strong></td>
<td><strong>6.NS.3</strong></td>
<td><strong>6.NS.3</strong></td>
<td><strong>6.NS.3</strong></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>Student response is incorrect. Justification does not include adding the given throw distances and determining the difference of that sum and the distance needed to tie for first place. The student response may show only addition.</td>
<td>Student response is incorrect, but attempts to determine the sum of the throw distances first and the difference of the sum and the distance needed to tie first place.</td>
<td>The response is incorrect due to slight miscalculations when adding and/or subtracting. It is evident that the student understands the process of adding the decimals first, then subtracting the sum from the other team’s final score.</td>
<td>Student response is correct. Student accurately determined the sum of the throw distances as 38.83 feet and the differences between that sum and the score needed to tie as 13.25 feet. It is evident that the student understands the process of adding the decimals first, then subtracting the sum from the other team’s final score.</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>Student response is incorrect or missing. The response depicts the use of an incorrect operation, such as addition or subtraction.</td>
<td>Student response is incorrect. The response shows understanding of multi-digit numbers, but lacks precision in place value, resulting in a product less than 3 or more than 262.</td>
<td>Student response of 26.235 square meters is correct, but shows little to no reasoning that multiplication is the accurate operation to choose to find the area of plastic to cover the sand pit.</td>
<td>Student response is correct and shows complete understanding of place value. The response of 26.235 square meters includes a picture that depicts finding the area through multiplication of the length and width of the sand pit.</td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>Student response is incorrect or missing. The response disregards finding the total price of the water.</td>
<td>Student response is incorrect. Student found the total price of the water only.</td>
<td>Student response is incorrect. Student determined the total price of the water and added it to the price of the lemonade and juice, but made minor computation errors.</td>
<td>Student response is correct. The student determined the total price of the water to be $20.40 and accurately added to the price of the lemonade and juice to determine a total cost of $233.92.</td>
</tr>
</tbody>
</table>
1. Yasmine is having a birthday party with snacks and activities for her guests. At one table, five people are sharing three-quarters of a pizza. What equal-sized portion of the pizza will each of the five people receive?

   a. Use a model (e.g., picture, number line, or manipulative materials) to represent the quotient.

   b. Write a number sentence to represent the situation. Explain your reasoning.

   Because there are 5 people, we found 1 out of the 5, which is \(\frac{1}{5}\). I can represent the situation as:

   \[
   \frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}
   \]

   c. If three-quarters of the pizza provided 12 pieces to the table, how many pieces were in the pizza when it was full? Support your answer with models.

   \[
   \frac{12}{3} = 4, \text{ each portion is } 4, \text{ and } 4 \times 4 = 16 \text{ pieces}
   \]
2. Yasmine needs to create invitations for the party. She has \(\frac{3}{4}\) of an hour to make the invitations. It takes her \(\frac{1}{12}\) of an hour to make each card. How many invitations can Yasmine create?

a. Use a number line to represent the quotient.

\[ \begin{array}{cccccccccc}
0 & \frac{1}{12} & \frac{2}{12} & \frac{3}{12} & \frac{4}{12} & \frac{5}{12} & \frac{6}{12} & \frac{7}{12} & \frac{8}{12} & \frac{9}{12} \\
\end{array} \]

Yasmine can make \(9\) invitations.

b. Draw a model to represent the quotient.

\[ \frac{3}{4} \div \frac{1}{12} = 9 \]

\[ \frac{3}{4} \cdot \frac{12}{1} = \frac{36}{4} = 9 \]

\(9\) twelfths

\(\frac{9}{1}\) twelfths

\(9\)

(c. Compute the quotient without models. Show your work.

\[ \frac{3}{4} \div \frac{1}{12} = \frac{9}{12} \div \frac{1}{12} = \frac{9}{1} = 9 \]
3. Yasmine is serving ice cream with the birthday cake at her party. She has purchased \(19 \frac{1}{2}\) pints of ice cream. She will serve \(\frac{3}{4}\) of a pint to each guest.

a. How many guests can be served ice cream?

\[
\frac{19 \frac{1}{2}}{\frac{3}{4}} = \frac{39}{2} \div \frac{3}{4} = \frac{39}{2} \cdot \frac{4}{3} = 13 \times 2 = 26
\]

Yasmine can serve 26 people.

b. Will there be any ice cream left? Justify your answer.

My answer, 26, is a whole number, so there will be no ice cream left over. If my answer was 26 \(\frac{1}{4}\) or any mixed number, there would be ice cream left over.
4. L.B. Johnson Middle School held a track and field event during the school year. Miguel took part in a four-person shot put team. Shot put is a track and field event where athletes throw (or “put”) a heavy sphere, called a “shot,” as far as possible. To determine a team score, the distances of all team members are added. The team with the greatest score wins first place. The current winning team’s final score at the shot put is 52.08 ft. Miguel’s teammates threw the shot put the following distances: 12.26 ft., 12.82 ft., and 13.75 ft. Exactly how many feet will Miguel need to throw the shot put in order to tie the current first place score? Show your work.

\[
\begin{array}{c}
12.26 \\
+ 12.82 \\
+ 13.75 \\
\hline
38.83 \\
\end{array}
\]

Miguel will need to throw the shot put 13.25 feet to tie the current first place score.

5. The sand pit for the long jump has a width of 2.75 meters and a length of 9.54 meters. Just in case it rains, the principal wants to cover the sand pit with a piece of plastic the night before the event. How many square meters of plastic will the principal need to cover the sand pit?

\[
\begin{array}{c}
2.75 \\
\times 9.54 \\
\hline
19.08 \\
67.70 \\
47.70 \\
\hline
26.2350 \\
\end{array}
\]

The principal needs 26.235 m² of plastic to cover the sand pit.
6. The chess club is selling drinks during the track and field event. The club purchased water, juice boxes, and pouches of lemonade for the event. They spent $138.52 on juice boxes and $75.00 on lemonade. The club purchased three cases of water. Each case of water cost $6.80. What was the total cost of the drinks?

\[
\begin{array}{c|c}
\text{water} & \text{juice} \\
$6.80 \times 3$ & $138.52$ \\
\hline
\text{lemonade} & \text{water} \\
$75.00$ & $20.40$ \\
\hline
\text{Total} & \text{Cost} \\
$20.40$ & $233.92$
\end{array}
\]

The total cost of the drinks was $233.92.
Topic C:
Dividing Whole Numbers and Decimals

6.NS.2, 6.NS.3

Focus Standard:  
6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Days: 4

Lesson 12: Estimating Digits in a Quotient
Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm
Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions
Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

In Topic C, students build from their previous learning to fluently divide numbers and decimals. They apply estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (6.NS.2). Students understand that when they “bring down” the next digit in the algorithm, they are distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (6.NS.3). They make connections to division of fractions and rely on mental math strategies in order to implement the division algorithm when finding the quotients of decimals.
Lesson 12: Estimating Digits in a Quotient

Student Outcomes

- Students connect estimation with place value in order to determine the standard algorithm for division.

Classwork

Opening Exercise (5 minutes)

Opening Exercise
Show an example of how you would solve $5,911 \div 23$. You can use any method or model to show your work. Just be sure that you can explain how you got to your solution.

There are many possible models. Here is one possible solution:

\[
\begin{array}{c|c}
23 & 4600 \\
200 & \\
50 & 1150 \\
5 & 115 \\
2 & 46 \\
\end{array}
\]

$5,911 \div 23 = 257$

We may want to check our work to see if our answer is reasonable. One way to do this is to estimate our answer.

- Estimate the quotient of 5,911 and 23.
  - Answers will vary. Sample solution: First I would round the numbers to 6,000 and 20.
  - Answer will vary. Sample solution: $6,000 \div 20 = 300$.
- Using your estimation, would you say that the answer you came up with in your model is reasonable?
  - Answers will vary. Sample solution: Yes, 257 is close to 300. This shows that my answer is reasonable.
- Would you expect your answer to be greater than or less than the actual answer? Why?
  - Answers will vary. Sample solutions: Since I chose to round 23 to 20, I expect that the answer will be greater than the actual quotient. If I had rounded up to 25, the answer would be smaller than the actual quotient. (Students could also respond that 5,911 was rounded to 6,000. Because the number was rounded up, we know that the estimate will be larger than the actual answer.)
Lesson 12

Estimating Digits in a Quotient

Date: 9/16/13

What other ways can we check our solution?

- I can use the inverse of division (multiplication) to check my work.
- $257 \times 23 = 5,911$

Example 1 (10 minutes)

Example 1

We can also use estimates before we divide to help us solve division problems. In this lesson we will be using estimation to help us divide two numbers using the division algorithm.

Estimate the quotient of $8,085 \div 33$. Then divide.

$8,100 \div 30 = 270$.

How could I round these numbers to get an estimate?

- There are many possible solutions. Here are an example of some responses: $8,000 \div 30$, $8,000 \div 35$, $8,100 \div 30$, $8,100 \div 35$.

Why is 8,100 and 30 the best option?

- 3 is not a factor of 8, but it is a factor of 81.

How can we use this to help us divide 8,085 and 33?

- When I begin to divide, I use 270 to help me choose what numbers to divide by. My actual answer should be near 270. The first number I used in my area model will be 200. Then I will see that the remainder is 1,485. I know that $30 \times 50 = 1,500$, which is too big. So I will choose one less ten and try 40.

Create a model to show the division of 8,085 and 33.

We can keep track of the areas in the model and what we have left by making a list and subtracting. We will create a list to keep track of the amounts the same way we created the diagram.

- $8085 - 6600 = 1485$
- $1485 - 1320 = 165$
- $165 - 165 = 0$

- $33 \times 200$
- $33 \times 40$
- $33 \times 5$
- $33 \times 245$
Now we can relate this model to the standard division algorithm.

\[
\begin{array}{c|c}
\hline
33 & 245 \\
-8085 & \\
\hline
-6600 & \\
-1485 & \\
-1320 & \\
\hline
-165 & \\
-165 & \\
\hline
0 & \\
\end{array}
\]

At this point, you are just showing how this work is the same as the work that is shown in the model.

- What does the 2 represent on top of the division bar?
  - We divided 80 hundreds by 33, so the 2 represents 2 hundreds.
- How did we use the 200 in the previous model?
  - \(33 \times 200 = 6,600\).
- What does the 4 represent in the division bar?
  - We divided 148 tens by 33, so the 4 represents 4 tens.
- How was the 40 used in the previous model?
  - \(33 \times 40 = 1,320\).
- Now let’s check our division. How can we use the quotient to check our work?
  - \(245 \times 33 = 8,085\).

**Example 2 (10 minutes)**

Example 2

Use estimation and the standard algorithm to divide.  \(1,512 \div 27\)

Students will estimate the quotient first and use the estimate to help them divide using the algorithm.

- Share an estimate that can be used to help us divide.
  - Answers may vary.  \(1,500 \div 30 = 50\).
- In the algorithm, we can show that there are fifty 27s by placing a 5 over the tens place. We know that 5 tens is 50. This is really showing that 151 tens \(\div 27 = 5\) tens.
What would typically be the next step if you were creating a model? (Students can create the model while solving to further solidify the connection.)

- I would multiply $50 \times 27$, which is 1,350. Then I would subtract 1,350 from 1,512.

```
\[
\begin{array}{c}
1512 \\
- 1350 \\
\hline
162 \\
- 162 \\
\hline
0
\end{array}
\]
```

We will show these same steps in the algorithm.

What would we do next?

- 162 ones $\div$ 27. I know that $30 \times 5$ is 150. So I am going to estimate that the answer is bigger than 5. Maybe there are six 27s in 162.

We will show the same steps again where we check our work by multiplying and subtracting.

Finally, we will check our work. Remind me one more time how we can check our quotient.

- We can multiply $27 \times 56$ and see if the product is 1,512.

Exercises 1–4 (10 minutes)

1. $1,008 \div 48$
   a. Estimate the quotient.

```
\[
1,000 \div 50 = 20
\]
```
Lesson 12: Estimating Digits in a Quotient

1. \( 2,400 \div 30 = 80 \)

2. \( 33 \times 76 = 2,508 \)

3. \( 28 \times 77 = 2,156 \)
4. \(4,732 \div 52\)
   a. Estimate the quotient.
      \[5,000 \div 50 = 100\]
   
   b. Use the algorithm to divide.
      \[
      \begin{array}{c|ccc}
      & 91 & \\
      \hline
      52 & 4732 & \\
        & -4680 & \\
      \hline
      & 52 & -52 \\
    \end{array}
      \]  
      \[52 \times 91 = 4,732\]
   
   c. Check your work.
      \[52 \times 91 = 4,732\]

Closing (5 minutes)

- How does estimation help you with the process of finding the exact quotient?
  - My estimate gives me an idea of what number my answer should be close to. For example, if I have an estimate of 75, I know not to try a number in the hundreds.

- In the previous problem we used 100 to approximate the quotient \(4,732 \div 52\). How did we know that our actual quotient would be in the 90s and not 100 as our approximation suggested? When using your estimate, how do you know if your estimate is too big?
  - I know that \(100 \times 52 = 5,200\) and this is greater than 4,732. This tells me to start with a 9 in the tens place.

