Modeling Word Problems with Tape Diagrams Grades 3 - 5

*** Note: Problems written in italics within the handout will not be solved as part of this session. The problems are in the handout as a reference for participants. An answer key with answers to all problems within the packet will be distributed at the end of the session. ***
Directions

Step 1  Read Problem 1 individually. Solve with a model, and then algebraically.
Step 2  Compare your model with a partner’s.

1. Last summer, at Camp Okey-Fun-Okey, the ratio of the number of boy campers to the number of girl campers was 8:7. If there were a total of 195 campers, how many boy campers were there? How many girl campers? (G6 M1 L5)

2. All the printing presses at a shop were scheduled to make copies of a novel and a cookbook. They printed the same number of copies of each book but the novel had twice as many pages as the cookbook.
   On the first day, all the presses worked printing novels.
   On the second day, the presses were split into two equally sized groups.
   The first group continued printing copies of the novel and finished printing all the copies by the evening.
   The second group worked on the cookbook but did not finish by evening.
   One press, working for two additional full days, finished printing the remaining copies of the cookbooks.
   If all presses worked (for both the novel and cookbook) at the same constant rate, how many printing presses are there at the print shop? (G9 M1 L25)
Directions

Step 1  Read and analyze the highlighted problem types silently for 2 minutes.

Step 2  Stand and find a partner from a different table. For 1 minute, using grade-appropriate measurement, whole number, or fractional units, create an \( A \times B = \) ____ situation.

Step 3  Change partners. For 1 minute, create a \( C \div A = \) ____ situation.

Step 4  Change partners. For 1 minute, create a \( C \div B = \) ____ situation.

Table 3: Multiplication and division situations

<table>
<thead>
<tr>
<th>Equal Product Unknown</th>
<th>Group Size Unknown</th>
<th>Number of Groups Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \times B =</td>
<td></td>
<td>( A \times</td>
</tr>
</tbody>
</table>

**Equal Groups of Objects**

<table>
<thead>
<tr>
<th>Array of Objects Unknown Product</th>
<th>Array of Objects Unknown Factor</th>
<th>Array of Objects Unknown Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are ( A ) bags with ( B ) plums in each bag. How many plums are there in all?</td>
<td>If ( B ) plums are shared equally into ( A ) bags, then how many plums will be in each bag?</td>
<td>If ( B ) plums are to be packed ( B ) to a bag, then how many bags are needed?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compare Larger Unknown Smaller Unknown</th>
<th>Compare Larger Unknown Smaller Unknown</th>
<th>Compare Larger Unknown Smaller Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>A blue hat costs ( B ). A red hat costs ( A ) times as much as the blue hat. How much does the red hat cost?</td>
<td>A red hat costs ( SC ) and that is ( A ) times as much as a blue hat. How much does a blue hat cost?</td>
<td>A red hat costs ( SC ) and a blue hat costs ( B ). How many times as much does the red hat cost as the blue hat?</td>
</tr>
</tbody>
</table>


Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of \( A \) and \( B \), so that the same array is described. For example: There are \( B \) columns of apples with \( A \) apples in each column. How many apples are there?
Reflection

• Compare Polya’s process and the RDW process.
• What obstacles might students encounter using each approach?
• Eureka Math has chosen to use the RDW process. Why do you think so?

Instead of

Polya’s Problem Solving Process

• Understand the problem.
• Devise a plan
• Carry out the plan.
• Look back.

We use the following process:

**RDW**: Read, Draw, Write

Read the problem.
Draw and label a model as you reread.
  • Can I draw something?
  • What can I draw?
  • What conclusions can I make from my drawing?
Write an equation or equations that help solve the problem.
Write a statement of the answer to the question.
Notes on Pedagogy:

The RDW process often involves moving back and forth between reading and drawing. Students might first read the problem entirely then reread the first sentence. Draw and label. Reread the second sentence. Draw and label, etc. Consider the following example:

*Mr. Peterson bought a case (24 boxes) of fruit juice. One-third of the drinks were grape and two-thirds were cranberry. How many boxes of cranberry drinks did Mr. Peterson buy? (G5 M2 L10)*

Read:

*Mr. Peterson bought a case (24 boxes) of fruit juice.*

Draw:

```
24 boxes
```

Read:

*One-third of the drinks were grape*

Draw:

```
&                                          &
&                                          &
24 boxes                                 &
```

Read:

*Two-thirds were cranberry.*

Draw:

```
&                                          &
&                                          &
&                                          &
24 boxes                                 &
```

Read:

*How many boxes of cranberry drinks did Mr. Peterson buy?*

Write

\[
\begin{align*}
1 \text{ unit} &= 24 \div 3 = 8 \\
2 \text{ units} &= 8 \times 2 = 16
\end{align*}
\]

Mr. Peterson bought 16 boxes of cranberry juice.

