



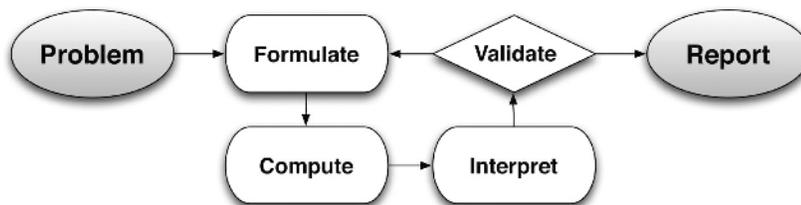
## Lesson 28: Federal Income Tax

### Student Outcomes

- Students create equations and inequalities in one variable and use them to solve problems.
- Students create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales.
- Students represent constraints by inequalities and interpret solutions as viable or non-viable options in a modeling context.

### Lesson Notes

This real-life descriptive modeling lesson (see page 61 of the CCLS or page 71 of the CCSS) is about using inequalities and graphs to understand the progressive federal tax system. Like the last lesson, this lesson again runs through the *problem, formulate, compute, interpret, validate, report* modeling cycle, but unlike the difficult modeling lesson on the *double and add 5* game, more autonomy can be given to students in this lesson. You might want to include more discussion of the words and process used in the modeling cycle:



### Materials

Please ensure that each student has a copy of the tax tables (on the next page) in their student materials or as a handout. Students will need a calculator and (your call) a spreadsheet program.

### Classwork

#### Mathematical Modeling Exercise (15 minutes): Formulating the Problem

Tell students: The federal income tax is not calculated by summing up all that an individual earns and then taking a fixed percentage of that income. Instead, the federal tax system is progressive. That means the more income an individual makes, the greater the percentage of it is taxed. In this lesson, we will analyze our tax system, graph the federal income tax versus income, and use the graph to compute effective tax rates for families with different incomes.

Important Tax Tables for this Lesson

*Exemption Deductions for Tax Year 2013*

Exemption Class	Exemption Deduction
Single	\$3,900
Married	\$7,800
Married with 1 child	\$11,700
Married with 2 children	\$15,600
Married with 3 children	\$19,500

*Standard Deductions Based Upon Filing Status for Tax Year 2013*

Filing Status	Standard Deduction
Single	\$6,100
Married filing jointly	\$12,200

*Federal Income Tax for Married Filing Jointly for Tax Year 2013*

If taxable income is over:	But not over:	The tax is:	Plus the Marginal Rate	Of the amount over:
\$0	\$17,850	10%		\$0
\$17,850	\$72,500	\$1,785.00	15%	\$17,850
\$72,500	\$146,400	\$9,982.50	25%	\$72,500
\$146,400	\$223,050	\$28,457.50	28%	\$146,400
\$223,050	\$398,350	\$49,919.50	33%	\$223,050
\$398,350	\$450,000	\$107,768.50	35%	\$398,350
\$450,000 +		\$125,846.00	39.6%	\$450,000

**TAXABLE INCOME:** The U.S. government considers the *income* of a family (or individual) to include the sum of any money earned from a husband’s or wife’s jobs, and money made from their personal businesses or investments. The taxes for a household (i.e., an individual or family) are not computed from the income; rather, they are computed from the household’s taxable income. For many families, the household’s *taxable income* is simply the household’s income minus exemption deductions and minus standard deductions:

$$(taxable\ income) = (income) - (exemption\ deduction) - (standard\ deduction)$$

All of the problems we will model in this lesson will use this equation to find a family’s taxable income. The only exception is if the family’s taxable income is less than zero, in which case we will say that the family’s taxable income is just \$0.

MP.2

Use this formula and the tables above to answer the following questions about taxable income:

**Exercise 1**

Find the taxable income of a single person with no kids, who has an income of \$55,000.

$55000 - 3900 - 6100 = 45000$ . *The person's taxable income is \$45,000.*

**Exercise 2**

Find the taxable income of a married couple with two children, who have a combined income of \$55,000.

$55000 - 15600 - 12200 = 27200$ . *The family's taxable income is \$27,200.*

**Exercise 3**

Find the taxable income of a married couple with one child, who have a combined income of \$23,000.

$23000 - 11700 - 12200 = -900$ . *The family's taxable income is \$0.*

**Federal Income Tax and the Marginal Tax Rate:** Below is an example of how to compute the federal income tax of a household using the Federal Income Tax table above.

**Example 1**

Compute the Federal Income Tax for the situation described in Exercise 1 (a single person with no kids making \$55,000).