- When using your estimate, how do you know if your estimate is too small?
  - When I subtract, the difference is bigger than the divisor.

Exit Ticket (5 minutes)
Lesson 12: Estimating Digits in a Quotient

Exit Ticket

1. Estimate the quotient: $1,908 \div 36$.

2. Use the division algorithm and your estimate to find the quotient: $1,908 \div 36$.

3. Use estimation to determine if $8,580 \div 78$ has a quotient in the $10$s, $100$s, or $1000$s.
Exit Ticket Sample Solutions

1. Estimate the quotient: \( 1,908 \div 36 \).
   \[ 2,000 \div 40 = 50 \]

2. Use the division algorithm and your estimate to find the quotient: \( 1,908 \div 36 \).
   \[ 
   \begin{array}{c|c}
   \text{36} & \text{53} \\
   \hline
   \text{1908} & \\
   \text{-1800} & \\
   \hline
   \text{108} & \\
   \text{-108} & \\
   \hline
   \text{0} & \\
   \end{array} 
   \]

3. Use estimation to determine if \( 8,580 \div 78 \) has a quotient in the 10s, 100s, or 1000s.
   
   I would round \( 8,580 \) to \( 8,800 \) and 78 to 80. \( 8,800 \div 80 = 110 \). I know that the quotient should be in the 100s.

Problem Set Sample Solutions

Complete the following steps for each problem:

a. Estimate the quotient.
b. Use the division algorithm to solve.
c. Show a model that supports your work with the division algorithm.
d. Be sure to check your work.

1. \( 3,312 \div 48 \)
   \[ 3,500 \div 50 = 70 \]

   \[ 
   \begin{array}{c|c|c}
   \text{48} & \text{69} & \text{48} \\
   \hline
   \text{3312} & \text{2880} & \\
   \text{-2880} & \\
   \hline
   \text{432} & \text{60} & \text{2880} \\
   \text{-432} & \\
   \hline
   \text{0} & \text{9} & \text{432} \\
   \end{array} 
   \]

   \[ 48 \times 69 = 3,312 \]
2. \(3,125 \div 25\)

\[
\begin{array}{c|c}
25 & 125 \\
\hline
25 & 125 \\
-2500 & -2500 \\
625 & 625 \\
-500 & -500 \\
125 & 125 \\
0 & 0 \\
\end{array}
\]

\(25 \times 125 = 3,125\)

3. \(1,344 \div 14\)

\[
\begin{array}{c|c}
14 & 96 \\
\hline
14 & 1344 \\
-1260 & -1260 \\
84 & 84 \\
-84 & -84 \\
0 & 0 \\
\end{array}
\]

\(14 \times 96 = 1,344\)
Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

Student Outcomes

- Students understand that the standard algorithm of division is simply a tally system arranged in place value columns.

Classwork

Example 1 (3 minutes)

The first example is a review from lesson 12 to start taking a deeper look at the division algorithm.

Example 1

a. Create a model to divide: $1,755 \div 27$.

\begin{align*}
27 & \quad 60 \\
\underline{1620} & \quad 5 \\
135 & \\
\end{align*}

\textit{Answers may vary. One possible solution:}

b. Use the division algorithm to show $1,755 \div 27$.

\begin{align*}
27 & \quad 65 \\
\underline{1755} & \quad -162 \\
\underline{135} & \\
135 & \\
0 & \\
\end{align*}

- Looking at your division work, where did the numbers 162 and 135 come from?

\begin{align*}
27 & \quad 65 \\
\underline{1755} & \quad -162 \\
\underline{135} & \\
-135 & \\
0 & \\
\end{align*}

\textit{The 162 comes from }27 \times 6. \textit{ This is really showing }27 \times 60, \textit{ which is 1,620. The 135 comes from }27 \times 5.
If you had to describe what is happening underneath the division bar in your own words, what would you say?
(For struggling students, have them think back to what they did when they used a model.)

- The work under the long division bar shows how I keep track of the parts that I have already divided out. Then I can see what remains to continue the process.

c. Check your work.
\[ 27 \times 65 = 1,755 \]

Example 2 (3 minutes)

Example 2
Find the quotient of \( 205,276 \div 38 \).

\[
\begin{array}{c}
38 \overline{205276} \\
-190 \\
\underline{152} \\
-152 \\
\underline{076} \\
-76 \\
\underline{0}
\end{array}
\]

- How can we use estimation to start solving this problem?

  - Answers will vary. I can start by determining how many times 40 will divide into 200 thousands. I know that \( 40 \times 5 = 200 \). So I will start the division process by placing a 5 over the 5 in 205,276. This 5 represents 5 thousands. So we will show in the long division that 200 thousands \( \div 38 = 5 \) thousands.

- Why did you divide 40 into 200 instead of 200,000?

  - I thought of this problem as 40 into 200 thousands, so that I could divide 40 into 200 to make the division simpler.

Give students a chance to complete the division. Students should also be creating a model to show the connection between the algorithm and the model.

- Aou divided by 400, what did you do next?

  - I had brought down the seven to complete the next step. However, 38 does not divide into 7 one or more times. This told me to put a 0 in the tens place and bring down the 6 to continue dividing.

\[
\begin{array}{c|c|c}
5000 & 190,000 \\
\hline
400 & 15,200 \\
\hline
2 & 76
\end{array}
\]
Imagine that your friend wrote 542 as the answer. How could you prove to your friends that 542 is not the solution?

- Answers may vary. I could use estimation. I would round the divisor and quotient and multiply them to see if I get an estimate that is close to the dividend $40 \times 500 = 20,000$. The estimate is about 10 times too small. So I can tell that the numbers are in the wrong place. I should have an estimate around 200,000.

Now, let’s use the algorithm to work through a division question that involves a much larger number.

Example 3 (3 minutes)

Example 3
Find the quotient of $17,216,673 \div 23$.

- When working with a dividend as large as this, what would happen if we tried to solve this question using a model?
  - Answers will vary. The model would be difficult to make because it could have many parts for the different place values.

- It might be difficult to figure out how many times 23 goes into this whole number. So we can break this into parts. Does 23 go into 1 one or more times? Does 23 go into 17 one or more times? Will 23 divide into 172 one or more times?
  - 23 does not go into 1 or 17 more than one time. However, 23 will go into 172.

- Could we use estimation to help us start the problem?
  - We could think about $175 \div 25$ or $180 \div 20$ (or other possible estimations that are backed by mathematical reasoning).

Remind students that what we are actually doing is $172$ hundred thousands $\div 23 = 7$ hundred thousands.

Each step can be represented using the units.

- Why would we place the 7 over the 2 and not somewhere else? What does the 7 represent?
  - The 7 shows how many times 23 goes into 172, but it really represents how many times 23 goes into 17,200,000. Because the 7 represents 700,000, we place the 7 over the 2 in the hundred thousands place.

- When we subtracted, we got an 11. What does this 11 represent?
  - The 11 shows the difference between 172 and 161, but it actually represents 1,100,000. This is the amount remaining after 23 groups of 700,000 are taken from 17,200,000.
Lesson 13
Dividing Multi-Digit Numbers Using the Algorithm

- After we have completed this first set of steps, where do we go next?
  - We could keep repeating the process until we reach the ones place.

When discussing the remaining steps you can refer to them as follows:

111 ten thousands ÷ 23: 4 ten thousands
196 thousands ÷ 23: 8 thousands
126 hundreds ÷ 23: 5 hundreds
117 tens ÷ 23: 5 tens
23 ones ÷ 23 = 1

- How can we determine if the answer is reasonable?
  - We could multiply the quotient with the divisor. $748,551 \times 23 = 17,216,673$
  - We could also use an estimate to check our work. $20,000,000 ÷ 20 = 1,000,000$. Our estimate is slightly larger because we rounded the dividend up.

Exercises 1–6 (20 minutes)

Give students a chance to practice using the division algorithm. Students may not be able to complete all questions in the time given.

Exercises

For each question you need to do the following:

a. Solve the question. Next to each line explain your work using place value.

b. Evaluate the reasonableness of your answer.

1. $891,156 ÷ 12$

<table>
<thead>
<tr>
<th>74,263</th>
</tr>
</thead>
<tbody>
<tr>
<td>89,1156</td>
</tr>
<tr>
<td>51</td>
</tr>
<tr>
<td>48</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>75</td>
</tr>
<tr>
<td>72</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

a. 89 ten thousands ÷ 12: 7 ten thousands
   51 thousands ÷ 12: 4 thousands
   31 hundreds ÷ 12: 2 hundreds
   75 tens ÷ 12: 6 tens
   36 ones ÷ 12: 3 ones

b. $74,263 \times 12 = 891,156$
Lesson 13
Dividing Multi-Digit Numbers Using the Algorithm

Date: 9/16/13

NYS COMMON CORE MATHEMATICS CURRICULUM

2. \[ \begin{array}{c}
484,692 \div 78 \\
\hline
78 | 484692 \\
-468 \\
\hline
166 \\
-156 \\
\hline
10 \\
-78 \\
\hline
28 \\
-26 \\
\hline
2 \\
\hline
6214
\end{array} \]
a. 484 thousands \div 78: 6 thousands
   166 hundreds \div 78: 2 hundreds
   109 tens \div 78: 1 ten
   312 ones \div 78: 4 ones
b. \( 6,214 \times 78 = 484,692 \)

3. \[ \begin{array}{c}
281,886 \div 33 \\
\hline
33 | 281886 \\
-264 \\
\hline
178 \\
-165 \\
\hline
13 \\
-12 \\
\hline
1 \\
-1 \\
\hline
8542
\end{array} \]
a. 281 thousands \div 33: 8 thousands
   178 hundreds \div 33: 5 hundreds
   138 tens \div 33: 4 tens
   66 ones \div 33: 2 ones
b. \( 8,542 \times 33 = 281,886 \)

4. \[ \begin{array}{c}
2,295,517 \div 37 \\
\hline
37 | 2295517 \\
-222 \\
\hline
75 \\
-74 \\
\hline
11 \\
-10 \\
\hline
1 \\
\hline
62041
\end{array} \]
a. 229 ten thousands \div 37: 6 ten thousands
   75 thousands \div 37: 2 thousands
   15 hundreds \div 37: 0 hundreds
   151 tens \div 37: 4 tens
   37 ones \div 37: 1 one
b. \( 62,041 \times 37 = 2,295,517 \)

5. \[ \begin{array}{c}
952,448 \div 112 \\
\hline
112 | 952448 \\
-896 \\
\hline
56 \\
-44 \\
\hline
128 \\
-128 \\
\hline
0
\end{array} \]
a. 952 thousands \div 112: 8 thousands
   564 hundreds \div 112: 5 hundreds
   44 tens \div 112: 0 tens
   448 ones \div 112: 4 ones
b. \( 8,504 \times 112 = 952,448 \)

6. \[ \begin{array}{c}
1,823,535 \div 245 \\
\hline
245 | 1823535 \\
-18235 \\
\hline
1058 \\
-980 \\
\hline
78 \\
-74 \\
\hline
735 \\
-735 \\
\hline
0
\end{array} \]
a. 1,823 thousands \div 245: 7 thousands
   1,085 hundreds \div 245: 4 hundreds
   1,053 tens \div 245: 4 tens
   735 ones \div 245: 3 ones
b. \( 7,443 \times 245 = 1,823,535 \)
Closing (2 minutes)

- Explain in your own words how the division algorithm works?
  - Answers will vary. Sample solution: The division algorithm shows, organized by place value, successive estimates of the quotient. Or students may answer, “The division algorithm breaks one large division problem into several smaller ones organized by place value.”

Exit Ticket (3 minutes)
Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

Exit Ticket

Divide using the division algorithm: \( 392,196 \div 87 \)
Exit Ticket Sample Solutions

Divide using the division algorithm: 392,196 ÷ 87

\[
\begin{array}{c|c}
87 & 392196 \\
-348 & -348 \\
441 & \\
-435 & -435 \\
696 & 696 \\
-696 & -696 \\
0 & \\
\end{array}
\]

Problem Set Sample Solutions

1. 459,054 ÷ 54

\[
\begin{array}{c|c}
54 & 459054 \\
-432 & -432 \\
270 & 270 \\
-270 & -270 \\
54 & 54 \\
-54 & -54 \\
0 & \\
\end{array}
\]

2. 820,386 ÷ 102

\[
\begin{array}{c|c}
102 & 820386 \\
-816 & -816 \\
438 & 438 \\
-408 & -408 \\
306 & 306 \\
-306 & -306 \\
0 & 0 \\
\end{array}
\]

3. 1,183,578 ÷ 227

\[
\begin{array}{c|c}
227 & 1183578 \\
-454 & -454 \\
317 & 317 \\
-227 & -227 \\
908 & 908 \\
-908 & -908 \\
0 & 0 \\
\end{array}
\]
Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Student Outcomes

- Students use the algorithm to divide multi-digit numbers with and without remainders. Students compare their answer to estimates to justify reasonable quotients.
- Students understand that when they “bring down” the next digit in the algorithm, they are distributing, recording, and shifting to the next place value.