Polya’s process culminates with “look back.” The RDW process, on the other hand, culminates with a statement and a labeled drawing, an illustration of the story. The statement puts the answer back into context. Does the statement make sense? Does it correspond correctly to the drawing? Does the drawing tell the story? This is MP.2 in action, “reasoning abstractly and quantitatively.” The drawing precipitates the reasoning. The student does not figure out the problem and then draw but rather decodes the relationships through the drawing. The abstract numbers are manipulated in the calculation and restored as quantities in the statement.
Directions

Step 1  Complete the first problem role-playing a student as the facilitator models the RDW process.
Step 2  Read the Problem Solving Protocol.
Step 3  Apply the protocol to Set 1, Group A.
Step 4  Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 1, Group A

1. Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use all together? (G3 M1 L2)

2. A scientist fills 5 test tubes with 9 milliliters of fresh water in each. She fills another 3 test tubes with 9 milliliters of salt water in each. How many milliliters of water does she use in all? (G3 M3 L12)
3. Three boxes weighing 128 pounds each and one box weighing 254 pounds were loaded onto the back of an empty truck. A crate of apples was then loaded onto the same truck. If the total weight loaded onto the truck was 2,000 pounds, how much did the crate of apples weigh? (G4 M3 L13)

4. 852 pounds of grapes were packed equally into 3 boxes for shipping. How many pounds of grapes will there be in 2 boxes? (G5 M2 L17)
Set 1, Group B

1. Shelly read her book for $\frac{1}{2}$ hour each afternoon for 9 days. How many hours did Shelly spend reading in all 9 days? (G4 M5 L24)

2. Rhonda exercised for $\frac{5}{6}$ hour every day for 5 days. How many total hours did Rhonda exercise? (G4 M5 L36)

3. Gail has two yards of fabric. It takes $8 \frac{3}{4}$ yards of fabric to make one quilt. She wants to make three quilts. How many more yards of fabric does she need to buy in order to make three quilts? (G4 M5 L39)
Set 1, Group C

1. There are 32 students in a class. Of the class, $\frac{3}{8}$ bring their own lunch. How many students bring their lunch? (G5 M4 L6)

2. Moussa delivered $\frac{3}{8}$ of the newspapers on his route in the first hour and $\frac{4}{5}$ of the rest in the second hour. What fraction of the newspapers did Moussa deliver in the second hour? (G5 M4 L15)

3. Mrs. Diaz makes 5 dozen cookies for her class. One-ninth of her 27 students are absent the day she brings the cookies. If she shares the cookies equally among the students who are present, how many cookies will each student get? (G5 M4 L11)
Modes of Instructional Delivery

A teacher also needs to make a decision about the mode of delivery of instruction each day as it is not dictated in the curriculum at all. Will students be encouraged to use a specific model to reason about the relationships within the problem (e.g. an array or tape diagram) or will any math drawing that makes sense be encouraged? Will this be a better problem to use a step-by-step guided approach because of new complexities, or will the students work independently and then share out their strategies? Will the students work independently or in pairs? In cooperative groups with a protocol or solo and then share with a partner? The chart below lays out three modes of delivery of instruction. There are many gradations within and between each one.

Directions

- **Step 1** Study the different modes of instructional delivery.
- **Step 2** Watch the video.
- **Step 3** Determine which type of delivery is being modeled in the video.
- **Step 4** Discuss what you would hear, see, and experience with the two other modes of delivery.
- **Step 5** Discuss the strengths and weaknesses of each mode of instructional delivery and when each might be the right choice.

### Chart: Modes of Delivery

<table>
<thead>
<tr>
<th>Modeling with Interactive Questioning</th>
<th>Guided Practice</th>
<th>Independent Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher models the whole process with interactive questioning, some choral response, and talk moves, such as “What did Monique say, everyone?” After completing the problem, students might reflect with a partner on the steps they used to solve the problem. “Students, think back on what we did to solve this problem. What did we do first?” Students might then be given the same or similar problem to solve for homework.</td>
<td>Each student has a copy of the question. Though guided by the teacher, they work independently at times and then come together again. Timing is important. Students might hear, “You have two minutes to do your drawing.” Or, “Put your pencils down. Time to work together again.” The Student Debrief might include selecting different student work to share.</td>
<td>Students are given a problem to solve and possibly a designated amount of time to solve it. The teacher circulates, supports, and is thinking about which student work to show to support the mathematical objectives of the lesson. When sharing student work, students are encouraged to think about the work with questions, such as “What do you see Jeremy did?” “What is the same about Jeremy’s work and Sara’s work?” “How did Jeremy show the $\frac{3}{7}$ of the students?” “How did Sara show the $\frac{3}{7}$ of the students?”</td>
</tr>
</tbody>
</table>


Directions

Step 1  Read the Deliberate Practice Protocol B.
Step 2  Follow the Deliberate Practice Protocol for Set 1, Group A, #3.
Step 3  Use the space below to write down planning notes and/or ideas that come from the deliberate practice.
Directions

Step 1  Complete the first problem role-playing a student as the facilitator models the RDW process.

Step 2  Apply the protocol to solve the problems of Set 2, Group A.

Step 3  Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 2, Group A

1. Andrew has 21 keys. He puts them in 3 equal groups. How many keys are in each group? (G3 M1 L4)

2. 1,624 shirts need to be sorted into 4 equal groups. How many will be in each group? (G4 M3 L31)
3. Jesse and his 3 friends buy snacks for a hike. They buy trail mix for $5.42, apples for $2.55, and granola bars for $3.39. If the 4 friends split the cost of the snacks equally, how much should each friend pay? (G5 M1 L16)

4. Four grade levels need equal time for indoor recess, and the gym is available for 3 hours. How many hours of recess will each grade level receive? (G5 M4 L4)

5. Mrs. Onusko made 60 cookies for a bake sale. She sold of them and gave of the remaining cookies to the students working at the sale. How many cookies did she have left? (G5 M4 L16)
6. Allen’s team was required to buy two pairs of uniform pants and two baseball caps which totaled $68. A pair of pants costs $12 more than a baseball cap. What is the cost of one cap? (G7 M2 L23)

