Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Student Outcomes

- Students use their knowledge of dividing multi-digit numbers to solve for quotients of multi-digit decimals.
- Students understand the mathematical concept of decimal placement in the divisor and the dividend and its connection to multiplying by powers of 10.

Classwork

Opening Exercise (5 minutes)

Divide $\frac{1}{2} \div \frac{1}{10}$. Use a tape diagram to support your reasoning.

This question is asking the following: $\frac{1}{2}$ is $\frac{1}{10}$ of what number?

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1 unit → $\frac{1}{2}$
10 units → $10 \times \frac{1}{2} = 5$
```

Relate the model to the invert and multiply rule.

```
\frac{1}{2} \div \frac{1}{10} = \frac{1}{2} \times \frac{10}{1} = \frac{10}{2} = 5
```
Example 1 (5 minutes)

- Let’s look at the Opening Exercise another way. We can represent \( \frac{1}{2} \div \frac{1}{10} \) using decimals.
- We can represent the fractions \( \frac{1}{2} \) and \( \frac{1}{10} \) with which decimals?
  - One-half can be represented with 0.5, and one-tenth can be represented by 0.1.
- Display the following:
  \[
  \frac{1}{2} \div \frac{1}{10} \\
  0.5 \div 0.1
  \]
- This expression can be represented with the same interpretation as the Opening Exercise: 5 tenths is 1 tenth of what number?
- Let’s model this question with a tape diagram.

- What do you notice about the diagram and the quotient?
  - The diagram is set up exactly the same as in the Opening Exercise with the fraction, and the quotient is the same. Since \( \frac{1}{2} \) is equivalent to 0.5, and \( \frac{1}{10} \) is equivalent to 0.1, the quotients must be the same.
- Look back at the problem: 0.5 ÷ 0.1. Rewrite this division expression as a fraction.
  - \( \frac{0.5}{0.1} \)
- How can we express the divisor as a whole number?
  - Multiply by a fraction equal to one.
- Choose a fraction to multiply in order to express the divisor as a whole number.
  - I could multiply \( \frac{0.5}{0.1} \) by \( \frac{10}{10} \) to represent the divisor as the whole number 1.
Lesson 14

The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

- Find the product of \( \frac{0.5}{0.1} \times \frac{10}{10} \)
  - \( \frac{0.5}{0.1} \times \frac{10}{10} = \frac{5}{1} = 5 \)

- What do you notice about the quotient?
  - It is the same as when we used a tape diagram to determine the quotient.

- Why do you think the quotients are the same?
  - They are the same because I multiplied the divisor and the dividend by the same power of ten.

- What conjecture can you make?
  - Because the divisor and the dividend both became ten times greater, when we write the numbers as ten times as much, we move the decimal to the right one place.

Example 1

Evaluate the expression. Use a tape diagram to support your answer.

\[ 0.5 \div 0.1 \]

Rewrite \( 0.5 \div 0.1 \) as a fraction.

\[
\begin{align*}
0.5 & \times 0.1 = 5 \\
10 \times 0.5 &= 5
\end{align*}
\]

Express the divisor as a whole number.

\[
\begin{align*}
0.5 \times \frac{10}{10} &= 5 \\
\frac{0.1}{10} &= 5
\end{align*}
\]

Exercises 1–3 (5 minutes)

Students can work on the exercises alone or in pairs. Students convert the decimal division expressions to fractional division questions in order to create a whole number divisor.

Exercises 1–3

Convert the decimal division expressions to fractional division expressions in order to create whole number divisors. You do not need to find the quotients. Explain the movement of the decimal point. The first exercise has been completed for you.

1. \( 18.6 \div 2.3 \)
   - \( 18.6 \times \frac{10}{10} = \frac{186}{23} \)
   - \( 186 \div 23 \)
   - I multiplied both the dividend and the divisor by ten, or by one power of ten, so each decimal point moved one place to the right because they grew larger by ten.
