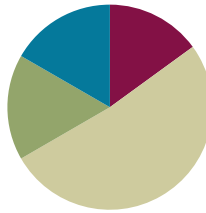


Lesson 2

Objective: Rotate tape diagrams vertically.

Suggested Lesson Structure

■ Fluency Practice	(9 minutes)
■ Application Problem	(10 minutes)
■ Concept Development	(31 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (9 minutes)

- Group Counting on a Vertical Number Line **3.OA.1** (3 minutes)
- Read Tape Diagrams **3.MD.4** (6 minutes)

Group Counting on a Vertical Number Line (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition.

T: (Project a vertical number line partitioned into intervals of 8, as shown. Cover the number line so that only the numbers 0 and 16 show.) What is halfway between 0 and 16?

S: 8.

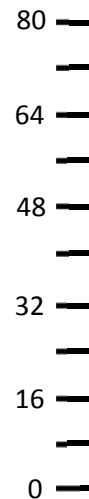
T: (Write 8 on the first hash mark.)

Continue for the remaining missing hashes so that the number line shows increments of 8 to 80.

T: Let's count by eights to 80.

Direct students to count forward and backward to 80, occasionally changing the direction of the count. Repeat the process using the following possible suggestions.

- Sixes to 60
- Sevens to 70
- Nines to 90



Read Tape Diagrams (6 minutes)

Materials: (S) Personal white boards

Note: This fluency activity reviews the relationship between the value of each unit in a tape diagram and the total value of the tape diagram. It also reviews comparison tape diagrams in preparation for today’s lesson.

T: (Project a tape diagram with 7 units.) Each unit in the tape diagram has a value of 4. Write a multiplication sentence that represents the total value of the tape diagram.

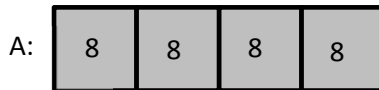
S: (Write $7 \times 4 = 28$.)

T: What is the total value of the tape diagram?

S: 28!

Use the same tape diagram. Repeat the process with the following suggested values for the units: 6, 3, 9, 7, and 8.

T: (Project tape diagrams as shown.)
What is the value of each unit in tape diagrams A and B?



S: 8!

T: Write a multiplication sentence that represents the total value of tape diagram A.



S: (Write $4 \times 8 = 32$.)

T: Write a multiplication sentence that represents the total value of tape diagram B.

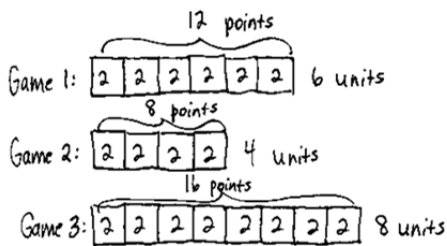
S: (Write $7 \times 8 = 56$.)

Continue with the following possible questions:

- What is the total value of both tape diagrams?
- How many more units of 8 are in tape diagram B?
- What is the total difference in the values of tape diagrams A and B?

Application Problem (10 minutes)

Reisha plays in three basketball games. She scores 12 points in Game 1, 8 points in Game 2, and 16 points in Game 3. Each basket that she made was worth 2 points. She uses tape diagrams with a unit size of 2 to represent the points she scored in each game. How many total units of 2 will it take to represent the points she scored in all three games?



Total units of 2: $6 + 4 + 8 = 18$
It will take 18 total units of 2 to represent the points scored in all 3 games.

Note: This problem reviews building tape diagrams with a unit size larger than 1 in anticipation of students using this same skill in the Concept Development. Ask students to solve this problem on personal white boards so that they can easily modify their work as they use it in the Concept Development. Invite students to discuss what the total number of units represents in relation to the three basketball games (18 total units of 2 is equal to 18 total baskets scored).

Concept Development (31 minutes)

Materials: (S) Tape diagrams from Application Problem, personal white boards

Problem 1: Rotate tape diagrams to make vertical tape diagrams with units of 2.

T: Turn your board so the bars on your tape diagrams are vertical like mine. (Model.) Erase the number of units labels, the brackets, and the points labels. How are these vertical tape diagrams similar to the picture graphs you made yesterday?

S: They both show us data. → There are labels on both of them. → Each unit on the vertical tape diagrams represents 2 points. → The symbol in the picture graph had a value greater than 1, and the unit in the vertical tape diagram has a value greater than 1.

T: How are the vertical tape diagrams different from the picture graphs?

S: The units are connected in the vertical tape diagrams. The pictures were separate in the picture graphs. → The units in the vertical tape diagrams are labeled, but in our picture graphs the value of the unit was shown on the bottom of the graph.