7. A motorcycle dealer paid a certain price for a motorcycle and marked it up by \( \frac{1}{5} \) of the price he paid. Later he sold it for $14,000. What was the original price? (G7 M1 L14)
8. Every day Heather practices soccer and piano. Each day she practices piano for 2 hours. If after 5 days she practiced both piano and soccer for a total of 20 hours, how many hours, \( h \), per day did Heather practice soccer? (G7 M3 L9)

9. Find 3 consecutive integers such that their sum is 1,623. (G8 M4 L1)
Set 2, Group B

1. Ms. Hayes has $\frac{1}{2}$ liter of juice. She distributes it equally to 6 students in her tutoring group. How many liters of juice will each student get? (G5 M4 L33)

2. You have $\frac{2}{5}$ of a cup of frosting to share equally among three desserts. How much frosting will be placed on each dessert? (G6 M2 L1)
3. Four baby socks can be made from $\frac{1}{3}$ skein of yarn. How many baby socks can be made from a whole skein? (G5 M4 L32)

4. Three gallons of water fills $\frac{1}{4}$ of the elephant’s pail at the zoo. How much water does the pail hold? (G5 M4 L25)
Directions
Step 1  Examine the models below and identify all the information that is given in the model.
Step 2  Individually, create at least one word problem that can be solved using each model.
Step 3  When signaled, compare your word problems with a partner.

1. (G4 M3 L12)

2. (G5 M4 L28)
3. (G6 M2 L6)

4. Draw a model and have your partner write a word problem.
Directions
- Step 1 Complete the first problem role-playing a student as the facilitator models the RDW process.
- Step 2 Apply the protocol to solve the problems of Set 3, Group A.
- Step 3 Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 3, Group A
1. Jackie buys 21 pizzas for a party. She places 3 pizzas on each table. How many tables are there? (G3 M1 L14)

2. Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make? (G3 M3 L19)
3. Monique needs exactly 4 plates on each table for the banquet. If she has 312 plates, how many tables is she able to prepare? (G4 M3 L31)

4. The cost of a babysitting service on a cruise is $10 for the first hour and $12 for each additional hour. If the total cost of babysitting baby Aaron was $58, how many hours was Aaron at the sitter? (G7 M2 L17)
Set 3, Group B

1. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up $\frac{3}{4}$ of a mile, how many family members are needed to clean the adopted section? (G6 M2 L2)

2. George bought 12 pizzas for a party. If each person will eat $\frac{5}{8}$ of a pizza, how many people can George feed with 12 pizzas? (G6 M2 L2)

3. A race begins with $2 \frac{1}{2}$ miles through town, continues through the park for $2 \frac{1}{3}$ miles, and finishes at the track after the last $\frac{1}{6}$ mile. A volunteer is stationed every quarter mile and at the finish line to pass out cups of water and cheer on the runners. How many volunteers are needed? (G5 M4 L26)
4. Xavier, Molly’s friend, purchased \(\frac{11}{8}\) cups of strawberries. If he eats \(\frac{3}{4}\) of a cup of strawberries per serving, how many servings will he have? (G6 M2 L5)

5. Tina uses \(\frac{1}{8}\) oz. of cinnamon each time she makes a batch of coffee cake topping. How many batches can she make if she has \(\frac{1}{2}\) oz. left in her spice jar? (G6 M2 L5)

6. Yasmine is serving ice cream with the birthday cake at her party. She has purchased \(19 \frac{1}{2}\) pints of ice cream. She will serve \(\frac{3}{4}\) of a pint to each guest. How many guests can be served? (G6 M2 MMA)
Directions

Step 1  Individually derive a description of partitive and measurement division based on your past experiences.
Step 2  Share descriptions with partners and revise as necessary.
Step 3  Share descriptions with the whole group.
Step 4  Examine the glossary, providing the precise descriptions from the curriculum.

Precise Descriptions of “Partitive Division” and “Measurement Division”
Directions

Step 1  Compare/contrast the student work.
Step 2  Discuss how you would help the struggling student.
Directions

Step 1  Read and analyze the highlighted problem situations silently for 1 minute.
Step 2  Stand and find a partner from a different table. For 1 minute, using grade-appropriate measurement, whole number, or fractional units, create a ‘Compare, Larger Unknown’ situation.
Step 3  Change partners. For 1 minute, create a ‘Compare, Smaller Unknown A > 1’ situation.
Step 4  Change partners. For 1 minute, create a ‘Compare, Smaller Unknown’ situation.
(Optional) Change partners. For 1 minute, create a ‘Compare, Larger Unknown A < 1’ situation.

Table 3: Multiplication and division situations

| A × B = | A × | = C and C ÷ A = | | × B = C and C ÷ B = |
|---|---|---|---|
| Equal Groups of Objects | Unknown Product | Group Size Unknown | Number of Groups Unknown |
| There are A bags with B plums in each bag. How many plums are there in all? | If C plums are shared equally into A bags, then how many plums will be in each bag? | If C plums are to be packed B to a bag, then how many bags are needed? |
| Arrays of Objects | Unknown Product | Unknown Factor | Unknown Factor |
| There are A rows of apples with B apples in each row. How many apples are there? | If C apples are arranged into A equal rows, how many apples will be in each row? | If C apples are arranged into equal rows of B apples, how many rows will there be? |
| Compare | Unknown Product | Unknown Factor | Unknown Factor |
| The apples in the grocery window are in A rows and B columns. How many apples are there? | If C apples are arranged into an array with A rows, how many columns of apples are there? | If C apples are arranged into an array with B columns, how many rows are there? |
| Equal groups language | Larger Unknown | A red hat costs $C and that is A times as much as a blue hat costs. How much does a blue hat cost? | Multiplier Unknown |
| A blue hat costs $B. A red hat costs A times as much as the blue hat. How much does the red hat cost? | A red hat costs $C and that is A of the cost of a blue hat. How much does a blue hat cost? | A red hat costs $C and a blue hat costs $B. What fraction of the cost of the blue hat is the cost of the red hat? |
| Smaller Unknown | Multiplier Unknown |
| A blue hat costs $B. A red hat costs A as much as the blue hat. How much does the red hat cost? | A red hat costs $C and that is A of the cost of a blue hat. How much does a blue hat cost? |


Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

Multiplicative Compare problems appear first in Grade 4, with whole-number values for A, B, and C, and with the “times as much”
Word Problems for Each Problem Situation
Directions

Step 1  Complete the first problem role-playing a student as the facilitator models the RDW process.
Step 2  Apply the protocol to solve the problems of Set 4, Group A.
Step 3  Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 4, Group A

1. Andre bought a stamp to mail a letter that cost 46 cents. He also mailed a package that cost 5 times as much as a stamp. How much did it cost to mail the package and the letter? (G4 M3 L8)

2. Jayden has 347 marbles. Elvis has 4 times as many as Jayden. Presley has 799 fewer than Elvis. How many marbles does Presley have? (G4 M3 L12)

3. Carol made punch. She used $12\frac{3}{8}$ cups of juice and then added three times as much ginger ale. Then she added 1 cup of lemonade. How many cups of punch did her recipe make? (G4 M5 L39)
4. In one month, Charlie read 814 pages. In the same month, his mom read 4 times as many pages as Charlie, and that was 143 pages more than Charlie’s dad read. What was the total number of pages read by Charlie and his parents? (G4 M3 L13)

5. Shelby is 7 times as old as Bonnie. If in 5 years the sum of Bonnie’s and Shelby’s ages is 98, find Bonnie’s present age. (G7 M3 L9)

6. 16 years from now, Pia’s age will be twice her age 12 years ago. Find her present age. (G9 M1 L25)
Set 4, Group B

1. Jasmine took \( \frac{2}{3} \) as much time to take a math test as Paula. If Paula took 2 hours to take the test, how long did it take Jasmine to take the test? Express your answer in minutes. (G5 M5 L23)

2. Carli has 90 apps on her phone. Braylen has \( \frac{1}{2} \) the amount of apps as Theiss. If Carli has 3 times the amount of apps as Theiss, how many apps does Braylen have? Let \( b \) represent the number of Braylen’s apps and \( t \) represent the number of Theiss’ apps. (G6 M4 L28)
Directions

Step 1  Choose one problem in the previous set that you found difficult to model.
Step 2  Share the model you drew with a partner and explain why the problem was difficult.
Directions

Step 1  Complete the first problem role-playing a student as the facilitator models the RDW process.
Step 2  Apply the protocol to solve the problems of Set 5, Group A.
Step 3  Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 5, Group A

1. Jamie drank 4 times as much juice as Brodie. Jamie drank 280 mL of juice. How much juice did Brodie drink? (G4 M3 L26)

2. The Hometown Hotel has a total of 480 guest rooms. That is 6 times as many rooms as the Travelers Hotel down the street. How many rooms are there in the Travelers Hotel? (G4 M3 L26)
3. The top surface of a desk has a length of 5.6 feet. The length is 4 times its width. What is the perimeter of the desk? (derived from G5 M2 L1)

4. Sylvia weighed 8 pounds when she was born. By her first birthday, her weight had tripled. By her second birthday, she had gained 12 more pounds. At that time, Sylvia’s father weighed 5 times as much as she did. What was Sylvia and her dad’s combined weight? (G4 M3 L13)
5. Marissa has twice as much money as Frank. Christina has $20 more than Marissa. If Christina has $100, how much money does Frank have? Let $f$ represent the amount of money Frank has in dollars and $m$ represent the amount of money Marissa has in dollars. 

(G6 M4 L28)

6. One angle is 5 less than 3 times the size of the other angle. Together, they have a sum of $143^\circ$. What are the sizes of the angles? (G8 M4 L5)
Set 5, Group B

1. Molly uses 9 cups of flour to bake bread. If this is \( \frac{3}{4} \) of the total amount of flour she started with, what was the original amount of flour? (G6 M2 L2)

2. Margo is freezing 8 cups of strawberries. If this is \( \frac{2}{3} \) of the total strawberries that were picked, how many cups of strawberries did Margo pick? (G6 M2 L2)
3. The measurement of an angle is \( \frac{2}{3} \) the measurement of its supplement. Find the measurement of the angle. (G7 M6 L4)

4. The total age of a woman and her son is 51 years. Three years ago, the women was eight times as old as her son. How old is her son now? (G9 M1 L25)
Directions

Step 1  Examine the models below and identify all the information that is given in the model.

Step 2  Individually, create at least one comparison word problem that can be solved using each model.

Step 3  Compare your word problems with a partner.

1. (G4 M5 L39)

   ![Diagram](image1)

   Natasha

   Maya

2. (G7 M6 L1)

   ![Diagram](image2)

   A

   B

   90°
Directions

Step 1  Complete the first problem role-playing a student as the facilitator models the RDW process.
Step 2  Read the Problem Solving Protocol.
Step 3  Apply the protocol to solve the problems of Set 1, Group A.
Step 4  Use both the multiplication/division and addition/subtraction situation charts (the latter when applicable) to analyze and classify the situations.

