Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes

- Students use visual models such as fraction bars and area models to divide fractions by fractions with different denominators.
- Students make connections between visual models and multiplication of fractions.

Classwork

Opening Exercise (2 minutes)

Begin class with a review of equivalent fractions. Ask each student for a new example of an equivalent fraction. Students need to share how they know that the new fraction is equivalent to the old fraction.

Opening Exercise

Write at least three equivalent fractions for each fraction below. Be sure to show how the two fractions are related.

a. \( \frac{2}{3} \)

Sample solutions include \( \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18} \).

b. \( \frac{10}{12} \)

Sample solutions include \( \frac{5}{6}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36} \).

Example 1 (3 minutes)

For the first example, students will be asked to solve a word problem using the skills they used in Lesson 3 to divide fractions with the same denominator.

- Molly purchased \( 1 \frac{3}{8} \) cups of strawberries. This can also be represented as \( \frac{11}{8} \). She eats \( \frac{2}{8} \) cups in a serving. How many servings did Molly purchase?

This question is really asking me how many \( \frac{2}{8} \) are in \( \frac{11}{8} \) or, in other words, to divide eleven eighths by two eighths. I can use a model to show that there are \( 5 \frac{1}{2} \) servings in the \( \frac{11}{8} \) cups of strawberries.
Example 1

Molly purchased 11/8 cups of strawberries. She eats 2/8 in a serving. How many servings did Molly purchase?

Use a model to prove your answer.

Example 2 (3 minutes)

- Now imagine that Molly’s friend Xavier purchased 11/8 cups of strawberries and that he eats 3/4 cup servings. How many servings has he purchased?
  - He has purchased 11/6 servings or 1 and 5/6 servings. (This would be answered last after a brief discussion using the questions that follow.)

- What is this question asking us to do?
  - I am being asked to divide 11/8 cups into 3/4 cup servings.

- How does the problem differ from the first example?
  - The denominators are different.

- What are some possible ways that we could divide these two fractions?
  - I could change 3/4 to 6/8. These fractions are equivalent. I scaled up from 3/4 by multiplying the top and bottom by 2.
Example 2

Now imagine that Molly’s friend Xavier purchased $\frac{11}{8}$ cups of strawberries and that he eats $\frac{3}{4}$ cup servings. How many servings has he purchased? Use a model to prove your answer.

There are $1$ and $\frac{5}{6}$ servings.

Example 3 (3 minutes)

MP.1

- $\frac{3}{4} \div \frac{2}{3}$
- What is this question asking us to do?
  - $\frac{2}{3}$ of what is $\frac{3}{4}$ or how many $\frac{2}{3}$ are in $\frac{3}{4}$

Lead students through a brief discussion about this example:

- Is your answer larger or smaller than one? Why?
  - Since $\frac{2}{3}$ is less than $\frac{3}{4}$, we will have an answer that is larger than 1.
- Why is this question more difficult to model than the questions in Lesson 3?
  - The fractions do not have common denominators.
- How can we rewrite this question to make it easier to model?
  - We can create equivalent fractions with like denominators and then model and divide.
  - We can also think of this as $\frac{9}{12} \div \frac{8}{12}$ or nine twelfths divided by eight twelfths. $9 \text{ units} \div 8 \text{ units} = \frac{9}{8}$ or $1\frac{1}{8}$ units.
Example 3

Find the quotient: \( \frac{3}{4} \div \frac{2}{3} \). Use a model to show your answer.

Exercises 1–5 (20 minutes)

Students will work in pairs or alone to solve more questions about division of fractions with unlike denominators.

Exercises 1–5

A model should be included in your solution.

1. \( \frac{6}{2} \div \frac{3}{4} \)

   We could rewrite this problem to ask \( \frac{12}{4} \div \frac{3}{4} = \frac{12}{3} = 4. \)

   ![Model diagram for Exercises 1–5]
2. \( \frac{2}{3} \div \frac{2}{5} \)

We could rewrite this problem to ask \( \frac{10}{15} \div \frac{6}{15} = \frac{10}{6} \) or \( 1 \frac{4}{6} \).

3. \( \frac{7}{8} \div \frac{1}{2} \)

We could rewrite this as \( \frac{7}{8} \div \frac{4}{8} = \frac{7}{4} \) or \( 1 \frac{3}{4} \).

4. \( \frac{3}{5} \div \frac{1}{4} \)

This can be rewritten as \( \frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2 \frac{2}{5} \).
5. \[
\frac{5}{4} \div \frac{1}{3}
\]

We can rewrite this as \[
\frac{15}{12} \div \frac{4}{12} = \frac{15}{12} \div \frac{4}{12} = \frac{15 \div 4}{12 \div 12} = \frac{3}{1} = 3 \frac{1}{4}.
\]

Closing (10 minutes)

- When dividing fractions, is it possible to get a whole number quotient?
- When dividing fractions, is it possible to get an answer that is larger than the dividend?
- When you are asked to divide two fractions with different denominators, what is one possible way to solve?

Exit Ticket (5 minutes)
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Exit Ticket

Draw a model to support your answer to the division questions.

1. \( \frac{9}{4} \div \frac{3}{8} \)

2. \( \frac{3}{5} \div \frac{2}{3} \)
Exit Ticket Sample Solutions

Draw a model to support your answer to the division questions.

1. \( \frac{9}{4} \div \frac{3}{8} \)
   
   This can be rewritten as \( \frac{18}{8} \div \frac{3}{8} = \frac{18}{3} = 6 \).

2. \( \frac{3}{5} \div \frac{2}{3} \)

   This can be rewritten as \( \frac{9}{15} \div \frac{10}{15} = \frac{9}{10} \) units.

   So this is equal to \( \frac{9}{10} \) units.
Problem Set Sample Solutions

The following problems can be used as extra practice or a homework assignment.

1. \(\frac{8}{9} \div \frac{4}{9}\)

   *Eight ninths divided by four ninths equals 2.*

2. \(\frac{9}{10} \div \frac{4}{10}\)

   *Nine tenths divided by four tenths equals \(2 \frac{1}{4}\).*

3. \(\frac{3}{5} \div \frac{1}{3}\)

   *Three fifths divided by one third equals nine fifteenths divided by five fifteenths equals \(\frac{9}{5} = \frac{4}{5}\).*
4. \( \frac{3}{4} \div \frac{1}{5} \)

\[
\frac{15}{20} \div \frac{4}{20} = \text{fifteen twentieths ÷ four twentieths} = \frac{15}{4}.
\]

\[
\begin{array}{cccc}
1 & + & 1 & + \\
\frac{4}{4} & + & \frac{4}{4} & + \\
\frac{3}{4} & + & \frac{3}{4} & = \frac{15}{4}
\end{array}
\]