T: Nice observations. Put your finger on the tape that shows data about Game 1. Now, write a multiplication equation to show the value of Game 1’s tape.

S: (Write $6 \times 2 = 12$.)

T: What is the value of Game 1’s tape?

S: 12 points!

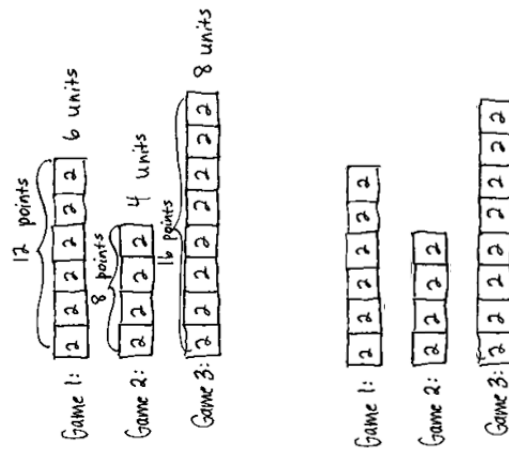
T: How did you know that the unit is points?

S: The Application Problem says Reisha scores 12 points in Game 1.

T: Let’s write a title on our vertical tape diagrams to help others understand our data. What does the data on the vertical tape diagrams show us?

S: The points Reisha scores in three basketball games.

T: Write *Points Reisha Scores* for your title. (Model appropriate placement of the title.)



Problem 2: Draw vertical tape diagrams with units of 4.

- T: Let’s use the same data from the Application Problem to create vertical tape diagrams where each unit has a value of 4 points instead of 2 points. Talk to a partner. How many units will I draw to represent Reisha’s points in Game 1? How do you know?
- S: Three units, because she scored 12 points in Game 1, and 3 units of 4 points equals 12 points. → Three units, because $3 \times 4 = 12$. → Three units, because $12 \div 4 = 3$. → Three units, because the value of each unit is now twice as much. Before we drew 6 units of 2, so now we draw half as many units. Each new unit has the value of two old units.
- T: Draw the 3 units vertically and label each unit 4. (Model.) What label do we need for this tape?
- S: Game 1.

Continue the process for Games 2 and 3.

- T: How many total units of 4 does it take to represent the points Reisha scored in all three games?
- S: 9 units!
- T: How does this compare to the total units of 2 it took to represent Reisha’s total points?
- S: It takes fewer total units when we used units of 4. → It took half as many total units when we used units of 4.
- T: Why does it take fewer units when you used units of 4?
- S: The units are bigger. → The units represent a larger amount.
- T: How can you use the vertical tape diagrams to write a multiplication number sentence that represents the total points Reisha scored in all three games?
- S: Multiply the total number of units times the value of each unit. → We can multiply 9 times 4.
- T: Write a multiplication number sentence to show the total points Reisha scored in all three games.
- S: (Write $9 \times 4 = 36$.)
- T: How many points did Reisha score in all three games?
- S: 36 points!

Continue with the following possible suggestions:

- How many more units of 4 did you draw for Game 1 than Game 2? How does this help you find how many



**NOTE ON
MULTIPLE MEANS OF
ENGAGEMENT:**

Although it is not possible to score 4 points at once in a basketball game, it can be helpful to rearrange the data in this way to see new things mathematically.



**NOTES ON
MULTIPLE MEANS OF
ENGAGEMENT:**

Students working above grade level and others may use parentheses and variables in their equations that represent the total points scored in all three games. Celebrate all true expressions, particularly those that apply the distributive property.

Students working below grade level and others may benefit from more scaffolded instruction for constructing and solving equations for three addends (number of units) and the total points.



**NOTES ON
MULTIPLE MEANS OF
REPRESENTATION:**

In Problem 1 of the Problem Set, some students with perceptual challenges may have difficulty tracking rows of stamps as they count. Have students place a straightedge below each row as they count by fours. In addition, students working below grade level may benefit from a fluency drill that reviews the fours skip-count.

MP.2

MP.2

more points Reisha scored in Game 1 than in Game 2?

- Suppose Reisha scored 4 fewer points in Game 3. How many units of 4 do you need to erase from Game 3's tape to show the new points?
- Reisha scores 21 points in a fourth game. Can you use units of 4 to represent the points Reisha scores in Game 4 on a vertical tape diagram?

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Rotate tape diagrams vertically.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- How does multiplication help you interpret the vertical tape diagrams on the Problem Set?
- Could you display the same data in Problem 1 in a vertical tape diagram with units of 6? Why or why not?
- If the value of the unit for your vertical tape diagrams in Problem 2 was 2 instead of 4, how would the number of units change? How about for 8?
- In what ways do vertical tape diagrams relate to picture graphs?