Set 6, Group A

1. Josie took a multiple-choice end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number she got correct is 2:9. If Josie missed 8 problems, how many problems did she get correct? (G6 M1 L3)

2. The ratio of Isabella’s money to Shane’s money is 3:11. If Isabella has $33, how much money do Shane and Isabella have together? Use diagrams to illustrate your answer. (G6 M1 L4)
3. The measures of two angles have a sum of 180 degrees. The measures of the angles are in a ratio of 5:1. Determine the measures of both angles. (G6 M4 L30)

4. On Saturday night, the ratio of the number of occupied rooms to the number of unoccupied rooms at a hotel is 2:5. On Sunday night the ratio of the number of occupied rooms to the number of unoccupied rooms is 6:1. If the hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night? (derived from G6 M1 L6)
5. In a school choir, \( \frac{1}{2} \) of the members were girls. At the end of the year, 3 boys left the choir, and the ratio of boys to girls became 3:4. How many boys remained in the choir? (G9 M1 L25)
Directions

Step 1  Examine the models below and identify all the information that is given in the model.
Step 2  Individually, create at least one ratio word problem that can be solved using each model.
Step 3  Compare your word problems with a partner.

1.  (G6 M1 L3)

```
  4
Mason

Laney

?```

2.  (G6 M1 L5)

```
?  
Boys

Girls

250```
Directions

Step 1  Attempt the opening problems again using your new knowledge of models.
Step 2  Compare your model with a partner.

1. Last summer, at Camp Okey-Fun-Okey, the ratio of the number of boy campers to the number of girl campers was 8:7. If there were a total of 195 campers, how many boy campers were there? How many girl campers?

2. All the printing presses at a shop were scheduled to make copies of a novel and a cookbook. They printed the same number of copies of each book but the novel had twice as many pages as the cookbook. On the first day, all the presses worked printing novels. On the second day, the presses were split into two equally sized groups. The first group continued printing copies of the novel and finished printing all the copies by the evening. The second group worked on the cookbook but did not finish by evening. One press, working for two additional full days, finished printing the remaining copies of the cookbooks. If all presses worked (for both the novel and cookbook) at the same constant rate, how many printing presses are there at the print shop? (G9 M1 L25)
Protocol A for Problem Solving: “Draw and Re-Draw”
Pencils are required for this protocol.

1. Independently model and solve the group’s problems on the left side of the space provided below each one.
2. Share your models and solutions with a partner or within a triad.
3. On the right side of the space provided, redo the models so as to clarify the relationships within each and the coherence from one problem to the other.

Protocol B for Deliberate Practice: “Teach and Repeat Using Feedback”
Time frames may be adjusted.

1. (90 seconds) The “teacher” and “students” plan for the instruction and prepare needed materials.
2. (60 seconds) The “teacher” stands and delivers instruction to other team members.
3. (90 seconds) The “students” make a positive statement about A’s teaching strategy and then suggest an improvement. Partner A “calls the shot” by restating the feedback that will be practiced.
4. (60 seconds) Partner A reteaches precisely the same segment of instruction, using the feedback.
5. (60 seconds) Partners give feedback again.
6. (60 seconds) Partner A uses the feedback again.

Protocol C for SHARE IT: Whole Group Participation

1. Write “Share It” on four brightly colored 3 x 5 or 4 x 6 cards.
2. As you, the teacher/facilitator, circulate during small group talk, listen for an interesting insight, thought, discussion that you think would be helpful for the whole group to hear.
3. Ask the students/participant(s) if they would be willing to share their thoughts with the whole group. If willing, hand over a “Share It” card.
4. Once the small group sharing has concluded, call on those to whom you passed the “Share It” cards.

Protocol D: Precise Descriptions of Terminology

1. Participants individually derive a description of these terms based on their experiences and understandings.
2. Participants share description with partners and revise as necessary.
3. Participants share their descriptions with the whole group.
4. Participants are guided to the glossary, providing the precise descriptions from the curriculum.
Possible Deliberate Practice Stems: “Push Me to My Best Practice!”

As adult learners, we communicate to each other and to our students that anyone’s teaching practice or learning can be improved with effective effort. Don’t give up on colleagues, on ourselves or on students!

Stems for Positive Feedback during Deliberate Practice:

- It was effective when you _____.
- The connection between the pictorial and the abstract was clarified when you _____.
- It helped when you gestured to show the relationship between _____ and _____.

Stems Suggesting Improvements during Deliberate Practice:

- One “push” I would give would be to _____.
- Next time, try _____.
- You might clarify the relationship between the pictorial representation and the number sentence by _____.
- You might remain silent and demonstrate (gesture) as you _____.
- You might say _____ instead of _____.
- You might shorten/lengthen the wait time after _____.
- _____ might be good to tell where _____ might be better to elicit.
- When you check for understanding you might ...
- After you asked ________, your cue for a response could be crisper.
- When you checked our personal boards, you might _____.
- Do it again. Do it better by _____.

Reflection on Practice: “How are we doing at practicing deliberately?”

- The distractions from the deliberate practice were ...
- You mention a jump between Problem ___ and Problem ____. What is a bridge problem you could inset between the two?
Glossary

Measurement Division: The whole and the size or measure of one group is known, but the number of groups needs to be determined.