2. \[14.04 \div 4.68\]
\[
14.04 \times \frac{100}{100} = \frac{1404}{486}
\]
\[
1.404 \div 486
\]
*I multiplied both the dividend and divisor by one hundred, or by two powers of ten, so each decimal point moved two places to the right because they grew larger by two powers of ten.*

3. \[0.162 \div 0.036\]
\[
0.162 \times \frac{1000}{1000} = \frac{162}{36}
\]
\[
162 \div 36
\]
*I multiplied both the dividend and divisor by one thousand, or three powers of ten, so each decimal point moved three places to the right because they grew larger by three powers of ten.*

**Example 2 (5 minutes)**

- We determined that when we multiply a divisor by a power of ten, the decimal point is moved to the right the number of times we multiply by a power of ten. How many places does the decimal point move to the right when we multiply the divisor by ten?
  - *It moves one place to the right.*
- Explain why the decimal point moves twice to the right when we multiply the divisor by one hundred?
  - *It moves two places to the right because we have multiplied by a power of ten twice.*
- We can use decomposition to explain.

Display the expression \[25.2 \div 0.72\].

- We can express the divisor as a whole number by multiplying it by one hundred.

Display the expression \[0.72 \times 100\].

- Because we can decompose 100 into \(10 \times 10\), we can also express the expression as \(0.72 \times 10 \times 10\).
- Multiply \(0.72 \times 10\).
  - 7.2
- What do you notice about the product?
  - *It is ten times greater than the original.*
- Multiply \(7.2 \times 10\).
  - 72
- What movement did the decimal point make in this process?
  - *It moved two places to the right.*
- What do you notice about the product?
  - *It is ten times greater than the last step and one hundred times greater than the original.*
- How many times does the decimal point move to the right when we multiply a number by \(1,000\)?
  - *It will move to the right three places because \(1,000 = 10 \times 10 \times 10\).*
Let’s use this information to divide $25.2 \div 0.72$. Express this as a fraction.

- \[
\begin{array}{c}
25.2 \\
- 0.72
\end{array}
\]

Now you can easily choose a fraction to multiply in order to express the divisor as a whole number.

- \[
\begin{array}{c}
25.2 \\
\times 100
\end{array}
\]

Use the division algorithm to find the quotient.

- \[
\frac{2520}{72} = 35
\]

Example 2

Evaluate the expression. First, convert the decimal division expression to a fractional division expression in order to create a whole number divisor.

\[
\frac{25.2}{0.72} = \frac{2520}{72}
\]

\[
\begin{array}{c|cc}
3 & 5 & \\
\hline
7 & 2 & 2 \\
- & 1 & 6 \\
\hline
& 3 & 6 \\
\hline
& 0 & 0
\end{array}
\]

Use the division algorithm to find the quotient.

\[25.2 \div 0.72 = 35\]

Using a calculator, multiply to check that the quotient is accurate.

- \[35 \times 0.72 = 25.2\]

Exercises 4–7 (10 minutes)

Students work on the exercises individually or in partners. Students convert the decimal division expressions to fractional division questions in order to create a whole number divisor. Quotients are computed using the division algorithm. Students are encouraged to check their work with a calculator.
Exercises 4–7

Convert the decimal division expressions to fractional division expressions in order to create whole number divisors. Compute the quotients using the division algorithm. Check your work with a calculator.