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•6

Name Gina Date _____

1. Find the total number of stamps each student has. Draw tape diagrams with a unit size of 4 to show the number of stamps each student has. The first one has been done for you.

Dana: 4 4 4 4
 Tanisha: 4 4
 Raquel: 4 4 4 4 4 4
 Anna: 4 4 4 4 4 4 4 4

2. Explain how you can create a vertical tape diagram to show this data.

I can create a vertical tape diagram by turning these tape diagrams. It shows the same data, but in a different way.

COMMON CORE Lesson 2: Rotate tape diagrams vertically. Date: 11/17/13 engageNY 6.A.20

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•6

3. Complete the vertical tape diagrams below using the data from Problem 1.

a. Dana Tanisha Raquel Anna

b. Dana Tanisha Raquel Anna

c. What is a good title for the vertical tape diagrams?
 A good title is Number of Stamps Collected.

d. How many total units of 4 are in the vertical tape diagram in Problem 3(a)?
 There are 20 total units of 4.

e. How many total units of 8 are in the vertical tape diagram in Problem 3(b)?
 There are 10 total units of 8.

f. Compare your answers to Problems (d) and (e). Why does the number of units change?
 The total number of units is less in (e) because the size of the unit in the vertical tape diagram in Problem 3(b) is greater.

g. Mattaeus looks at the vertical tape diagram in 3(b) and finds the total number of Anna and Raquel's stamps by writing the equation, $7 \times 8 = 56$. Explain his thinking.
 Mattaeus counted the number of units for Anna and Raquel, which is 7 and multiplied that by the value of each unit, 8.

COMMON CORE Lesson 2: Rotate tape diagrams to create bar graphs. Date: 10/11/13 engageNY 6.A.8

- How did today's Application Problem relate to our new learning?
- In what ways did the fluency activities prepare you for today's lesson?

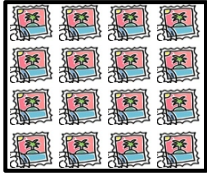
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____

Date _____

1. Find the total number of stamps each student has. Draw tape diagrams with a unit size of 4 to show the number of stamps each student has. The first one has been done for you.



Dana




Tanisha



Raquel



Anna

Each  represents 1 stamp.



Tanisha:

Raquel:

Anna:

2. Explain how you can create a vertical tape diagram to show this data.

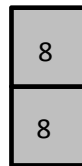
3. Complete the vertical tape diagrams below using the data from Problem 1.

a.



Dana Tanisha Raquel Anna

b.



Dana Tanisha Raquel Anna

- c. What is a good title for the vertical tape diagrams?
- d. How many total units of 4 are in the vertical tape diagram in Problem 3(a)?
- e. How many total units of 8 are in the vertical tape diagram in Problem 3(b)?
- f. Compare your answers to Parts (d) and (e). Why does the number of units change?
- g. Mattaeus looks at the vertical tape diagram in 3(b) and finds the total number of Anna and Raquel’s stamps by writing the equation, $7 \times 8 = 56$. Explain his thinking.

Name _____

Date _____

1. The chart below shows a survey of the book club's favorite type of books.

Book Club's Favorite Type of Books	
Type of Book	Number of Votes
Mystery	12
Biography	16
Fantasy	20
Science Fiction	8

- a. Draw tape diagrams with a unit size of 4 to represent the book club's favorite type of books.

- b. Use your tape diagrams to draw vertical tape diagrams that represent the data.

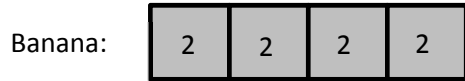
Name _____

Date _____

1. Adi surveys third-graders to find out what their favorite fruits are. The results are in the table below.

Favorite Fruits of Third-Graders	
Fruit	Number of Student Votes
Banana	8
Apple	16
Strawberry	12
Peach	4

Draw units of 2 to complete the tape diagrams to show the total votes for each fruit. The first one has been done for you.



Apple:

Strawberry:

Peach:

2. Explain how you can create a vertical tape diagram to show this data.

3. Complete the vertical tape diagrams below using the data from Problem 1.

a.



Banana Apple Strawberry Peach

b.



Banana Apple Strawberry Peach

- c. What is a good title for the vertical tape diagrams?
- d. Compare the number of units used in each vertical tape diagram. Why does the number of units change?
- e. Write a multiplication number sentence to show the total number of votes for strawberry in the vertical tape diagram in Problem 3(a).
- f. Write a multiplication number sentence to show the total number of votes for strawberry in the vertical tape diagram in Problem 3(b).
- g. What changes in your multiplication number sentences in (e) and (f)? Why?