Partitive Division: The whole and the number of groups being created are known, but the size of the groups needs to be determined.
# Appendix A: Addition and Subtraction Situations

<table>
<thead>
<tr>
<th>Table 1: Addition and subtraction situations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Add To</td>
</tr>
<tr>
<td>A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now?</td>
</tr>
<tr>
<td>C apples were on the table. I ate B apples. How many apples are on the table now?</td>
</tr>
<tr>
<td>Take From</td>
</tr>
<tr>
<td>Put Together/Take Apart</td>
</tr>
<tr>
<td>A red apples and B green apples are on the table. How many apples are on the table?</td>
</tr>
<tr>
<td>Grandma has C flowers. How many can she put in her red vase and how many in her blue vase?</td>
</tr>
<tr>
<td>Compare</td>
</tr>
<tr>
<td>“How many more?” version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?</td>
</tr>
<tr>
<td>“How many fewer?” version. Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie?</td>
</tr>
<tr>
<td>“More” version suggests operation. Lucy has B more apples than Lucy. Julie has A apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>“Fewer” version suggests wrong operation. Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Lucy have?</td>
</tr>
</tbody>
</table>

In each type (shown as a row), any one of the three quantities in the situation can be unknown, leading to the subtypes shown in each cell of the table. The table also shows some important language variants which, while mathematically the same, require separate attention. Other descriptions of the situations may use somewhat different names. Adapted from CCSS, p. 86, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33.

1 This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the right help children understand that “does not always mean “makes” or “results in” but always means “is the same number as.” Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

2 Either addend can be unknown; both variations should be included.
Appendix B: Word Problems from Models

In the lesson, the model is provided and the students are asked to write a word problem. The sample responses are given here:

G4 M3 L12
Patti’s sandals weight 1,167 grams. She bought 3 pairs, all different colors! All 3 pairs of sandals together weigh 239 grams more than her winter boots. What is the weight of Patti’s winter boots?

G5 M4 L28
Victor has $\frac{1}{4}$ of ice cream to equally share among himself and four friends. How much ice cream will each person receive?

G6 M2 L6
Zolanda spent $\frac{5}{8}$ of her class period taking notes. If Zolanda spent 25 minutes taking notes, how long is her class period?

In the lesson, the word problems are given and the students may use a model to solve the problem.

G4 M5 L39
Natasha’s sculpture was $5\frac{3}{16}$ inches tall. Maya’s was 4 times as tall. How much shorter was Natasha’s sculpture than Maya’s?

G7 M6 L1
In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measurements of the two angles.

G6 M1 L3
Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3. If Mason ran 4 miles, how far did Laney run?

G6 M1 L5
At a country concert, the ratio of the number of boys to the number of girls is 2:7. If there are 250 more girls than boys, how many boys are at the concert?
Appendix C: Stages of Model Drawing

The foundation of the tape diagram begins in Kindergarten. As students advance through the years, there are different ways to model and to scaffold the labeling, as you can see in the sequence on the next page.

The primary concern is that the model supports the student in decoding the situation and makes sense in relationship to the problem. To differentiate you might label pictorially, with the number inside the tape, or the number outside the tape. In deciding how to model for a whole class the question is, “What is the most sophisticated method I can use that students can readily access?”

You will notice that in Stages 1-5 the tapes are labeled with the whole on the bottom of the tape. It has advantages in that it readily transitions to the number line. Does this mean it is always correct? Not necessarily. Consider a situation such as: *Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers.* The natural way might be to draw one unit first, label the unit as having a value of 3 and then draw 3 more like units. It would make sense to label the ‘3’ on the bottom rather than the top if labeling outside the tape just because the student is seeing the unit while labeling rather than covering it with his hand. At the end of drawing, the student might then label the unknown at the top of the tape.

That said, consistent approaches can be helpful as students are learning to model, e.g. labeling the total on the top or the bottom, inside or outside the tape, but take care that initial restrictions and consistencies don’t turn into rigid rules that become meaningless procedures rather than tools for reasoning about relationships. ¹ A rule of thumb might be to let the situation guide the drawing as you work chunk by chunk with an eye towards labeling so that students will understand. As you use the tape diagram be aware of the choices you are making. Over time, expect students to label and model in their own ways when working independently.

¹ The progression outlined is derived from and inspired by an excellent resource, “A Handbook for Mathematics Teachers in Primary School” by Julianan Ng Chye Huat and Lim Kian Huat
6 Stages of Model Drawing
Directions: Analyze and notate the new complexities at each stage.

Addition and Subtraction

Stage 1

Stage 2

Stage 3

Stage 4

Stage 5

Stage 6

Equal Groups

Stage 1

Stage 2

Stage 3

Stage 4

Stage 5

Stage 6

Jen

Risa
Appendix D:  Analysis of Evidence of the Standards for Mathematical Practice within a Grade 2 Classroom

Directions:
Watch the short video clip of the Grade 2 student’s thinking about his work.

Discuss with your partner: What Standards for Mathematical Practice are evidenced? How are they evidenced?

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.
STUDENT A

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{align*}
152 & - 80 \\
72 & \div 8 = 9
\end{align*}
\]

Mia makes nine bracelets.

STUDENT B

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{align*}
152 & - 80 \\
72 & \div 8 = 9
\end{align*}
\]

Mia makes nine bracelets.
STUDENT C

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{align*}
152 & \rightarrow 80 = 72 \\
152 & \rightarrow \frac{100 - 80 = 20}{52 + 20 = 72} \\
72 & \div 8 = 9
\end{align*}
\]

Mia made 9 bracelets.

STUDENT D

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{align*}
152 & \div 80 = 132 \\
52 + 20 & = 72 \\
72 & \div 8 = 9
\end{align*}
\]

Mia makes 9 bracelets.
STUDENT E

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{array}{c}
152 \\
- 80 \\
\hline
72
\end{array}
\]

Mia can make 9 bracelets with 72 beads.

STUDENT F

Use the RDW process to solve.