4. \( 2,000 \div 3.2 \)

\[
2,000 \div 3.2 = \frac{2,000}{3.2} = \frac{20,000}{32}
\]

\[
\begin{array}{c|cccc}
& 6 & 2 & 5 \\
\hline
3.2 & 2 & 0 & 0 & 0 \\
\hline
- & 1 & 9 & 2 \\
\hline
& 8 & 0 \\
\hline
- & 6 & 4 \\
\hline
& 1 & 6 & 0 \\
\hline
- & 1 & 6 & 0 \\
\hline
& 0 & 0 & 0 & 0 \\
\end{array}
\]

\( 20,000 \div 32 = 625 \)
\( 625 \times 32 = 20,000 \)
\( 2,000 \div 3.2 = 625 \)
\( 625 \times 3.2 = 2,000 \)

5. \( 3,581.9 \div 4.9 \)

\[
3,581.9 \div 4.9 = \frac{3,581.9}{4.9} = \frac{35,819}{49}
\]

\[
\begin{array}{c|ccc}
& 7 & 3 & 1 \\
\hline
4.9 & 3 & 5 & 8 \hspace{1cm} 1 \hspace{1cm} 9 \\
\hline
- & 3 & 4 & 3 \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} 6 \\
\hline
& 1 & 5 & 1 \\
\hline
- & 1 & 4 & 7 \\
\hline
& 4 & 9 \\
\hline
- & 4 & 9 \\
\hline
& 0 & 0 & 0 & 0 \\
\end{array}
\]

\( 35,819 \div 49 = 731 \)
\( 731 \times 49 = 35,819 \)
\( 3,581.9 \div 4.9 = 731 \)
\( 731 \times 4.9 = 3,581.9 \)
6. \(893.76 \div 0.21\)

\[
\begin{array}{c}
893.76 \\
- 0.21 \\
\hline
100 \\
\hline
21 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
2 & 1 & 8 & 9 & 3 & 7 & 6 \\
- & 8 & 4 & & & & & \\
\hline
5 & 3 & & & & & & \\
- & 4 & 2 & 1 & 1 & 7 & & \\
\hline
1 & 2 & 6 & & & & & \\
- & 1 & 2 & 6 & & & & \\
\hline
0 & & & & & & & \\
\end{array}
\]

\[
893.76 \div 21 = 42.56 \\
4.256 \times 21 = 89.376 \\
893.76 \div 0.21 = 4256 \\
4.256 \times 0.21 = 893.76
\]

7. \(6.194 \div 0.326\)

\[
\begin{array}{c}
6.194 \\
- 0.326 \\
\hline
1000 \\
\hline
326 \\
\end{array}
\]

\[
\begin{array}{cccc}
3 & 2 & 6 \\
- & 3 & 2 & 6 \\
\hline
6 & 1 & 9 & 4 \\
- & 6 & 1 & 9 & 4 \\
\hline
0 & & & & \\
\end{array}
\]

\[
6.194 \div 326 = 19 \\
19 \times 326 = 6.194 \\
6.194 \div 0.326 = 19 \\
19 \times 0.326 = 6.194
\]

Example 3 (5 minutes)

This example is to provide students a real-world context for division of decimals. Students recall information from Module 1 and apply unit rate.

- What is this question asking us to find?
  - This question is asking me to find the unit rate. I need to divide the number of miles by the number of hours so that I can determine the number of miles the plane flew in 1 hour, as we did in Module 1.
- Rewrite the expression $3625.26 \div 6.9$ as a fraction.
  \[
  \frac{3625.26}{6.9}
  \]
- How can we express the divisor as a whole number?
  - Multiply by a fraction equal to one.
- Choose a fraction to multiply in order to express the divisor as a whole number.
  - I could multiply $\frac{3625.26}{6.9} \times 10 \div 10$ to represent the divisor as the whole number $169$.
- Find the product of $\frac{3625.26}{6.9} \times 10 \div 10$.
  \[
  \frac{36252.6}{69}
  \]
- Show how we can rewrite $3625.26$ (362,526 hundredths) and $6.9$ (69 tenths) using the same units.
  - $36,252.6$ tenths $\div 69$ tenths.
- Notice that the decimal in both the dividend and the divisor shifted one place to the right. This resulted in a whole number divisor. Now we can evaluate by dividing $36,252.6 \div 69$ using the division algorithm.

**Example 3**

A plane travels $3625.26$ miles in $6.9$ hours. What is the plane’s unit rate?

Represent this situation with a fraction.

\[
\frac{3625.26}{6.9}
\]

Represent this situation using the same units.

$36,252.6$ tenths $\div 69$ tenths

Estimate the quotient.

$35,000 \div 70 = 500$

Express the divisor as a whole number.

\[
\frac{3625.26}{6.9} \times 10 \div 10 = \frac{36252.6}{69}
\]
Use the division algorithm to find the quotient.

\[
\begin{array}{c|cccc}
& 5 & 2 & 5 & 4 \\
\hline
6 & 9 & 5 & 2 & 6 \\
\hline
& 3 & 4 & 5 & 2 \\
\hline
& 3 & 7 & 2 & 0 \\
\hline
\end{array}
\]

Use multiplication to check your work.

\[525.4 \times 69 = 36,252.6\]

- Let’s check our answer to ensure that it is reasonable. What are some different ways that we can do this?
  - We can multiply the quotient with the original divisor and see if we get the original dividend. 
    \[6.9 \times 525.4 = 3,625.26\].