Classwork

Example 1 (4 minutes)

Students will review how to divide a whole number by a number that is not a factor resulting in a non-whole number quotient. They will first estimate the quotient. Then they will use the division algorithm to get an exact answer. Finally, they will compare the two to decide if the answer is reasonable.

Example 1

Divide: $31,218 \div 132$

As we divide, we can use our knowledge of place value to guide us.

- $312 \text{ hundreds} \div 132: 2 \text{ hundreds}$
- $481 \text{ tens} \div 132: 3 \text{ tens}$
- $858 \text{ ones} \div 132: 6 \text{ ones}$
- $660 \text{ tenths} \div 132: 5 \text{ tenths}$

- Estimate the quotient.
  - Answers may vary. Possible estimates include the following: $30,000 \div 100 = 300$ or $30,000 \div 150 = 200$.
- How was solving this question similar to the questions you solved in Lessons 12 and 13?
  - Answers may vary. To get the quotient in all questions, I used the division algorithm where I divided two whole numbers.
How was solving this question different than the questions you solved in Lessons 12 and 13?

- Answers may vary. In this example, the divisor is not a factor of the dividend. I know this because the quotient was not a whole number. When I got to the ones place, I still had a remainder, so I placed a zero in the tenths place so that I could continue dividing. Then I divided 660 tenths by 132 ones. The answer to this question had a decimal in the quotient where the other lessons had whole number quotients.

Example 2 (4 minutes)

We have seen questions with decimals in the quotient. Now let’s discuss how we would divide when there are decimals in the dividend and divisor. (Please note that this question is quite difficult. Students will most likely struggle with this question for quite some time. You may want to offer this question as a challenge.)

| Example 2 |
| Divide: 974.835 ÷ 12.45 |

Point out that all whole number division has involved dividing two quantities that are ultimately counting with the same unit: ones (e.g., 32,218 ones divided by 132 ones)

Now let’s take a look at what this question is asking including the units.

- 974 ones and 835 thousandths, 12 ones and 45 hundredths
- What do you notice about these two numbers?
  - They do not have the same unit.
- How could we rewrite these numbers, so that they have the same units?
  - 974,835 ÷ 12,450
  - 974,835 thousandths, 12,450 thousandths

Now, the division problem that we need to solve is 974,835 thousandths ÷ 12,450 thousandths
Example 3 (4 minutes)

Example 3
A plane travels 3.625.26 miles in 6.9 hours. What is the plane’s unit rate?

- What is this question asking us to do?
  - This question is asking me to divide the miles by hours so that I can find out how many miles the plane went in 1 hour, like we did in Module 1.

- How can we rewrite 3,625.26 (362,526 hundredths) and 6.9 (69 tenths) using the same units?
  - First, I would rewrite the question as 3,625.26 ÷ 6.90. This is the same as 362,526 hundredths ÷ 690 hundredths.
  - Now we can solve by dividing 362,526 ÷ 690.

\[
\begin{array}{c|c}
690 & 3625.26 \\
-3450 & \\
-1380 & 1752 \\
-1380 & 3726 \\
-3450 & \\
-2760 & 0 \\
\end{array}
\]

- Let’s check our answer to ensure that it is reasonable. What are some different ways that we can do this?
  - We can multiply the quotient with the original divisor and see if we get the original dividend.
    \[6.9 \times 525.4 = 3,625.26\]
  - We could also estimate to check our answer. 3,500 ÷ 7 = 500. Because we rounded down, we should expect our estimate to be a little less than the actual answer.

Exercises 1–7 (20 minutes)

Students can work on the problem set alone or in partners. Students should be estimating the quotient first and using the estimate to justify the reasonableness of their answer.

Exercises

1. Daryl spent $4.68 on each pound of trail mix. He spent a total of $14.04. How many pounds of trail mix did he purchase?
   
   \[\text{Estimate } 15 \div 5 = 3\]
   
   \[14.04 \div 4.68 \rightarrow 1,404 \text{ hundredths} ÷ 468 \text{ hundredths}\]
   
   \[1,404 ÷ 468 = 3\]  Daryl purchased 3 pounds of trail mix.

   \[\text{Our estimate of 3 shows that our answer of 3 is reasonable.}\]
2. Kareem purchased several packs of gum to place in gift baskets for $1.26 each. He spent a total of $8.82. How many packs of gum did he buy?

Estimate $9 = 9$

$8.82 \div 1.26 \rightarrow 882 \text{ hundredths} \div 126 \text{ hundredths}$

$882 \div 126 = 7 \text{ packs of gum}$

Our estimate of 9 shows that our answer of 7 is reasonable.

3. Jerod is making candles from beeswax. He has $132.72$ ounces of beeswax. If each candle uses $8.4$ ounces of beeswax, how many candles can he make? Will there be any wax left over?

Estimate $120 \div 8 = 15$

$132.72 \div 8.4 \rightarrow 13,272 \text{ hundredths} \div 84 \text{ tenths}$

$13,272 \div 840 = 15 \text{ candles with wax leftover}$

Our estimate of 15 shows that our answer of 15 is reasonable.

4. There are $20.5$ cups of batter in the bowl. If each cupcake uses $0.4$ cups of batter, how many cupcakes can be made?

Estimate $20 \div 0.5 = 40$

$20.5 \div 0.4 \rightarrow 205 \text{ tenths} \div 4 \text{ tenths}$

$51.25$ cups of batter can be made. There is not quite enough for $52$.

Our estimate of 40 shows that our answer of 51.25 is reasonable.

5. In Exercises 3 and 4, how were the remainders, or extra parts, interpreted?

In both Exercises 3 and 4, the remainders show that there was not quite enough to make another candle or cupcake. In the candle example, there was wax left over that could be saved for the next time there is more wax. However, in the cupcake example, the leftover batter could be used to make a smaller cupcake, but it would not count as another whole cupcake.
6. \(159.12 \div 6.8\)

*Estimate \(160 \div 8 = 20\)*

\[
159.12 \div 6.8 \rightarrow 15,912 \text{ hundredths} \div 68 \text{ tenths} \rightarrow 15,912 \text{ hundredths} \div 680 \text{ hundredths}
\]

\[
\begin{array}{c|cccc}
& 2 & 3 & . & 4 \\
\hline
680 & 15912.0 \\
-1360 & -1360 & \\
\hline
2312 \\
-2040 & -2040 & \\
\hline
2720 & 2720 & \\
-2720 & -2720 & \\
\hline
0
\end{array}
\]

*Our estimate of 20 shows that our answer of 23.4 is reasonable.*

7. \(167.67 \div 8.1\)

*Estimate \(160 \div 8 = 20\)*

\[
167.67 \div 8.1 \rightarrow 16,767 \text{ hundredths} \div 81 \text{ tenths} \rightarrow 16,767 \text{ hundredths} \div 810 \text{ hundredths}
\]

\[
\begin{array}{c|cccc}
& 2 & 0 & . & 7 \\
\hline
810 & 16767.0 \\
-1620 & -1620 & \\
\hline
5670 \\
-5670 & -5670 & \\
\hline
0
\end{array}
\]

*Our estimate of 20 shows that our answer of 20.7 is reasonable.*

**Closing (3 minutes)**

- Describe the steps that you use to change a division question with decimals to a division question with whole numbers?
  - *If the divisor and or the dividend are not whole numbers, we find the largest common unit, smaller than one, that allows us to rewrite each as a whole number multiple of this common unit.*
  - *Example:*
    - \(1,220.934 \text{ ones} \div 54.34 \text{ ones}\)
    - \(12,209.34 \text{ tenths} \div 543.4 \text{ tenths}\)
    - \(122,093.4 \text{ hundredths} \div 5,434 \text{ hundredths}\)
    - \(1,220,934 \text{ thousandths} \div 54,340 \text{ thousandths}\)
  - *We could keep going, and both the dividend and divisor would still be whole numbers, but we were looking for the largest common unit that would make this happen.*

**Exit Ticket (5 minutes)**
Name ___________________________________________  Date________________

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Exit Ticket

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $14.70. How many pounds of almonds did she purchase?

2. Divide 125.01 ÷ 5.4. Then check your answer for reasonableness.
Exit Ticket Sample Solutions

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $14.70. How many pounds of almonds did she purchase?

\[
\begin{array}{c}
\text{350} \\
\hline
4.2 \\
\end{array}
\]

\[
\begin{array}{c}
1470.0 \\
-1400 \\
\hline
700 \\
-700 \\
\hline
0 \\
\end{array}
\]

Lisa purchased 4.2 pounds of almonds.

2. Divide: 125.01 ÷ 5.4

\[
\begin{array}{c}
\text{540} \\
\hline
12501.00 \\
-1080 \\
\hline
1701 \\
-1620 \\
\hline
810 \\
-540 \\
\hline
2700 \\
-2700 \\
\hline
0 \\
\end{array}
\]

The quotient of 125.01 and 5.4 is 23.15.

Estimate 125 ÷ 5 = 25

My estimate of 25 is near 23, which shows that my answer is reasonable.

Problem Set Sample Solutions

1. Aslan purchased 3.5 lbs. of his favorite mixture of dried fruits to use in a trail mix. The total cost was $16.87. How much does the fruit cost per pound?

\[
\begin{array}{c}
\text{350} \\
\hline
4.82 \\
\end{array}
\]

\[
\begin{array}{c}
1687.00 \\
-1400 \\
\hline
2870 \\
-2800 \\
\hline
700 \\
-700 \\
\hline
0 \\
\end{array}
\]

The dried fruit costs $4.82 per pound.
2. Divide: $994.14 \div 18.9$

$994.14 \div 18.9 \rightarrow 99.414 \text{ hundredths} \div 1.890 \text{ hundredths}$

\[
\begin{array}{c|c}
\text{1890} & \text{99414.0} \\
\text{9450} & \text{9450} \\
\hline
\text{-3780} & \text{4914} \\
\text{11340} & \text{11340} \\
\hline
\text{-11340} & \text{0}
\end{array}
\]

$994.14 \div 18.9 = 52.6$
Lesson 15: The Division Algorithm—Converting Decimal Division to Whole Number Division Using Mental Math

Student Outcomes

- Students use their knowledge of dividing multi-digit numbers to solve for quotients of multi-digit decimals.
- Students understand the mathematical concept of decimal placement in the divisor and the dividend and its connection to multiplying by powers of 10.

Classwork

Opening Questions (10 minutes)

This is an optional section for those who would like to include a warm-up exercise in their lessons.

These questions can be asked in discussion form with the whole group to introduce the students to the idea of multiplying the divisor or dividend by powers of ten.

- Let’s take a look at what happens when we change our division problem.

Opening Exercise

Start by finding the quotient of $1,728$ and $32$.

\[
\begin{array}{c|c}
32 & 1728 \\
\hline
54 & 1728 \\
-160 & -160 \\
128 & -128 \\
0 & 0 \\
\end{array}
\]

What would happen if we multiplied the divisor by 10? $1,728 \div 320$

\[
\begin{array}{c|c}
320 & 1728.0 \\
\hline
5.4 & -1600 \\
1280 & -1280 \\
0 & 0 \\
\end{array}
\]

When the divisor is ten times bigger, the quotient is ten times smaller.
What would happen if we multiplied the dividend by 10?  
\[ 17,280 \div 32 \]

\[
\begin{array}{r}
32 & \overline{17280} \\
\underline{-160} & 128 \\
\underline{0} & \underline{128} \\
\underline{0} & \underline{0} \\
\end{array}
\]

*When the dividend is ten times bigger, the quotient is ten times bigger.*

What would happen if we multiplied both the divisor and dividend by 10?  
\[ 17,280 \div 320 \]

\[
\begin{array}{r}
320 & \overline{17280} \\
\underline{-1600} & 1280 \\
\underline{-1280} & 0 \\
\end{array}
\]

*When both the divisor and dividend are multiplied by 10, the quotient does not change.*

What would happen if we multiplied both the divisor and dividend by 100?  
\[ 172,800 \div 3,200 \]

\[
\begin{array}{r}
3200 & \overline{172800} \\
\underline{-16000} & 12800 \\
\underline{-12800} & 0 \\
\end{array}
\]

*When both the divisor and dividend are multiplied by 100, the quotient does not change.*

What would happen if we multiplied both the divisor and the dividend by 1,000, 10,000, or 100,000?  What do you predict would happen?

*As long as both the divisor and the dividend are multiplied by the same amount, the quotient will not change.*

How can we use this to help us divide when there are decimals in the divisor?  For example, how can we use this to help us divide 172.8 and 3.2?

172.8 is the same as 1,728 tenths and 3.2 is the same as 32 tenths.  So to get the quotient, we could divide 1,728 by 32.  If we multiply both the divisor and the dividend by 10, we get the same two numbers that we need to divide, 1,728 and 32.  Because we multiplied both numbers by the same amount, the answer will not change.
Example 1 (3 minutes)

Example 1

Using our discoveries from the discussion, let’s divide 537.1 by 8.2.

How can we rewrite this problem using what we learned in Lesson 14?