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?

\[
\begin{array}{c}
152 \\
- 80 \\
\hline
72
\end{array}
\]

Mia used 152 beads.
Modeling Word Problems with Tape Diagrams (3-5) – Answer Key

Note: The solutions provided within this document show how the problems could be solved. Because there is not a rigid way to draw tape diagrams, solution processes may vary.

Step 1 Read Problem 1 individually. Solve with a model, and then algebraically.
Step 2 Compare your model with a partner’s.

1. Last summer, at Camp Okey-Fun-Okey, the ratio of the number of boy campers to the number of girl campers was 8:7. If there were a total of 195 campers, how many boy campers were there? How many girl campers? (G6 M1 L5)

   \[
   \begin{align*}
   \text{Boys} & : 8x \\
   \text{Girls} & : 7x \\
   \text{8x} + 7x & = 195 \\
   15x & = 195 \\
   x & = 13 \\
   \end{align*}
   \]

   15 units = 195 campers
   1 unit = \( \frac{195}{15} \) campers
   boys = 8(13) = 104
   girls = 7(13) = 91

   There were 104 boy campers and 91 girl campers.

2. All the printing presses at a shop were scheduled to make copies of a novel and a cookbook. They printed the same number of copies of each book but the novel had twice as many pages as the cookbook.
   On the first day, all the presses worked printing novels.
   On the second day, the presses were split into two equally sized groups.
   The first group continued printing copies of the novel and finished printing all the copies by the evening.
   The second group worked on the cookbook but did not finish by evening.
   One press, working for two additional full days, finished printing the remaining copies of the cookbooks.
   If all presses worked (for both the novel and cookbook) at the same constant rate, how many printing presses are there at the print shop? (G9 M1 L25)

   There are 8 printing presses at the print shop.
Set 1, Group A

1. Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use all together? (G3 M1 L2)

\[ \begin{array}{c}
3 \\
3 \\
3 \\
3 \\
\end{array} \quad 3+3+3+3=12 \\
4\times3=12 \\
\text{Jordan uses 12 lemons all together.} \\
\]

2. A scientist fills 5 test tubes with 9 milliliters of fresh water in each. She fills another 3 test tubes with 9 milliliters of salt water in each. How many milliliters of water does she use in all? (G3 M3 L12)

\[ \begin{array}{c}
\text{Fresh Water} \\
9 \\
9 \\
9 \\
\end{array} \quad \begin{array}{c}
\text{Salt Water} \\
9 \\
9 \\
9 \\
\end{array} \quad \text{Unknown product} \\
\]

\[(5\times9)+(3\times9)=45+27 \quad \text{or} \quad 8\times9=72 \]
\[=5\times22 \]
\[=72 \]

The scientist uses 72 milliliters of water in all.
3. Three boxes weighing 128 pounds each and one box weighing 254 pounds were loaded onto the back of an empty truck. A crate of apples was then loaded onto the same truck. If the total weight loaded onto the truck was 2,000 pounds, how much did the crate of apples weigh? (G4 M3 L13)

![Diagram showing the calculation of the total weight](image)

The crate of apples weighed 1,362 pounds.

4. 852 pounds of grapes were packed equally into 3 boxes for shipping. How many pounds of grapes will there be in 2 boxes? (G5 M2 L17)

![Diagram showing the calculation of grape weight](image)

There will be 568 pounds of grapes in 2 boxes.
Set 1, Group B

1. Shelly read her book for \( \frac{3}{4} \) hour each afternoon for 9 days. How many hours did Shelly spend reading in all 9 days? (G4 M5 L24)

\[
\begin{array}{c}
\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \\
\end{array}
\]

\[9 \times \frac{1}{2} = 9 \times 1 \text{ half} = (9 \times 1) \text{ half} = 9 \text{ halves} = \frac{9}{2} = 4 \frac{1}{2}\]

Shelly spent \( 4 \frac{1}{2} \) hours reading in all 9 days.

2. Rhonda exercised for \( \frac{5}{6} \) hour every day for 5 days. How many total hours did Rhonda exercise? (G4 M5 L36)

\[
\begin{array}{c}
\frac{5}{6} \quad \frac{5}{6} \quad \frac{5}{6} \quad \frac{5}{6} \quad \frac{5}{6} \\
\end{array}
\]

\[5 \times \frac{5}{6} = 5 \times 5 \text{ sixths} = (5 \times 5) \text{ sixths} = 25 \text{ sixths} = \frac{25}{6} = 4 \frac{1}{6}\]

Rhonda exercised for \( 4 \frac{1}{6} \) hours.

3. Gail has two yards of fabric. It takes \( 8 \frac{3}{4} \) yards of fabric to make one quilt. She wants to make three quilts. How many more yards of fabric does she need to buy in order to make three quilts? (G4 M5 L39)

\[
\begin{array}{c}
\frac{2}{1} \quad \frac{2}{1} \quad \frac{2}{1} \quad \frac{2}{1} \quad \frac{2}{1} \\
\end{array}
\]

\[2 \times \left( 3 \times 8 \frac{3}{4} \right) - 2 = \left( 3 \times 8 \right) + \left( 3 \times 8 \frac{3}{4} \right) - 2 = 24 + 9 \frac{3}{4} - 2 = 24 + 2 \frac{1}{4} - 2 = 26 \frac{1}{4} - 2 = 24 \frac{1}{4}\]

Gail needs to buy \( 24 \frac{1}{4} \) yards of fabric in order to make 3 quilts.
Set 1, Group C

1. There are 32 students in a class. Of the class, \(\frac{3}{8}\) bring their own lunch. How many students bring their lunch? (G5 M4 L6)  

\[
\begin{align*}
8 \text{ units} &= 32 \text{ students} \\
1 \text{ unit} &= 32 \text{ students} \div 8 \\
1 \text{ unit} &= 4 \text{ students} \\
3 \text{ units} &= 3 \times 4 \text{ students} \\
&= 12 \text{ students}
\end{align*}
\]

12 students bring their lunch.