  - We could also compare the quotient to estimate to check our answer. \[3,500 \div 7 = 500\]. Because we rounded down, we should expect our estimate to be a little less than the actual answer, 525.4.

Closing (5 minutes)

- Explain a strategy to convert a decimal divisor into a whole number divisor.
  - In order to convert a decimal divisor into a whole number divisor, we could multiply by powers of ten. 
  - We continue to multiply by a power of ten until the decimal divisor is a whole number divisor. We must also multiply the dividend by the same power of ten for equality.
  - Must we always multiply a decimal divisor by a power of ten to convert it to a whole number divisor?
    - Answers may vary. Allow students to project inferences.

Present the following problem: \[105 \div 3.5\].

- What could I multiply the divisor by to convert it to a whole number divisor?
  - Answers may vary. Elicit responses, where some students may suggest multiplying by ten, as they have practiced in this lesson.

- Is there a more efficient means of converting the decimal divisor into a whole number divisor?
  - Answers may vary. Elicit responses, where some students may suggest multiplying the divisor and dividend by 2.
If we multiplied the divisor by 2, that would convert the decimal to the whole number, 7. Since we multiplied the divisor by 2, we must also multiply the dividend by 2, which is 210. What do you find convenient with this strategy?

- Answers may vary. Responses should include that using mental math helped create a less complex problem in comparison to multiplying by powers of ten.

Preview that the next lesson practices using mental math strategies to efficiently divide whole numbers and decimals.

Exit Ticket (5 minutes)
Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Exit Ticket

Estimate quotients. Convert decimal division expressions to fractional division expressions to create whole number divisors. Compute the quotient using the division algorithm. Check your work with a calculator and your estimate.

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $24.50. How many pounds of almonds did she purchase?

2. Divide: 125.01 ÷ 5.4.
Exit Ticket Sample Solutions

Estimate quotients. Convert decimal division expressions to fractional division expressions to create whole number divisor. Compute the quotient using the division algorithm. Check your work with a calculator and your estimate.

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $24.50. How many pounds of almonds did she purchase?

\[
\begin{align*}
24.50 & \times \frac{100}{100} = 245 \\
3.50 & \times \frac{10}{10} = 35 \\
\text{Estimate: } 270 \div 30 = 9
\end{align*}
\]

\[
\begin{array}{c}
7 \\
3.5 \\
- 3 \\
\hline
0
\end{array}
\]

Lisa purchased 7 pounds of almonds. This is close to my estimate of 9.

\[
7 \times 35 = 245 \\
7 \times 3.5 = 24.5
\]

2. Divide: \(125.01 \div 5.4\).

\[
\begin{align*}
125.01 & \times \frac{10}{10} = 1250.1 \\
5.4 & \times \frac{10}{10} = 54 \\
\text{Estimate: } 125 \div 5 = 25
\end{align*}
\]

\[
\begin{array}{c}
2 \ 3 \ 1 \ 5 \\
5.4 \ 4 \\
- 1 \ 0 \ 8 \\
\hline
1 \ 7 \ 0 \\
\frac{1}{4} \\
- 1 \ 6 \ 2 \\
\hline
8 \ 1 \\
- 5 \ 4 \\
2 \ 7 \ 0 \\
\frac{3}{2} \\
- 2 \ 7 \ 6 \\
\hline
0
\end{array}
\]

The quotient of 125.01 and 5.4 is 23.15. This is close to my estimate of 25.

\[
23.15 \times 54 = 1250.1 \\
23.15 \times 5.4 = 125.01
\]
Problem Set Sample Solutions

Convert decimal division expressions to fractional division expressions to create whole number divisors.