*We can rewrite this problem as 5.371 tenth ÷ 82 tenths. These two numbers already have the same unit, so we would divide 5.371 by 82.*

How could we use the short cut from our discussion to change the original numbers to 5.371 and 82?

*We can multiply both numbers by 10, so that the divisor will change from 8.2 to 82. Then we will have a whole number to divide by. So the new problem will become 5.371 ÷ 82.*

\[
\begin{array}{c|c}
82 & 5371.0 \\
-492 & \\
\hline
451 & \\
-410 & \\
\hline
410 & \\
-410 & \\
\hline
0 & \\
\end{array}
\]

Example 2 (3 minutes)

Example 2

Now let’s divide 742.66 by 14.2.

How can we rewrite this division problem so that the divisor is a whole number and the quotient remains the same?

*We can multiply both numbers by 10, so that the divisor will change from 14.2 to 142. Then we will have a whole number to divide by. So the new problem will become 7426.6 ÷ 142.*

\[
\begin{array}{c|c}
142 & 7426.6 \\
-710 & \\
\hline
326 & \\
-284 & \\
\hline
426 & \\
-426 & \\
\hline
0 & \\
\end{array}
\]
Exercises (18 minutes)

Exercises
Students will participate in a game called pass the paper. Students will work in groups of no more than four. There will be a different paper for each player. When the game starts, each student solves the first problem on his paper then passes the paper clockwise to the second person who uses multiplication to check the work that was done by the previous student. Then the paper is passed clockwise again to the third person who solves the second problem. The paper is then passed to the fourth person who checks the second problem. This process continues until all of the questions on every paper are complete or time runs out.

The four different player pages are attached. Answers are below.

Player A Answers
1. \( \frac{15.5}{6.2} = 2.5 \)
2. \( \frac{28.08}{7.8} = 3.6 \)
3. \( \frac{44.888}{3.62} = 12.4 \)
4. \( \frac{3,912.99}{15.9} = 246.1 \)
5. \( \frac{865.1475}{47.25} = 18.31 \)

Player B Answers
1. \( \frac{32.4}{7.2} = 4.5 \)
2. \( \frac{49.14}{6.3} = 7.8 \)
3. \( \frac{39.321}{2.57} = 15.3 \)
4. \( \frac{8,578.02}{24.6} = 348.7 \)
5. \( \frac{439.0464}{35.18} = 12.48 \)

Player C Answers
1. \( \frac{25.9}{7.4} = 3.5 \)
2. \( \frac{25.48}{5.2} = 4.9 \)
3. \( \frac{61.962}{4.49} = 13.8 \)
4. \( \frac{16.437.42}{31.8} = 516.9 \)
5. \( \frac{1,238.8048}{52.76} = 23.48 \)

Player D Answers
1. \( \frac{63.7}{9.8} = 6.5 \)
2. \( \frac{32.68}{8.6} = 3.8 \)
3. \( \frac{142.912}{8.12} = 17.6 \)
4. \( \frac{23.344.58}{57.4} = 406.7 \)
5. \( \frac{2,498.743}{39.65} = 63.02 \)
Closing (3 minutes)

- Based upon our work today, discuss ways you would alter the problem $4,509 \div 0.03$ to make it easier to use the long division algorithm and yield the same answer.
  - As long as I multiply both dividend and divisor by the same number the quotient will not change. If I multiply by powers of 10, I will be able to ultimately get to a point where both dividend and divisor are whole numbers. In this case, if I multiply by $10^2$, the problem will become $4,509 \div 3$.

Exit Ticket (5 minutes)
Lesson 15: The Division Algorithm—Converting Decimal Division to Whole Number Division Using Mental Math

Exit Ticket

State the power of 10 you would use to convert the given decimal division to whole number division. Then complete the multiplication on the dividend and divisor.

1. $133.84 \div 5.6$

2. $12.4 \div 1.036$

3. $38.9 \div 2.91$

4. $45 \div 1.5$
Exit Ticket Sample Solutions

State the power of 10 you would use to convert the given decimal division to whole number division. Then complete the multiplication on the dividend and divisor.

1. \(133.84 \div 5.6\)
   
   \(100 \text{ or } 10^2, 13.384 \text{ and } 560\)

2. \(12.4 \div 1.036\)
   
   \(1,000 \text{ or } 10^3, 12,400 \text{ and } 1,036\)

3. \(38.9 \div 2.91\)
   
   \(100 \text{ or } 10^2, 3,890 \text{ and } 291\)

4. \(45 \div 1.5\)
   
   \(10 \text{ or } 10^1, 450 \text{ and } 15\)

Problem Set Sample Solutions

These extra problems can be used for homework or for students that need extra practice.

1. \(118.4 \div 6.4\)
   
   \(1.184 \div 64\)

   \[
   \begin{array}{c|c}
   \hline
   18.5 & \\
   \\
   64 & 1184.0 \\
   - 64 & -64 \\
   544 & 544 \\
   - 512 & -512 \\
   320 & 320 \\
   - 320 & -320 \\
   0 & 0 \\
   \hline
   \end{array}
   \]

2. \(314.944 \div 3.7\)
   
   \(3.149.44 \div 37\)

   \[
   \begin{array}{c|c}
   \hline
   85.12 & \\
   \\
   37 & 3149.44 \\
   - 296 & -296 \\
   189 & 189 \\
   - 185 & -185 \\
   44 & 44 \\
   - 37 & -37 \\
   74 & 74 \\
   - 74 & -74 \\
   0 & 0 \\
   \hline
   \end{array}
   \]

© 2013 Common Core, Inc. Some rights reserved. commoncore.org

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.
3. \(1.840.5072 \div 23.56\)

\[
\begin{array}{c|cccc}
2356 & 184050.72 & \downarrow & 78.12 \\
& 16422 & & \\
\hline
& 19130 & & \\
-18848 & & & \\
\hline
& 282 & & \\
-2356 & & & 4712 \\
\hline
& 4712 & & 0 \\
\hline
\end{array}
\]

\(184.050.72 \div 2.356\)
Player A

1. \(15.5 \div 6.2\)
   Check:

2. \(28.08 \div 7.8\)
   Check:

3. \(44.888 \div 3.62\)
   Check:

4. \(3,912.99 \div 15.9\)
   Check:

5. \(865.1475 \div 47.25\)
   Check:
**Lesson 15**

The Division Algorithm — Converting Decimal Division to Whole Number Division Using Mental Math

**Date:** 9/16/13

**Player B**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Check:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$32.4 \div 7.2$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$49.14 \div 6.3$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$39.321 \div 2.57$</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>$8,578.02 \div 24.6$</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>$439.0464 \div 35.18$</td>
<td></td>
</tr>
</tbody>
</table>
## Player C

1. \(25.9 \div 7.4\)

   **Check:**

2. \(25.48 \div 5.2\)

   **Check:**

3. \(61.962 \div 4.49\)

   **Check:**

4. \(16437.42 \div 31.8\)

   **Check:**

5. \(1,238.8048 \div 52.76\)

   **Check:**
## Player D

1. \(63.7 \div 9.8\)  
   Check:

2. \(32.68 \div 8.6\)  
   Check:

3. \(142.912 \div 8.12\)  
   Check:

4. \(23,344.58 \div 57.4\)  
   Check:

5. \(2,498.743 \div 39.65\)  
   Check:
Topic D:

Number Theory—Thinking Logically About Multiplicative Arithmetic

6.NS.4

Focus Standard: 6.NS.4

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Instructional Days: 4

Lesson 16: Even and Odd Numbers
Lesson 17: Divisibility Tests for 3 and 9
Lesson 18: Least Common Multiple and Greatest Common Factor
Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Students have previously developed facility with multiplication and division. They now begin to reason logically about them in Topic D. Students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid’s Algorithm is a more efficient means to finding the greatest common factor of larger numbers and determine that Euclid’s Algorithm is based on long division.
Lesson 16: Even and Odd Numbers

Student Outcomes

- Students apply odd and even numbers to understand factors and multiples.

Lesson Notes

Students will need poster paper and markers to complete the exercises.

Classwork

Discussion (15 minutes)

Present each question and then allow students to share their thinking. Also have students record notes in their student materials.

Opening Exercise

What is an even number?

Possible student responses:

- An integer that can be evenly divided by 2.
- A number whose last digit is 0, 2, 4, 6, or 8.
- All the multiples of 2.

List some examples of even numbers.

Answers will vary.

What is an odd number?

Possible student responses:

- An integer that CANNOT be evenly divided by 2.
- A number whose last digit is 1, 3, 5, 7, or 9.
- All the numbers that are NOT multiples of 2.

List some examples of odd numbers.

Answers will vary.

Present each question and then discuss the answer using models.

What happens when we add two even numbers? Will we always get an even number?
Before holding a discussion about the process to answer these questions, have students write or share their predictions.

### Exercises

1. **Why is the sum of two even numbers even?**
   
   a. Think of the problem $12 + 14$. Draw dots to represent each number.

   
   ![Dot Representation of 12 and 14](image)

   b. Circle pairs of dots to determine if any of the dots are left over.

   ![Circle Pairs of Dots](image)

   **There are no dots leftover; the answer will be even.**

   c. Will this be true every time two even numbers are added together? Why or why not?

   Since 12 is represented by 6 sets of two dots and 14 is represented by 7 sets of two dots, the sum will be 13 sets of two dots. This will be true every time two even numbers are added together because even numbers will never have dots left over when we are circling pairs. Therefore, the answer will always be even.

2. **Why is the sum of two odd numbers even?**

   a. Think of the problem $11 + 15$. Draw dots to represent each number.

   ![Dot Representation of 11 and 15](image)

   b. Circle pairs of dots to determine if any of the dots are left over.

   ![Circle Pairs of Dots](image)

   **Because each addend is an odd number, when we circle groups of two dots there is one dot remaining in each representation. When we look at the sum, however, the two remaining dots can form a pair, leaving us with a sum that is represented by groups of two dots. The sum is, therefore, even. Since each addend is odd, there is one dot for each addend that does not have a pair. However, these two dots can be paired together, which means there are no dots without a pair, making the sum an even number.**

   c. Will this be true every time two odd numbers are added together? Why or why not?

   This will be true every time two odd numbers are added together because every odd number will have one dot remaining when we circle pairs of dots. Since each number will have one dot remaining, these dots can be combined to make another pair. Therefore, no dots will be remaining resulting in an even sum.
Lesson 16: Even and Odd Numbers

3. Why is the sum of an even number and an odd number odd?
   a. Think of the problem 14 + 11. Draw dots to represent each number.
   b. Circle pairs of dots to determine if any of the dots are left over.

Students draw dots to represent each number. After circling pairs of dots, there will be one dot left for the number 11 and the number 14 will have no dots remaining. Since there is one dot leftover, the sum will be odd because every dot does not have a pair.

c. Will this be true every time an even number and an odd number are added together? Why or why not?

   This will always be true when an even number and an odd number are added together because only the odd number will have a dot remaining after we circle pairs of dots. Since this dot does not have a pair, the sum will be odd.

d. What if the first addend was odd and the second was even. Would the sum still be odd? Why or why not?

   For example, if we had 11 + 14, would the sum be odd?

   The sum will still be odd for two reasons. First, the commutative property states that changing the order of an addition problem does not change the answer. Because an even number plus an odd number is odd, then an odd number plus an even number is also odd. Second, it does not matter which addend is odd, there will still be one dot remaining making the sum odd.

If students are struggling, encourage them to draw dots to prove their answer. It may also be helpful to remind students of the commutative property to help them prove their answer.

Sum up the discussion by having students record notes in their handbooks.

Let’s sum it up:

   Even + even = even
   Odd + odd = even
   Odd + even = odd

Exploratory Challenge (20 minutes–12 minutes for group work; 8 minutes for gallery walk and discussion)

Divide students into small groups. Each group will be asked to determine on its poster paper whether one of the following products is odd or even: The product of two even numbers, the product of two odd numbers, or the product of an even number and an odd number. Encourage students to use previous knowledge about even and odd numbers, the connection between addition and multiplication, and visual methods (e.g., dots) in their proofs.

Scaffolding:
The teacher could also ask students if the same rules apply to subtraction. Using the same method for addition, have students determine if the rules apply to subtraction.

Scaffolding:

   ▪ If students are struggling with the proofs, the teacher can present each proof as students take notes in their handbooks. Or, allow students the time to explore and have a few groups who did not struggle present at the end.
   ▪ Ask early finishers if the same rule applies to division.

MP.8

MP.7
Exploratory Challenge

1. The product of two even numbers is even.
   
   Answers will vary, but two example answers are provided.
   
   Using the problem $6 \times 14$, students know that this is equivalent to the sum of six fourteens or $14 + 14 + 14 + 14 + 14 + 14$. Students also know that the sum of two even numbers is even; therefore, when adding the addends two at a time, the sum will always be even. This means the sum of six even numbers will be even, making the product even, since it will be equivalent to the sum.

   Using the problem $6 \times 14$, students can use the dots from previous examples.

   
   
   From here, students can circle dots and see that there will be no dots remaining, so the answer must be even.