From the answer in Exercise 1, the taxable income is \$45,000. Looking up \$45,000 in the tax table above, we see that \$45,000 corresponds to the second row because it is between \$17,850 and \$72,500:

If taxable income is over:	But not over:	The tax is:	Plus the Marginal Rate	Of the amount over:
\$17,850	\$72,500	\$1,785.00	15%	\$17,850

To calculate the tax, add \$1,785 plus 15% of the amount of \$45,000 that is over \$17,850. Since  $45000 - 17850 = 27150$ , and 15% of 27,150 is \$4,072.50, the total federal income tax on \$45,000 of taxable income is \$5,857.50.

**Exercise 4**

Compute the Federal Income Tax for a married couple with two children making \$127,800.

*The taxable income is  $\$127,800 - \$15,600 - \$12,200 = \$100,000$ .*

*Looking up \$100,000 in the tax table, we see that \$100,000 corresponds to the third row because it is between \$72,500 and \$146,000:*

If taxable income is over:	But not over:	The tax is:	Plus the Marginal Rate	Of the amount over:
\$72,500	\$146,400	\$9,982.50	25%	\$72,500

*To calculate the tax, add \$9,982.50 plus 25% of the amount over \$72,500. Since  $100,000 - 72,500 = 27,500$ , we take 25% of 27,500 to get \$6,875. Thus, the total federal income tax on \$100,000 of taxable income is \$16,857.50.*

Taxpayers sometimes misunderstand *marginal tax* to mean: “If my taxable income is \$100,000, and my marginal tax rate is 25%, my federal income taxes are \$25,000.” This statement is not true—they would not owe \$25,000 to the federal government. Instead, a marginal income tax charges a progressively higher tax rate for successively greater levels of income. Therefore, they would really owe:

- 10% on the first \$17,850, or \$1,785 in taxes for the interval from \$0 to \$17,850;
- 15% on the next \$54,650, or \$8,197.50 in taxes for the interval from \$17,850 to \$72,500;
- 25% on the last \$27,500, or \$6,875.00 in taxes for the interval from \$72,500 to \$100,000;

for a total of \$16,857.50 of the \$100,000 of taxable income. Thus, their *effective federal income tax rate* is 16.8575%, not 25% as they claimed. Note that the tax table above incorporates the different intervals so that only one calculation needs to be made (the answer to this problem is the same as the answer in Exercise 5).

### Exercises 5–7 (15 minutes)

Students are now ready to formulate and create a graph of federal income taxes versus income. In Exercise 5, the creation of the table and the graph involves many of the ideas that students have been learning throughout this module. The hope here is that they can work through this problem on their own (or in groups of two) with *minimum help from you*. However, since these tax terms are new, you may need to walk around the room and help explain words like income, taxable income, exemption, standard deduction, and federal income tax (as well as marginal tax rate, filing status, and deduction).

#### Exercise 5

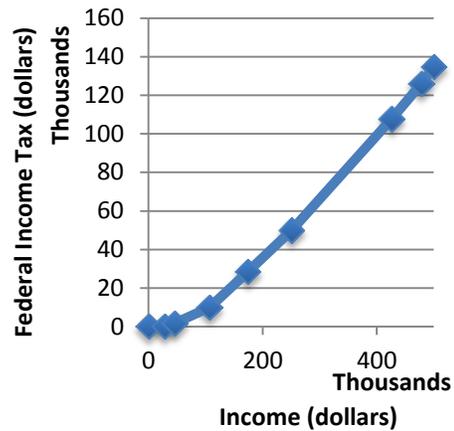
Create a table and a graph of federal income tax versus income for a married couple with two children between \$0 of income and \$500,000 of income.

*The first step in creating the graph is to determine the equation for taxable income. A married couple with two children has a standard deduction of \$12,200 and an exemption deduction of \$15,600, for a total deduction of \$27,800. If we let the real number,  $TI$ , stand for the family's taxable income, and the real number,  $I$ , stand for the family's income, we get the following equation for taxable income:*

$$TI = \begin{cases} I - 27800 & I \geq 27800 \\ 0 & 0 \leq I < 27800 \end{cases}$$

Help students to create the following table using the intervals in the federal income tax table:

Income (\$)	Taxable Income (\$)	Federal Income Tax (\$)
0	0	0
27,800	0	0
45,650	17,850	1785.00
100,300	72,500	9,982.50
174,200	146,400	28,457.50
250,850	223,050	49,919.50
426,150	398,350	107,768.50
477,800	450,000	125,846.00
500,000	472,200	134,637.20



Use column 1 and column 3 in this table to create the graph on the right.