1. \(35.7 \div 0.07\)
   \[
   \begin{align*}
   35.7 & \times 100 = 3570 \\
   0.07 & \times 10 = 0.7 \\
   \end{align*}
   \]

2. \(486.12 \div 0.6\)
   \[
   \begin{align*}
   486.12 & \times 10 = 4861.2 \\
   0.6 & \times 10 = 6 \\
   \end{align*}
   \]

3. \(3.43 \div 0.035\)
   \[
   \begin{align*}
   3.43 & \times 1000 = 3430 \\
   0.035 & \times 1000 = 35 \\
   \end{align*}
   \]

4. \(5418.54 \div 0.009\)
   \[
   \begin{align*}
   5418.54 & \times 1000 = 5418540 \\
   0.009 & \times 1000 = 9 \\
   \end{align*}
   \]

5. \(812.5 \div 1.25\)
   \[
   \begin{align*}
   812.5 & \times 100 = 81250 \\
   1.25 & \times 100 = 125 \\
   \end{align*}
   \]

6. \(17.343 \div 36.9\)
   \[
   \begin{align*}
   17.343 & \times 10 = 173.43 \\
   36.9 & \times 10 = 369 \\
   \end{align*}
   \]

Estimate quotients. Convert decimal division expressions to fractional division expressions to create whole number divisors. Compute the quotients using the division algorithm. Check your work with a calculator and your estimates.

7. Norman purchased 3.5 lb. of his favorite mixture of dried fruits to use in a trail mix. The total cost was $16.87. How much does the fruit cost per pound?
   \[
   \begin{align*}
   16.87 & \times 10 = 168.7 \\
   3.5 & \times 10 = 35 \\
   \end{align*}
   \]
   *Estimate*: 16 ÷ 4 = 4

   The dried fruit costs $4.82 per pound. This is close to my estimate of 4.

8. Divide: \(994.14 \div 18.9\)
   \[
   \begin{align*}
   994.14 & \times 10 = 9941.4 \\
   18.9 & \times 10 = 189 \\
   \end{align*}
   \]
   *Estimate*: 100 ÷ 2 = 50

   The quotient is 52.6. This is close to my estimate of 50.
9. Daryl spent $4.68 on each pound of trail mix. He spent a total of $14.04. How many pounds of trail mix did he purchase?

\[
\begin{align*}
14.04 & \times 100 = 1404 \\
4.68 & \times 100 = 468 \\
\text{Estimate: } 15 \div 5 = 3
\end{align*}
\]

Daryl purchased 3 pounds of trail mix. This is my estimate.

10. Mamie saved $161.25. This is 25% of the amount she needs to save. How much money does Mamie need to save?

\[
\begin{align*}
161.25 & \times 100 = 16,125 \\
0.25 & \times 100 = 25 \\
\text{Estimate: } 1,600 \div 2 = 800
\end{align*}
\]

Mamie needs to save $645. This is close to my estimate of 800.

11. Kareem purchased several packs of gum to place in gift baskets for $1.26 each. He spent a total of $8.82. How many packs of gum did he buy?

\[
\begin{align*}
8.82 & \times 100 = 882 \\
1.26 & \times 100 = 126 \\
\text{Estimate: } 9 \div 1 = 9
\end{align*}
\]

Kareem bought 7 packs of gum. This is close to my estimate of 9.

12. Jerod is making candles from beeswax. He has 132.72 ounces of beeswax. If each candle uses 8.4 ounces of beeswax, how many candles can he make? Will there be any wax left over?

\[
\begin{align*}
132.72 & \times 10 = 1,327.2 \\
8.4 & \times 10 = 84 \\
\text{Estimate: } 120 \div 8 = 15
\end{align*}
\]

The quotient is 15.8. This means that Jerod can make 15 candles. This is close to my estimate. There will be wax left over.

13. There are 20.5 cups of batter in the bowl. This represents 0.4 of the entire amount of batter needed for a recipe. How many cups of batter are needed?

\[
\begin{align*}
20.5 & \times 10 = 205 \\
0.4 & \times 10 = 4 \\
\text{Estimate: } 200 \div 4 = 50
\end{align*}
\]

The number of cups of batter needed for the recipe is 51.25. This is close to my estimate.

14. Divide: 159.12 ÷ 6.8

\[
\begin{align*}
159.12 & \times 10 = 1,591.2 \\
6.8 & \times 10 = 68 \\
\text{Estimate: } 160 \div 8 = 20
\end{align*}
\]

The quotient is 23.4. This is close to my estimate.

15. Divide: 167.67 ÷ 8.1

\[
\begin{align*}
167.67 & \times 10 = 1,676.7 \\
8.1 & \times 10 = 81 \\
\text{Estimate: } 160 \div 8 = 20
\end{align*}
\]

The quotient is 20.7. This is close to my estimate.