2. The product of two odd numbers is odd.
   
   Answers will vary, but two example answers are provided.
   
   Using the problem $5 \times 15$, students know that this is equivalent to the sum of seven fifteens or $15 + 15 + 15 + 15 + 15 + 15$. Students also know that the sum of two odd numbers is even and the sum of an odd and even number is odd. When adding two of the addends together at a time, the answer will rotate between even and odd. When the final two numbers are added together, one will be even and the other odd. Therefore, the sum will be odd, which makes the product odd since it will be equivalent to the sum.

   Using the problem $5 \times 15$, students may also use the dot method.

   
   After students circle the pairs of dots, one dot from each set of $15$ will remain, for a total of $5$ dots. Students can group these together and circle more pairs, as shown below.

   Since there is still one dot remaining, the product of two odd numbers is odd.

3. The product of an even number and an odd number is even.
   
   Answers will vary, but two example answers are provided.
   
   Using the problem $6 \times 7$, students know that this is equivalent to the sum of six sevens or $7 + 7 + 7 + 7 + 7 + 7$. Students also know that the sum of two odd numbers is even and the sum of two even numbers is even. Therefore, when adding two addends at a time, the results will be an even number. Calculating the sum of these even numbers will also be even, which means the total sum will be even. This also implies the product will be even since the sum and product are equivalent.

   Using the problem $6 \times 7$, students may also use the dot method.

   
   After students circle the pairs of dots, one dot from each set of $7$ will remain, for a total of $6$ dots. Students can group these together and circle more pairs, as shown below.

   Since there are no dots remaining, the product of an even number and an odd number is even.
After students complete their posters, hang the posters up around the room. Conduct a gallery walk to allow groups to examine each poster and take notes in their student materials. In the end, students should have a proof for all three exercises in their student handbook.

Allow time for a discussion and an opportunity for students to ask any unanswered questions.

Closing (5 minutes)

- How does knowing whether a sum or product will be even or odd assist in division?
  - Possible student response: When dividing, it is helpful to know whether the sum or product of two numbers is even or odd, because it narrows down the possible factors. For example, if a dividend is odd then we know the factors must also be odd because the product of two odd numbers is odd.

Lesson Summary

Adding:
- The sum of two even numbers is even.
- The sum of two odd numbers is odd.
- The sum of an even number and an odd number is odd.

Multiplying:
- The product of two even numbers is even.
- The product of two odd numbers is odd.
- The product of an even number and an odd number is even.

Exit Ticket (5 minutes)
Lesson 16: Even and Odd Numbers

Exit Ticket

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. \(56,426 + 17,895\)

2. \(317,362 \times 129,324\)

3. \(10,481 + 4,569\)

4. \(32,457 \times 12,781\)

5. Show or explain why \(12 + 13 + 14 + 15 + 16\) will result in an even sum.
Exit Ticket Sample Solutions

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. \(56,426 + 17,895\)
   \begin{align*}
   \text{Odd, because the sum of an even number and an odd number is odd.}
   \end{align*}

2. \(317,362 \times 129,324\)
   \begin{align*}
   \text{Even, because the product of two even numbers is even.}
   \end{align*}

3. \(10,481 + 4,569\)
   \begin{align*}
   \text{Even, because the sum of two odd numbers is even.}
   \end{align*}

4. \(32,457 \times 12,781\)
   \begin{align*}
   \text{Odd, because the product of two odd numbers is odd.}
   \end{align*}

5. Show or explain why \(12 + 13 + 14 + 15 + 16\) will result in an even sum.
   \begin{align*}
   12 + 13 & \text{ will be odd because even + odd is odd.} \\
   \text{Odd number} + 14 & \text{ will be odd because odd + even is odd.} \\
   \text{Odd number} + 15 & \text{ will be even because odd + odd is even.} \\
   \text{Even number} + 16 & \text{ will be even because even + even is even.}
   \end{align*}

Problem Set Sample Solutions

Tell whether each sum or product is even or odd. Explain your reasoning.

1. \(346 + 721\)
   \begin{align*}
   \text{Odd, because the sum of an even and an odd number is odd.}
   \end{align*}

2. \(4,690 \times 141\)
   \begin{align*}
   \text{Even, because the product of an even and an odd number is even.}
   \end{align*}

3. \(1,462,891 \times 745,629\)
   \begin{align*}
   \text{Odd, because the product of two odd numbers is odd.}
   \end{align*}

4. \(425,922 + 32,481,064\)
   \begin{align*}
   \text{Even, because the sum of two even numbers is even.}
   \end{align*}
5. 32 + 45 + 67 + 91 + 34 + 56

The first two addends will be odd, because even and an odd is odd.

Odd number + 67 will be even because the sum of two odd numbers is even.

Even number + 91 will be odd because the sum of an even and an odd number is odd.

Odd number + 34 will be odd because the sum of an odd and an even number is odd.

Odd number + 56 will be odd because the sum of an odd and an even number is odd.

Therefore, the final sum will be odd.
Lesson 17: Divisibility Tests for 3 and 9

Student Outcomes

- Students apply divisibility rules, specifically for 3 and 9, to understand factors and multiples.

Lesson Notes

Students already have knowledge on the divisibility rules of 2, 4, 6, 8, and 10. Although those are not a focus for this lesson, they are revisited throughout the lesson. Also emphasize the difference between factors and multiples throughout the lesson.

Classwork

Opening Exercise (5 minutes)

The Opening Exercise will help students review the divisibility tests for the numbers 2, 4, 5, 8, and 10.

Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. You will place some numbers more than once. For example if 32 were on the list, you would place it in the circles with 2, 4, and 8 because they are all factors of 32.

24; 36; 80; 115; 214; 360; 975; 4,678; 29,785; 414,940

2

24; 36; 80; 214; 360; 4,678; 414,940

4

24; 36; 80; 360; 414,940

5

80; 115; 360; 975; 29,785; 414,940

8

24; 80; 360

10

80; 360; 414,940
Discussion (8 minutes)

Discuss students’ results from the opening exercise. Students can either share their answers or the teacher can conduct a poll (raising hands, standing up, electronically) to determine where students placed each number.

After sharing which numbers go in each circle, have students examine the numbers in the opening activity. Ask students to find shortcuts to determine in which group the number belongs just by looking at it.

Ask students to share their short cuts or rules and discuss the divisibility rules for each number. Have students take notes in their handbooks.

Discussion

- Divisibility rule for 2: If and only if its last digit is 0, 2, 4, 6, or 8.
- Divisibility rule for 4: If and only if its last two digits are a number divisible by 4.
- Divisibility rules for 5: If and only if its last digit is 0 or 5.
- Divisibility rule for 8: If and only if its last three digits are a number divisible by 8.
- Divisibility rule for 10: If and only if its last digit is 0.

Explain that students will learn two new divisibility rules today. The rules will be used to determine if numbers are divisible by 3 or 9. Start with a number students already know have factors of 3 and 9, so they can see that the rule works.

- What do the numbers 12, 18, 30, 66, and 93 all have in common?
  - They are divisible by 3.
- Calculate the sum of the digits for each given number. For example, the sum of the digits in the number 12 is 3 because $1 + 2 = 3$.
  - Provide time for students to find the sums. Record sums on the board.
- What do all these sums have in common?
  - They are divisible by 3.
- When the sum of a number’s digits is divisible by 3, the entire number is divisible by 3. Now let’s examine a different set of numbers: 27, 36, 54, 72, and 99. What do these numbers have in common?
  - They are divisible by 9.
- Calculate the sum of the digits for each given number.
  - Provide time for students to find the sums. Record sums on the board.
- What do all the sums have in common?
  - They are divisible by 9.
- When the sum of the digits is divisible by 3 and 9, the entire number is divisible by 9. Let’s try to use this knowledge to determine if a large number is divisible by 3, 9 or both. The number 765 is divisible by both 3 and 9. (Show students on the calculator). Find the sum of the digits.
  - Student Response: $7 + 6 + 5 = 18$.
- Are 3 and 9 both factors of 18?
  - Yes.
- Calculating the sum of a number’s digits helps us to determine if the number is divisible by 3 or 9 or both.
Introduce the divisibility rules for 3 and 9. Have students record the rules in their handbooks.

- Divisibility rule for 3: If the sum of the digits is divisible by 3, then the number is divisible by 3.
- Divisibility rule for 9: If the sum of the digits is divisible by 9, then the number is divisible by 9.

Through further discussion, explain to students that if a number is divisible by 9, it is also divisible by 3.
- Because $9 = 3 \times 3$, any number that is divisible by 9 will also be divisible by 3.

**Example 1 (5 minutes)**

Example 1
This example will show you how to apply the two new divisibility rules we just discussed.

Is 378 divisible by 3 or 9? Why or why not?

a. What are the three digits in the number 378?
   - 3, 7, and 8

b. What is the sum of the three digits?
   - $3 + 7 + 8 = 18$; the sum of the three digits is 18.

c. Is 18 divisible by 9?
   - Yes

d. Is the entire number 378 divisible by 9? Why or why not?
   - The number 378 is divisible by 9 because the sum of the digits is divisible by 9.

Scaffolding:
- If needed, the teacher can also ask if 18 is divisible by 3. Students may still struggle with the connection between the multiples of 3 and 9.
- If students struggled with the opening exercise, the divisibility rules for 2, 4, 5, 8, and 10 can be reviewed in this example as well.

This may be the place to help students recognize the difference between factors and multiples. Nine is a factor of 378 because it is the product of 9 and 42; therefore, 378 is a multiple of 9.

e. Is the number 378 divisible by 3? Why or why not?
   - Three is a factor of 378 because if 9 is a factor of 378, then 3 will also be a factor. OR
   - The number 378 is divisible by 3 because the sum of the digits is divisible by 3.

**Example 2 (5 minutes)**

The students have now seen one example of the two new divisibility rules. Allow students to work with a partner to decide whether a given number is divisible by 3 and 9. If a majority of students are still struggling, the teacher may ask the same leading questions found in Example 1.

- Is 3,822 divisible by 3 or 9? Why or why not?
Encourage students to check 9 first because if 9 is a factor then students know that 3 is also a factor. If 3,822 is not divisible by 9, then students must check to see if 3,822 is divisible by 3.

Example 2
Is 3,822 divisible by 3 or 9? Why or why not?

The number 3,822 is divisible by 3, but not by 9 because the sum of the digits is 3 + 8 + 2 + 2 = 15 and 15 is divisible by 3 but not by 9.

This is another opportunity to emphasize the difference between factors and multiples. Three is a factor of 3,822 because the product of 3 and 1,274 is 3,822; therefore, 3,822 is a multiple of 3.

Exercises 1–5 (13 minutes)
Students may work with partners or individually to complete the exercises. Remind students that they may circle more than one answer.

Exercises
Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. Is 2,838 divisible by
   - 3
   - 9
   - 4

   Explain your reasoning for your choices.
   The number 2,838 is divisible by 3 because 3 is a factor of 2,838. I know this because the sum of the digits is 21 which is divisible by 3. The number 2,838 is not divisible by 9 because 21 is not divisible by 9, and 2,838 is not divisible by 4 because the last two digits (38) are not divisible by 4.

2. Is 34,515 divisible by
   - 3
   - 9
   - 5

   Explain your reasoning for your choices.
   The number 34,515 is divisible by 3 and 9 because both 3 and 9 are factors of 34,515. I know this because the sum of the digits is 18 and 18 is divisible by both 3 and 9. The number 34,515 is also divisible by 5 because the last digit is a 5.

3. Is 10,534,341 divisible by
   - 3
   - 9
   - 2

   Explain your reasoning for your choices.
   The number 10,534,341 is divisible by 3 but not 9 because 3 is a factor of 10,534,341, but 9 is not. I know this because the sum of the digits is 21, which is divisible by 3 but not 9. The number 10,534,341 is not divisible by 2 because it does not end with 0, 2, 4, 6, or 8.
Lesson 17
Divisibility Tests for 3 and 9

4. Is 4,320 divisible by
   - 3
   - 9
   - 10

Explain your reasoning for your choices.

The number 4,320 is divisible by 3 and 9 because 3 and 9 are factors of 4,320. I know this because the sum of the digits is 9, which is divisible by 3 and 9. The number 4,320 is also divisible by 10 because 10 is a factor of 4,320.

5. Is 6,240 divisible by
   - 3
   - 9
   - 8

Explain your reasoning for your choices.

The number 6,240 is divisible by 3 but not divisible by 9 because 3 is a factor of 6,240, but 9 is not a factor. I know this because the sum of the digits is 12, which is divisible by 3 but not divisible by 9. The number 6,240 is divisible by 8 because 8 is a factor of 6,240 because the last three digits (240) is divisible by 8.

Closing (4 minutes)

- Without completing the division, how can you determine if a number is divisible by 3?
  - Calculate the sum of the digits; if the sum of the digits is divisible by 3, the entire number is divisible by 3.
- If a number is divisible by 9, will it be divisible by 3? Explain your answer.
  - If a number is divisible by 9, the sum of the digits will be divisible by 9. Any number that is divisible by 9 is also divisible by 3 since $9 = 3 \times 3$.
- If a number is divisible by 3, will it be divisible by 9? Explain your answer.
  - If a number is divisible by 3, it may not be divisible by 9 because 3 has more multiples than 9.