**Exercise 6**

Interpret and validate the graph you created in Exercise 5. Does your graph provide an approximate value for the federal income tax you calculated in Exercise 4?

*Yes. The graph suggests that the federal income tax for a married couple with two children with an income of \$127,800 should be between \$15,000 and \$20,000, which is close to the actual amount of \$16,857.50.*

**Exercise 7**

Use the table you created in Exercise 5 to report on the effective federal income tax rate for a married couple with two children, who makes:

- a. \$27,800
- b. \$45,650
- c. \$500,000

*Note to teacher:* Answer the first two incomes with your class, using them as examples to explain the meaning of effective federal income tax rate. Let them find the effective federal income tax rate for \$500,000 as an exercise.

*The effective federal income tax rate is found by writing the number (federal income tax)/(income) as a percentage. The effective federal income tax rate for a married couple with two children making:*

- a. \$27,800 is 0%,
- b. \$45,650 is about 4%,
- c. \$500,000 is about 27%.

**Exit Ticket (5 minutes)**

Adjust this problem based upon the remaining class time.

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 28: Federal Income Tax

### Exit Ticket

A famous movie actress made \$10 million last year. She is married and has no children, and her husband does not earn any income. Assume that she computes her taxable income using the following formula:

$$(\text{taxable income}) = (\text{income}) - (\text{exemptions}) - (\text{standard deductions})$$

Find her taxable income, her federal income tax, and her effective federal income tax rate.

## Exit Ticket Sample Solutions

A famous movie actress made \$10 million last year. She is married and has no children, and her husband does not earn any income. Assume that she computes her taxable income using the following formula:

$$(\text{taxable income}) = (\text{income}) - (\text{exemptions}) - (\text{standard deductions})$$

Find her taxable income, her federal income tax, and her effective federal income tax rate.

$$\text{Taxable Income: } \$10\,000\,000 - \$7800 - \$12200 = \$9\,980\,000$$

**Federal Income Tax:**

If taxable income is over:	But not over:	The tax is:	Plus the Marginal Rate	Of the amount over:
\$450,000 +		\$125,846.00	39.6%	\$450,000

39.6% of  $9\,980\,000 - 450\,000$ , or 39.6% of  $9\,530\,000$ , is  $\$3\,773\,880$  in tax over the first  $\$450,000$ . Add the tax of  $\$125,846$  on the first  $\$450,000$  of taxable income, to get a total federal income tax of  $\$3\,899\,726$ .

$$\text{Effective Federal Income Tax Rate: } \frac{3\,899\,726}{10\,000\,000} \cdot 100 \approx 39\%$$

## Problem Set Sample Solutions

Use the formula and tax tables given in the lesson to perform all computations.

- Find the taxable income of a married couple with two children, who have a combined income of \$75,000.  
**\$47,200**
- Find the taxable income of a single person with no children, who has an income of \$37,000.  
**\$27,000**
- Find the taxable income of a married couple with three children, who have a combined income of \$62,000.  
**\$30,300**
- Find the federal income tax of a married couple with two children, who have a combined income of \$100,000.  
**\$9,937.50**
- Find the federal income tax of a married couple with three children, who have a combined income of \$300,000.  
**\$64,852**
- Find the effective federal income tax rate of a married couple with no children, who have a combined income of \$34,000.  
**4.1%**

7. Find the effective federal income tax rate of a married couple with one child who have a combined income of \$250,000.

*Approximately 20.4%*

8. The latest report on median household (family) income in the United States is \$50,502 per year. Compute the federal income tax and effective federal income tax rate for a married couple with three children, who have a combined income of \$50,502.

*Federal income tax: \$1,927.80*

*Effective federal income tax rate: Approximately 3.8%*

9. Extend the table you created in Exercise 6 by adding a column called, "Effective federal income tax rate." Compute the effective federal income tax rate to the nearest tenth of a percent for each row of the table, and create a graph that shows effective federal income tax rate versus income using the table.

Income	Taxable Income	Federal Income Tax	Effective Federal Income Tax Rate
0	0	0	0%
27,800	0	0	0%
45,650	17,850	1,785.00	3.9%
107,550	72,500	9,982.50	9.3%
174,200	146,400	28,457.50	16.3%
250,850	223,050	49,919.50	19.9%
426,150	398,350	107,768.50	25.3%
477,800	450,000	125,846.00	26.3%
500,000	472,200	134,637.20	26.9%