Lesson Summary

To determine if a number is divisible by 3 or 9:

- Calculate the sum of the digits.
- If the sum of the digits is divisible by 3, the entire number is divisible by 3.
- If the sum of the digits is divisible by 9, the entire number is divisible by 9.

Note: If a number is divisible by 9, the number is also divisible by 3.

Exit Ticket (5 minutes)
Lesson 17: Divisibility Tests for 3 and 9

Exit Ticket

1. Is 26,341 divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.

2. Is 8,397 divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.

3. Explain why 186,426 is divisible by both 3 and 9.
Exit Ticket Sample Solutions

1. Is 26,341 divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.
   The number 26,341 is not divisible by 3 because the sum of the digits is 16, which is not divisible by 3.

2. Is 8,397 divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.
   The number 8,397 is divisible by 9 because the sum of the digits is 27, which is divisible by 9. Nine is a factor of 8,397 because $9 \times 933 = 8,397$.

3. Explain why 186,426 is divisible by both 3 and 9.
   The number 186,426 is divisible by both 3 and 9 because the sum of the digits is 27, which is divisible by both 3 and 9.

Problem Set Sample Solutions

1. Is 32,643 divisible by both 3 and 9? Why or why not?
   The number 32,643 is divisible by both 3 and 9 because the sum of the digits is 18, which is divisible by 3 and 9.

2. Circle all the factors of 424,380 from the list below.
   $$\begin{array}{cccccccc}
   2 & 3 & 4 & 5 & 8 & 9 & 10
   \end{array}$$

3. Circle all the factors of 322,875 from the list below.
   $$\begin{array}{cccccccc}
   2 & 3 & 4 & 5 & 8 & 9 & 10
   \end{array}$$

4. Write a 3 digit number that is divisible by both 3 and 4. Explain how you know this number is divisible by 3 and 4.
   Answers will vary. e.g., The sum of the digits is divisible by 3, and that's how I know the number is divisible by 3. And the last 2 digits are divisible by 4, so the entire number is divisible by 4.

5. Write a 4 digit number that is divisible by both 5 and 9. Explain how you know this number is divisible by 5 and 9.
   Answers will vary. e.g., The number ends with a 5 or 0, so the entire number is divisible by 5. And the sum of the digits is divisible by 9, so the entire number is divisible by 9.
Lesson 18: Least Common Multiple and Greatest Common Factor

Student Outcomes
- Students find the least common multiple and greatest common factor and apply factors to the Distributive Property

Lesson Notes
Least common multiple and greatest common factor are terms that are easily confused by young learners. A clear definition of both with several examples of each should be posted in the classroom before, during, and after the lesson. Furthermore, these skills should be practiced regularly, so the concepts do not fade or blend together from lack of use.

During this lesson, students will move around in groups to various stations where a topic is presented on chart paper. At each station, students read the directions, choose a problem, and then work collaboratively to solve the problem. Group students prior to the lesson using the most appropriate ability or social grouping.

There are four different topics: Factors and GCF, Multiples and LCM, Using Prime Factors to Determine GCF, and Applying Factors to the Distributive Property.

If two sets of chart paper are prepared for each topic, there will be 8 stations. This makes manageable groups of 3–4 students. Further, if the stations are placed in order around the room (1, 2, 3, 4, 1, 2, 3, 4) it will not matter where a group starts, and the group will finish after only three rotations. Groups should spend five minutes at each station.

Suggested Student Roles:

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marker</td>
<td>This student records the group’s work on the chart paper. Note: Each group should use a different (unique) color when adding its work to the chart paper.</td>
</tr>
<tr>
<td>Recorder</td>
<td>This student records the group’s work in his/her student materials and later shares this work with the other members of the group ensuring that all students finish the activity with their student materials completed.</td>
</tr>
<tr>
<td>Calculator Operator/Master Mathematician</td>
<td>This student uses a calculator when necessary and helps clarify the group’s response</td>
</tr>
</tbody>
</table>

Materials: 8 pieces of chart paper with directions attached, one calculator per group, a different colored chart marker for each group, a multiplication table posted at Stations 2 and 4.
Classwork

Opening Exercise (8 minutes)

Point out the definitions on the student pages and work through the examples before assigning and releasing groups.

The Greatest Common Factor of two whole numbers $a$ and $b$, written $GCF(a,b)$, is the greatest whole number, which is a factor of both $a$ and $b$.

The Least Common Multiple of two nonzero numbers $a$ and $b$, written $LCM(a,b)$, is the least whole number (larger than zero), which is a multiple of both $a$ and $b$.

Example 1 (3 minutes)

Example 1

Greatest Common Factor: Find the greatest common factor of 12 and 18.

- Listing these factors as pairs can help you not miss any. Start with one times the number.
- Circle all factors that appear on both lists.
- Place a triangle around the greatest of these common factors.

$GCF(12, 18) = 6$

\[
\begin{array}{|c|}
\hline
12 \\
1 & 12 \\
2 & 6 \\
3 & 4 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|}
\hline
18 \\
1 & 18 \\
2 & 9 \\
3 & 6 \\
\hline
\end{array}
\]
Example 2 (5 minutes)

Least Common Multiple: Find the least common multiple of 12 and 18.

LCM 12, 18

- List the first 10 multiples of both numbers.
- Circle all multiples that appear on both lists.
- Place a triangle around the least of these common multiples.
  - Student circles 36, 72, and 108 and draws a triangle around 36.
- Did you really need to write out 10 multiples of each number?
  - No, we could have stopped as soon as a multiple appeared on both lists.
- Should we start by writing the multiples of the larger or the smaller of the two numbers? Which will lead us to find the LCM most efficiently?
  - If we start writing the multiples of the larger of the two numbers, we can stop when we find the first one that is a multiple of the smaller of the two numbers. In the example given, we would list the multiples of 18 first and stop at 36 because 36 is a multiple of 12. In this case we will have found the LCM 12, 18 after listing only two numbers.

Discussion (5 minutes)

- Today you will be visiting several stations around the room. At each station, you will have 5 minutes to read and follow directions. Only the Recorder's paper will be used at the station. Later the Recorder will share his copy of the work so each of you will leave with a record of today's classwork.
- Another person, the Marker, in the group will have the chart marker for writing on the chart paper, and a third person will serve as Calculator Operator/Master Mathematician who will use a calculator when necessary and help clarify your group's response before putting it on the chart paper.
At each station the directions start out the same way: Choose one of the problems that have not yet been solved. Solve it together on the Recorder’s page. The Marker should copy your group’s work neatly on the chart paper and cross out the problem your group solved, so the next group solves a different problem.

**Exploratory Challenge 1: Factors and GCF (5 minutes)**

Groups will choose from the following number problems:

Find the greatest common factor of one of these pairs: 30, 50; 30, 45; 45, 60; 42, 70; 96, 144

Next, groups will choose from the following application problems:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same ratio of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b. The Ski Club members are preparing identical welcome kits for the new skiers. The Ski Club has 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of identical kits they can prepare using all of the hand warmer and foot warmer packets?

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representatives and senators could be formed from all of Congress, if we want the largest groups possible?

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

<table>
<thead>
<tr>
<th>Exploratory Challenge 1: Factors and GCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.</td>
</tr>
</tbody>
</table>

**GCF 30, 50**

**Factors of 30**: 1, 2, 3, 5, 6, 10, 15, 30.  
**Common Factors**: 1, 2, 5, 10.  
**Factors of 50**: 1, 2, 5, 10, 25, 50.  
**Greatest Common Factor**: 10.

**GCF 30, 45**

**Factors of 30**: 1, 2, 3, 5, 6, 10, 15, 30.  
**Common Factors**: 1, 3, 5, 15.  
**Factors of 45**: 1, 3, 5, 9, 15, 45.  
**Greatest Common Factor**: 15.

**GCF 45, 60**

**Factors of 45**: 1, 3, 5, 9, 15, 45.  
**Common Factors**: 1, 3, 5, 15.  
**Factors of 60**: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.  
**Greatest Common Factor**: 15.

**GCF 42, 70**

**Factors of 42**: 1, 2, 3, 6, 7, 14, 21, 42.  
**Common Factors**: 1, 2, 7, 14.  
**Factors of 70**: 1, 2, 5, 7, 10, 14, 35, 70.  
**Greatest Common Factor**: 14.

**GCF 96, 144**

**Factors of 96**: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96.  
**Common Factors**: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.  
**Factors of 144**: 1, 2, 3, 4, 6, 8, 9, 12, 16, 24, 36, 48, 72, 144.  
**Greatest Common Factor**: 48.
Next, choose one of these problems that have not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?
   
   6 teams, each having 3 girls and 4 boys.

b. The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand warmer and foot warmer packets?
   
   12 welcome kits, each having 5 hand warmer packets and 4 foot warmer packets.

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representative and senators could be formed from all of Congress, if we want the largest groups possible?
   
   5 groups, each with 87 representatives and 20 senators.

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.
   
   Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.
   
   No. Factors are, by definition, less than or equal to the number. Therefore, the GCF cannot be greater than both numbers.

Exploratory Challenge 2: Multiples and LCM (5 minutes)

Groups will choose from the following number problems:

Find the least common multiple of one of these pairs: 9,12; 8,18; 4,30; 12,30; 20,50

Next, groups will choose from the following application problems:

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?

b. Starting at 6:00 a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.
Exploratory Challenge 2: Multiples and LCM

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

LCM 9, 12

Multiples of 9: 9, 18, 27, 36. 
Least Common Multiple: 36.

Multiples of 12: 12, 24, 36.

LCM 8, 18

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72.
Least Common Multiple: 72.

Multiples of 18: 36, 54, 72.

LCM 4, 30

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60.
Least Common Multiple: 60.

Multiples of 30: 30, 60.

LCM 12, 30

Multiples of 12: 12, 24, 36, 48, 60
Least Common Multiple: 60.

Multiples of 30: 30, 60.

LCM 20, 50

Multiples of 20: 20, 40, 60, 80, 100.
Least Common Multiple: 100.

Multiples of 50: 50, 100.

Next, choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so the next group solves a different problem.

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?

4 packages of hot dogs = 40 hot dogs. 5 packages of buns = 40 buns. LCM 8, 10 = 40.

b. Starting at 6:00 a.m., A bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

7:00 a.m., which is 60 minutes after 6:00 a.m. LCM 12, 15 = 60.

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

5 revolutions. During this time 120 teeth will pass by. The second gear will revolve 3 times. LCM 24, 40 = 120.

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

No. Multiples are, by definition, equal to or greater than the number.
Exploratory Challenge 3: Using Prime Factors to Determine GCF (5 minutes)

Note: If the classroom has Internet access, a Factor Tree applet is available at:
http://nlvm.usu.edu/en/nav/frames_asid_202_g_3_t_1.html#factor

Choose “Two Factor Trees” and “User Defined Problems”. When both numbers are prime factored, each common prime factor is dragged into the middle of a two-circle Venn diagram. Unique prime factors are separated into the other two regions of the Venn diagram. Introducing the applet before the lesson and allowing exploration time will strengthen understanding and make this lesson more cohesive.

Groups will choose from the following number problems:
Use prime factors to find the greatest common factor of one of the following pairs of numbers:
30, 50; 30, 45; 45, 60; 42, 70; 96, 144.

Next, groups will choose from the following application problems:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?
b. Find the GCF of your original pair of numbers.
c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?
d. Glenn’s favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The prime factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn’s number? When was Sarah born?
Next, choose one of these problems that has not yet been solved:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?
   
   *Accept opinions. Students should defend their answer, and use accurate mathematical terms in their response.*

b. Find the GCF of your original pair of numbers.
   
   *See answers listed in Exploratory Challenge 1.*

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?
   
   *In all cases, GCF \(a, b \cdot \text{LCM } a, b = a \cdot b.*

d. Glenn’s favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn’s number? When was Sarah born?
   
   \[2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2310\]  
   *Birthdate 2/3/10.*

**Exploratory Challenge 4: Applying Factors to the Distributive Property (5 minutes)**

Study these examples of how factors apply to the Distributive Property:

\[8 + 12 = 4(2) + 4(3) = 4(2 + 3) = 20\] \[15 + 25 = 5(3) + 5(5) = 5(3 + 5) = 40\]

Students will factor out the GCF from the two numbers and rewrite the sum using the Distributive Property.

Groups will choose from the following problems:

a. \(12 + 18 =\)

b. \(42 + 14 =\)

c. \(36 + 27 =\)

d. \(16 + 72 =\)

e. \(44 + 33 =\)

Next, students will add their own examples to one of two statements applying factors to the Distributive Property:

\[n(a) + n(b) = n(a + b)\]
\[n(a) - n(b) = n(a - b)\]
Exploratory Challenge 4: Applying Factors to the Distributive Property

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the GCF from the two numbers and rewrite the sum using the Distributive Property

1. \( 12 + 18 = \)
   \( 6(2) + 6(3) = 6(2 + 3) = 6(5) = 30 \)
2. \( 42 + 14 = \)
   \( 7(6) + 7(2) = 7(6 + 2) = 7(8) = 56 \)
3. \( 36 + 27 = \)
   \( 9(4) + 9(3) = 9(4 + 3) = 9(7) = 63 \)
4. \( 16 + 72 = \)
   \( 8(2) + 8(9) = 8(2 + 9) = 8(11) = 88 \)
5. \( 44 + 33 = \)
   \( 11(4) + 11(3) = 11(4 + 3) = 11(7) = 77 \)

Next, add another new example to one of these two statements applying factors to the Distributive Property.

Choose any numbers for \( n, a, \) and \( b. \)
\( n (a) + n (b) = n (a + b) \)

Accept all student responses that are mathematically correct.

\( n (a) − n (b) = n (a − b) \)

The Distributive Property holds for addition as well as subtraction. Accept all student responses that are mathematically correct.

Closing (6 minutes)

- Use this time to discuss each station. Assign the problem set which asks students to revisit each topic independently.

Exit Ticket (4 minutes)
Lesson 18: Least Common Multiple and Greatest Common Factor

Exit Ticket

1. Find the LCM and GCF of 12 and 15.

2. Write two numbers, neither of which is 8, whose GCF is 8.

3. Write two numbers, neither of which is 28, whose LCM is 28.

Rate each of the stations you visited today. Use this scale:
3 – Easy – I’ve got it; I don’t need any help.
2 – Medium – I need more practice, but I’m understanding some of it.
1 – Hard – I’m not getting this yet.

Complete the following chart:

<table>
<thead>
<tr>
<th>Station</th>
<th>Rating (3, 2, 1)</th>
<th>Comment to the Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 1: Factors and GCF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 2: Multiples and LCM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 3: Using Prime Factors for GCF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 4: Applying factors to the Distributive Property</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exit Ticket Sample Solutions

1. Find the LCM and GCF of 12 and 15.
   \[ \text{LCM: 60; GCF: 3} \]

2. Write two numbers, neither of which is 8, whose GCF is 8.
   \[ \text{Answers will vary, i.e., 16 and 24, or 24 and 32} \]

3. Write two numbers, neither of which is 28, whose LCM is 28.
   \[ \text{Answers will vary, i.e., 4 and 14, or 4 and 7} \]

Rate each of the stations you visited today. Use this scale:
3 – Easy – I've got it, I don’t need any help.
2 – Medium – I need more practice, but I'm understanding some of it.
1 – Hard – I'm not getting this yet.

Complete the following chart:

<table>
<thead>
<tr>
<th>Station</th>
<th>Rating (3, 2, 1)</th>
<th>Comment to the Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station 1 Factors and GCF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 2 Multiples and LCM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 3 Using Prime Factors for GCF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station 4 Applying factors to the Distributive Property</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem Set Sample Solutions

Students should complete the remaining exploratory challenge problems from class.
Exploratory Challenge Reproducible

Exploratory Challenge 1: Factors and GCF

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the greatest common factor of one of these pairs: 30, 50  30, 45  45, 60  42, 70  96, 144.

Next, choose one of these problems that have not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b. The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand warmer and foot warmer packets?

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups could be formed from all the senators and representatives?

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.
Exploratory Challenge 2: Multiples and LCM

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the least common multiple of one of these pairs: 9, 12  8, 18  4, 30  12, 30  20, 50.

Next, choose one of these problems that have not yet been solved:

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy?

b. Starting at 6:00 a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Solve it together on your student page. Then use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so the next group solves a different problem.
Exploratory Challenge 3: Using Prime Factors to Determine GCF

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Use Prime Factors to find the Greatest Common Factor of one of the following pairs of numbers:

30, 50  30, 45  45, 60  42, 70  96, 144

Next, choose one of these problems that have not yet been solved:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

b. Find the GCF of your original pair of numbers.

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?

d. Glenn’s favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat and are all the prime numbers less than 12. What is Glenn’s number? When was Sarah born?
Exploratory Challenge 4: Applying Factors to the Distributive Property

Study these examples of how factors apply to the Distributive Property:

\[8 + 12 = 4(2) + 4(3) = 4(2+3) = 20\]  
\[15 + 25 = 5(3) + 5(5) = 5(3+5) = 40\]  
\[36 – 24 = 4 (9) – 4 (6) = 4 (9–6) = 12\]

\[4(2) + 4(3) = 4(5) = 20\]  
\[5(3) + 5(5) = 5(8) = 40\]  
\[4 (9) – 4 (6) = 4 (3) = 12\]

Choose one of these problems that have not yet been solved. Solve it together on your student page. Then use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so the next group solves a different problem.

Find the GCF from the two numbers and rewrite the sum using the Distributive Property.

a. \(12 + 18 = \)

b. \(42 + 14 = \)

c. \(36 + 27 = \)

d. \(16 + 72 = \)

e. \(44 + 33 = \)

Next, add another new example to one of these two statements applying factors to the Distributive Property.

Choose any numbers for \(n, a,\) and \(b.\)

\(n(a) + n(b) = n(a + b)\)

\(n(a) – n(b) = n(a – b)\)
Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Student Outcomes

- Students explore and discover that Euclid’s Algorithm is a more efficient means to finding the greatest common factor of larger numbers and determine that Euclid’s Algorithm is based on long division.

Lesson Notes

MP.7

Students look for and make use of structure, connecting long division to Euclid’s Algorithm.

Students look for and express regularity in repeated calculations leading to finding the greatest common factor of a pair of numbers.

These steps are contained in the Student Materials and should be reproduced, so they can be displayed throughout the lesson:

Euclid’s Algorithm is used to find the greatest common factor (GCF) of two whole numbers.

1. Divide the larger of the two numbers by the smaller one.
2. If there is a remainder, divide it into the divisor.
3. Continue dividing the last divisor by the last remainder until the remainder is zero.
4. The final divisor is the GCF of the original pair of numbers.

In application, the algorithm can be used to find the side length of the largest square that can be used to completely fill a rectangle so that there is no overlap or gaps.

Classwork

Opening (5 minutes)

Lesson 18 Problem Set can be discussed before going on to this lesson.
Opening Exercise (4 minutes)

- There are 3 division warm-ups on your Student Material page today. Please compute them now. Check your answer to make sure it is reasonable.

**Opening Exercise**

Euclid’s Algorithm is used to find the greatest common factor (GCF) of two whole numbers.

1. Divide the larger of the two numbers by the smaller one.
2. If there is a remainder, divide it into the divisor.
3. Continue dividing the last divisor by the last remainder until the remainder is zero.
4. The final divisor is the GCF of the original pair of numbers.

\[ \frac{383}{4} = 95.75 \]
\[ \frac{432}{12} = 36 \]
\[ \frac{403}{13} = 11 \]

Discussion (2 minutes)

- Our opening exercise was meant for you to recall how to determine the quotient of two numbers using long division. You have practiced the long division algorithm many times. Today’s lesson is inspired by Euclid, a Greek mathematician, who lived around 300 BCE. He discovered a way to find the greatest common factor of two numbers that is based on long division.

**Example 1 (10 minutes): Euclid’s Algorithm Conceptualized**

- What is the GCF of 60 and 100?
  - 20
- What is the interpretation of the GCF in terms of area? Let’s take a look.
  Project the following diagram:

![Example 1: Euclid’s Algorithm Conceptualized](image)
- Notice that we can use the GCF of 20 to create the largest square tile that will cover the rectangle without any overlap or gaps. We used a $20 \times 20$ tile.

- But, what if we didn’t know that? How could we start? Let’s guess! What is the biggest square tile that we can guess?
  - $60 \times 60$

Display the following diagram:

- It fits, but there are 40 units left over. Do the division problem to prove this.

Teacher note: with each step in this process, please write out the long division algorithm that accompanies it.

It is important for students to make a record of this as well. The remainder becomes the new divisor and continues until the remainder is zero.

- What is the leftover area?
  - $60 \times 40$

- What is the largest square tile that we can fit in the leftover area?
  - $40 \times 40$
Display the following diagram:

- What is the leftover area?
  - $20 \times 40$
- What is the largest square tile that we can fit in the leftover area?
  - $20 \times 20$
- When we divide 40 by 20, there is no remainder. We have tiled the whole rectangle!
- If we had started tiling the whole rectangle with squares, the largest square we could have used would be 20 by 20.

Take a few minutes to allow discussion, questions, and clarification.

**Example 2 (5 minutes): Lesson 18 Classwork Revisited**

During Lesson 18, students found the GCF of several pairs of numbers.

- Let’s apply Euclid’s Algorithm to some of the problems from our last lesson.

**Example 2: Lesson 18 Classwork Revisited**

a. Let’s apply Euclid’s Algorithm to some of the problems from our last lesson.
   i. What is the GCF of 30 and 50?
      
      10
   ii. Using Euclid’s Algorithm, we follow the steps that are listed in the opening exercise.

   $\begin{align*}
   30 & \div 50 = 1 \text{ R } 20 \\
   50 & \div 20 = 2 \text{ R } 10 \\
   20 & \div 10 = 2 \text{ R } 0 \\
   \end{align*}$

   *When the remainder is zero, the final divisor is the GCF.*
Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

b. What is the GCF of 30 and 45?

\[
\begin{array}{c|cc}
30 & 45 \\
\hline
1 & 15 \\
-30 & -30 \\
\hline
15 & 00 \\
\end{array}
\]

Example 3 (5 minutes): Larger Numbers

Example 3: Larger Numbers

GCF (96, 144)

\[
\begin{array}{c|cc}
96 & 144 \\
\hline
1 & 48 \\
-96 & -96 \\
\hline
048 & 00 \\
\end{array}
\]

GCF (660, 840)

\[
\begin{array}{c|cc}
660 & 840 \\
\hline
1 & 180 \\
-660 & -120 \\
\hline
120 & 00 \\
\end{array}
\]

Example 4 (5 minutes): Area Problems

Example 4: Area Problems

The greatest common factor has many uses. Among them, the GCF lets us find out the maximum size of squares that will cover a rectangle. Whenever we solve problems like this, we cannot have any gaps or any overlapping squares. Of course, the maximum size squares will be the minimum number of squares needed.

A rectangular computer table measures 30 inches by 50 inches. We need to cover it with square tiles. What is the side length of the largest square tile we can use to completely cover the table, so that there is no overlap or gaps?
Direct students to consider the GCF (30, 50), which they already calculated. It should become clear that squares of 10 inches by 10 inches will tile the area.

c. If we use squares that are 10 by 10, how many will we need?  
   \[3 \cdot 5, \text{ or } 15 \text{ squares}\]

d. If this were a giant chunk of cheese in a factory, would it change the thinking or the calculations we just did?  
   \[\text{No.}\]

e. How many 10 inch \times 10 inch squares of cheese could be cut from the giant 30 inch \times 50 inch slab?  
   \[15\]

Closing (3 minutes)

- Euclid’s Algorithm is used to find the greatest common factor (GCF) of two whole numbers. The steps are listed on your Student Page and need to be followed enough times to get a zero remainder. With practice, this can be a quick method for finding the GCF.
- Let’s use the steps to solve this problem: Kiana is creating a quilt made of square patches. The quilt is 48 inches in length and 36 inches in width. What is the largest size of square length of each patch?
- Divide the larger of the two numbers by the smaller one. What is the quotient?  
  \[1\]
- If there is a remainder, divide it into the divisor. What is the remainder?  
  \[12\]
- What is the divisor?  
  \[36\]
- Divide 36 by 12. What is the quotient?  
  \[3\]
- Continue dividing the last divisor by the last remainder until the remainder is zero. Is the remainder zero?  
  \[\text{Yes}\]
- The final divisor is the GDF of the original pair of numbers. What is the final divisor?  
  \[12\]

Exit Ticket (5 minutes)

Scaffolding:
- To find the GCF of two numbers, give students rectangular pieces of paper using the two numbers as length and width (e.g., 6 by 16 cm.)
- Challenge students to mark the rectangles into the largest squares possible, cut them up, and stack them to assure their squareness.

Scaffolding:
- Ask students to compare the process of Euclid’s Algorithm with a subtraction-only method of finding the GCF:
  1. List the two numbers.
  2. Find their difference.
  3. Keep the smaller of the two numbers; discard the larger.
  4. Use the smaller of the two numbers and the difference to form a new pair.
  5. Repeat until the two numbers are the same. This is the GCF.

Teacher resource:  
http://www.youtube.com/watch?v=2HsIpFAXvKk
Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Exit Ticket

Use Euclid’s Algorithm to find the Greatest Common Factor of 45 and 75.
Exit Ticket Sample Solutions

Use Euclid’s Algorithm to find the greatest common factor of 45 and 75.

\[
\begin{array}{c|c}
45 & 1 \\
\hline
75 & 30 \\
\hline
-45 & -30 \\
\hline
30 & -30 \\
\hline
15 & 0 \\
\end{array}
\]

The GCF of 45 and 75 is 15.

Problem Set Sample Solutions

1. Use Euclid’s Algorithm to find the greatest common factor of the following pairs of numbers:
   a. GCF of 12 and 78.
      \[
      \begin{array}{c|c}
      12 & 6 \\
      \hline
      78 & 12 \\
      \hline
      -66 & -6 \\
      \hline
      6 & 0 \\
      \end{array}
      \]
      GCF of 12 and 78 = 6
   b. GCF of 18 and 176
      \[
      \begin{array}{c|c}
      18 & 9 \\
      \hline
      176 & 14 \\
      \hline
      -162 & -13 \\
      \hline
      54 & 0 \\
      \end{array}
      \]
      GCF of 18 and 176 = 2

2. Juanita and Samuel are planning a pizza party. They order a rectangular sheet pizza which measures 21 inches by 36 inches. They tell the pizza maker not to cut it because they want to cut it themselves.
   a. All pieces of pizza must be square with none left over. What is the length of the side of the largest square pieces into which Juanita and Samuel can cut the pizza?
      \[\text{LCM of 21 and 36} = 3. \text{ They can cut the pizza into 3 by 3 inch squares.}\]
   b. How many pieces of this size will there be?
      \[7 \cdot 12 = 84. \text{ There will be 84 pieces.}\]

3. Shelly and Mickelle are making a quilt. They have a piece of fabric that measures 48 inches by 168 inches.
   a. All pieces of fabric must be square with none left over. What is the length of the side of the largest square pieces into which Shelly and Mickelle can cut the fabric?
      \[\text{GCF of 48 and 168} = 24.\]
   b. How many pieces of this size will there be?
      \[2 \cdot 7 = 14. \text{ There will be 14 pieces.}\]
1. L.B. Johnson Middle School held a track and field event during the school year. The chess club sold various drink and snack items for the participants and the audience. All together they sold 486 items that totaled $2,673.

   a. If the chess club sold each item for the same price, calculate the price of each item.

   b. Explain the value of each digit in your answer to 1(a) using place value terms.
2. The long jump pit was recently rebuilt to make it level with the runway. Volunteers provided pieces of wood to frame the pit. Each piece of wood provided measured 6 feet, which is approximately 1.8287 meters.

\[
\begin{align*}
2.75 \text{ meters} \\
9.54 \text{ meters}
\end{align*}
\]

a. Determine the amount of wood, in meters, needed to rebuild the frame.

b. How many boards did the volunteers supply? Round your calculations to the nearest thousandth and then provide the whole number of boards supplied.
3. Andy ran 436.8 meters in 62.08 seconds.
   
   a. If Andy ran at a constant speed, how far did he run in one second? Give your answer to the nearest tenth of a second.

   b. Use place value, multiplication with powers of 10, or equivalent fractions to explain what is happening mathematically to the decimal points in the divisor and dividend before dividing.

   c. In the following expression, place a decimal point in the divisor and the dividend to create a new problem with the same answer as in 3(a). Then explain how you know the answer will be the same.

     \[ \frac{436.8}{62.08} \]
4. The PTA created a cross-country trail for the meet.

   a. The PTA placed a trail marker in the ground every four hundred yards. Every nine hundred yards the PTA set up a water station. What is the shortest distance a runner will have to run to see both a water station and trail marker at the same location?

      Answer: ................................................ hundred yards

   b. There are 1,760 yards in one mile. About how many miles will a runner have to run before seeing both a water station and trail marker at the same location? Calculate the answer to the nearest hundredth of a mile.

   c. The PTA wants to cover the wet areas of the trail with wood chips. They find that one bag of wood chips covers a $3\frac{1}{2}$ yards section of the trail. If there is a wet section of the trail that is approximately $50\frac{1}{4}$ yards long, how many bags of wood chips are needed to cover the wet section of the trail?
5. The Art Club wants to paint a rectangle-shaped mural to celebrate the winners of the track and field meet. They designed a checkerboard background for the mural where they will write the winners’ names. The rectangle measures 432 inches in length and 360 inches in width. What is the side length of the largest square they can use to fill the checkerboard pattern completely without overlap or gaps?
### A Progression Toward Mastery

<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
<th>STEP 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> 6.NS.2</td>
<td>Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</td>
<td>Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
<td>A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</td>
</tr>
<tr>
<td>a</td>
<td>Student response is missing or depicts inaccurate operation choice.</td>
<td>Student response is inaccurate and does not represent the correct place value.</td>
<td>Student response is inaccurate through minor calculation errors, however place value is represented accurately.</td>
<td>Student response is correct. The price of each item is determined as $5.50, where place value is represented accurately.</td>
</tr>
<tr>
<td>b 6.NS.2</td>
<td>Student response is incorrect or missing. Place value is not depicted in the response.</td>
<td>Student response depicts place value only in monetary denominations, such as dollars and cents.</td>
<td>Student response depicts place value accurately, but makes little to no correlation to monetary denominations.</td>
<td>Student response is accurate. Each place value is labeled accurately AND shows correlation to the monetary denominations each place value represents. For example: 5 dollars is labeled with 5 ones and 5 dimes, 50 cents is labeled as 5 tenths and 5 dimes, and the zero in the hundredths place is labeled with zero hundredths and &quot;no pennies.&quot;</td>
</tr>
<tr>
<td><strong>2</strong> 6.NS.3</td>
<td>Student response is incorrect or missing. Students merely included one length and one side in their calculation.</td>
<td>Student response is incorrect based on place value.</td>
<td>Student response depicts understanding of the addition algorithm, but minor calculation errors hindered the correct sum of 24.58 meters.</td>
<td>Student calculations include all sides of the sand pit. Student applied the standard algorithm of addition of decimals to determine the correct sum of 24.58 meters.</td>
</tr>
</tbody>
</table>
### End-of-Module Assessment Task

**6•2**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number</th>
<th>6.NS.3</th>
<th>6.NS.3</th>
<th>6.NS.3</th>
<th>6.NS.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>3 a</td>
<td>Student response is incorrect or missing. Calculations disregard place value.</td>
<td>Student response is incorrect. Response depicts inaccurate place value where the divisor is represented by a whole number, but the dividend remains a decimal.</td>
<td>Student response is correct, but the quotient of 7.03 is not rounded to the nearest tenth, OR calculations are incorrect, but represent knowledge of place value.</td>
<td>Student response is correct, depicting accurate place value in order to generate a whole number dividend. Calculations are flawless AND the answer, 7.0, is represented to the nearest tenth.</td>
</tr>
<tr>
<td>b</td>
<td>4 a</td>
<td>Student response either incorrectly depicts place value or is missing.</td>
<td>Student response depicts some place value knowledge, but not enough to sufficiently describe why and how a whole number divisor is generated.</td>
<td>Student response correctly includes accurate place value through the use of equivalent fractions to demonstrate how and to generate a whole number divisor.</td>
<td>Student response is correct and includes multiplying by a power of ten to determine an equivalent fraction with a whole number denominator. Student determines that the quotient of the decimals is equivalent to the quotient of the whole numbers generated through the use of place value.</td>
</tr>
<tr>
<td>c</td>
<td>6.NS.3</td>
<td>Student response is missing.</td>
<td>Student response is incorrect or indicates the same decimal placements from the previous problem.</td>
<td>Student response accurately places decimals in the divisor and dividend with no explanation or justification.</td>
<td>Student response accurately places decimals within the divisor (6.208) and dividend (43.68) to generate a quotient of 7.03 AND justifies placement through the use of either place value, powers of ten, or equivalent fractions.</td>
</tr>
</tbody>
</table>
| 4 a     | 6.NS.4 | Student response is incorrect or missing. Response is a result of finding the sum of or the difference between 9 and 4. | Student response is incorrect, OR the response is simply the product of 4 and 9 with no justification. | Student response accurately finds the least common multiple of 4 and 9, but the response is determined as 36, instead of 36 hundred or 3,600 yards, OR the response is accurately determined through finding the least common multiple. The response represents understanding of the unit “hundred” as a
<table>
<thead>
<tr>
<th></th>
<th>6.NS.2</th>
<th>6.NS.1</th>
<th>6.NS.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b</strong></td>
<td>Student response is missing. Or the response utilizes incorrect operations, such as addition, subtraction or multiplication.</td>
<td>Student response shows little reasoning through the use of division to determine the quotient. Student response depicts division of 1,760 yards by a divisor of 2, derived from counting the two stations. The student response did not include values from the previous problem.</td>
<td>Student response is incorrect, but did include values from the previous problem. Instead of using 3,600, however, the response chose 36 as the dividend, resulting in an incorrect quotient.</td>
</tr>
<tr>
<td><strong>c</strong></td>
<td>Student response is incorrect or missing. Response includes inappropriate operations, such as addition, subtraction, or multiplication.</td>
<td>The student response is incorrect due to inaccurate calculations when converting mixed numbers OR when finding the quotients of the fractions.</td>
<td>Student response is correctly determined through mixed number conversion and division of fractions, but is inaccurately left as a mixed number (14 \frac{5}{14}).</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>Student response is incorrect or missing. Response includes inappropriate operations, such as addition, subtraction, or multiplication.</td>
<td>Student response is incorrect, but depicts reasoning leading to finding the greatest common factor. OR student response incorrectly utilizes division to determine the quotient of (1 \frac{5}{72}).</td>
<td>Student response determines that the greatest common factor of 432 and 360 is 72 through means other than the Euclidean Algorithm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Student response efficiently utilizes the Euclidean Algorithm to determine the greatest common factor of 432 and 360 as 72. Response correlates the GCF to the side length of the largest square.</td>
</tr>
</tbody>
</table>
1. L.B. Johnson Middle School held a track and field event during the school year. The chess club sold various drink and snack items for the participants and the audience. All together they sold 486 items that totaled $2,673.

   a. If the chess club sold each item for the same price, calculate the price of each item. (6.NS.2 – Lesson 13)

   
   ![Calculation]

   Each item's price is $5.50.

   b. Explain the value of each digit in your answer to 1(a) using place value terms. (6.NS.2 – Lesson 13)
2. The long jump pit was recently rebuilt to make it level with the runway. Volunteers provided pieces of wood to frame the pit. Each piece of wood provided measured 6 feet, which is approximately 1.8287 meters.

\[
\begin{align*}
2.75 \text{ meters} & \\
+ 2.75 & \\
9.54 \text{ meters} & 
\end{align*}
\]

a. Determine the amount of wood, in meters, needed to rebuild the frame. (6.NS.3 – Lesson 9)

b. How many boards did the volunteers supply? Round your calculations to the nearest thousandth and then provide the whole number of boards supplied. (6.NS.3 – Lessons 9, 14, and 15)
3. Andy ran 436.8 meters in 62.08 seconds.

a. If Andy ran at a constant speed, how far did he run in one second? Give your answer to the nearest tenth of a second. (6.NS.3 – Lessons 14 and 15)

b. Use place value, multiplication with powers of 10, or equivalent fractions to explain what is happening mathematically to the decimal points in the divisor and dividend before dividing. (6.NS.3 – Lessons 14 and 15)

c. In the following expression, place a decimal point in the divisor and the dividend to create a new problem with the same answer as in 3(a). Then explain how you know the answer will be the same. (6.NS.3 – Lesson 15)
4. The PTA created a cross-country trail for the meet.

   a. The PTA placed a trail marker in the ground every four hundred yards. Every nine hundred yards the PTA set up a water station. What is the shortest distance a runner will have to run to see both a water station and trail marker at the same location? (6.NS.4 – Lesson 18)

   b. There are 1,760 yards in one mile. About how many miles will a runner have to run before seeing both a water station and trail marker at the same location? Calculate the answer to the nearest hundredth of a mile. (6.NS.2 – Lessons 12, 14 and 15)

   c. The PTA wants to cover the wet areas of the trail with wood chips. They find that one bag of wood chips covers a 3 \( \frac{1}{2} \) yards section of the trail. If there is a wet section of the trail that is approximately 50 \( \frac{1}{4} \) yards long, how many bags of wood chips are needed to cover the wet section of the trail? (6.NS.1 – Lesson 8)
5. The Art Club wants to paint a rectangle-shaped mural to celebrate the winners of the track and field meet. They designed a checkerboard background for the mural where they will write the winners’ names. The rectangle measures 432 inches in length and 360 inches in width. What is the side length of the largest square they can use to fill the checkerboard pattern completely without overlap or gaps? (6.NS.4, Lessons 18 and 19)

\[
\begin{align*}
\text{length} & \quad 432 \text{ inches} \\
\text{width} & \quad 360 \text{ inches} \\
360 & \overline{)432} \\
-360 & \underline{\phantom{0}} \\
72 & \\
432 & = 360 \cdot 1 + 72 \\
\end{align*}
\]

\[
\begin{align*}
360 & \overline{)432} \\
-360 & \underline{\phantom{0}} \\
5 & \quad \text{or} \\
12 \overline{)360} \\
-360 & \underline{\phantom{0}} \\
0 & \\
\end{align*}
\]

\[
\text{GCF}(432, 360) = \text{GCF}(360, 72) = 72
\]

The side length of the largest square they can use is 72 inches.