2. Moussa delivered \(\frac{3}{8}\) of the newspapers on his route in the first hour and \(\frac{4}{5}\) of the rest in the second hour. What fraction of the newspapers did Moussa deliver in the second hour? (G5 M4 L15)  

\[
\begin{align*}
\text{Moussa delivered} & \quad \frac{4}{8} \text{ of the newspapers in the second hour.}
\end{align*}
\]

3. Mrs. Diaz makes 5 dozen cookies for her class. One-ninth of her 27 students are absent the day she brings the cookies. If she shares the cookies equally among the students who are present, how many cookies will each student get? (G5 M4 L11)  

\[
\begin{align*}
\text{5 dozen} &= 60 \text{ cookies} \\
\frac{12}{60} &= \frac{1}{5} \\
9 \text{ units} &= 27 \\
1 \text{ unit} &= 27 \div 9 = 3 \\
8 \text{ units} &= 8 \times 3 = 24 \\
24 \text{ students were present.}
\end{align*}
\]

\[
\begin{align*}
\frac{60}{24} &= \frac{60 \div 12}{24 \div 12} = \frac{5}{2} = 2\frac{1}{2} \\
\text{Each student will get} & \quad 2\frac{1}{2} \text{ cookies.}
\end{align*}
\]
Set 2, Group A

1. Andrew has 21 keys. He puts them in 3 equal groups. How many keys are in each group? (G3 M1 L4)

\[ 21 \div 3 = 7 \]

There are 7 keys in each group.

2. 1,624 shirts need to be sorted into 4 equal groups. How many will be in each group? (G4 M3 L31)

\[ \frac{1624}{4} = 406 \]

406 shirts will be in each group.

3. Jesse and his 3 friends buy snacks for a hike. They buy trail mix for $5.42, apples for $2.55, and granola bars for $3.39. If the 4 friends split the cost of the snacks equally, how much should each friend pay? (G5 M1 L16)

\[ 5.42 + 2.55 + 3.39 = 11.36 \]

\[ \frac{11.36}{4} = 2.84 \]

Each friend should pay $2.84.
4. Four grade levels need equal time for indoor recess, and the gym is available for 3 hours. How many hours of recess will each grade level receive? (G5 M4 L4)

\[
\begin{align*}
3 \text{ hours} \\
x
\end{align*}
\]

\[
\frac{0.3}{4} \quad \frac{4}{3}
\]

4 units = 3 hours
1 unit = \frac{3}{4} hour
Each grade level will receive \frac{3}{4} hour of recess.

5. Mrs. Onusko made 60 cookies for a bake sale. She sold \frac{2}{3} of them and gave \frac{3}{4} of the remaining cookies to the students working at the sale. How many cookies did she have left? (G5 M4 L16)

\[
\begin{align*}
60 \text{ cookies} \\
\text{sold} \\
\text{given away} \\
\text{left}
\end{align*}
\]

12 units = 60 cookies
1 unit = \frac{60}{12} cookies
1 unit = 5 cookies
Mrs. Onusko had 5 cookies left.

6. Allen's team was required to buy two pairs of uniform pants and two baseball caps which totaled $68. A pair of pants costs $12 more than a baseball cap. What is the cost of one cap? (G7 M2 L23)

\[
\begin{align*}
Pants & \quad Pants & \quad Caps & \quad Caps \\
C & \quad 12 & \quad C & \quad 12 & \quad C & \quad C
\end{align*}
\]

\[
\begin{align*}
& \quad \text{\$68} \\
\end{align*}
\]

\[
4C + 24 = 68
\]
\[
4C = 44
\]
\[
C = 11
\]

The cost of one cap is $11.
7. A motorcycle dealer paid a certain price for a motorcycle and marked it up by $\frac{1}{5}$ of the price he paid. Later he sold it for $14,000. What was the original price? (G7 M1 L14)

$6 \times 2,333 \frac{2}{3} = 14,000$

$1 \times 2,333 \frac{2}{3} = 1,166 \frac{2}{3}$

The original price was $11,666 \frac{3}{6}$.

8. Every day Heather practices soccer and piano. Each day she practices piano for 2 hours. If after 5 days she practiced both piano and soccer for a total of 20 hours, how many hours, $h$, per day did Heather practice soccer? (G7 M3 L9)

$(5 \times 2) + (5 \times h) = 20$

$10 + 5h = 20$

$5h = 10$

$h = 2$

Heather practiced soccer for 2 hours each day.

9. Find 3 consecutive integers such that their sum is 1,623. (G8 M4 L1)

$3n + 3 = 1,623$

$3n = 1,620$

$n = 540$

$n + 1 = 541$

$n + 2 = 542$
Set 2, Group B

1. Ms. Hayes has $\frac{1}{2}$ liter of juice. She distributes it equally to 6 students in her tutoring group. How many liters of juice will each student get? (G5 M4 L33)

\[
\frac{1}{2} \div 6 = \frac{1}{12}
\]

Each student will get $\frac{1}{12}$ liter of juice.

2. You have $\frac{2}{5}$ of a cup of frosting to share equally among three desserts. How much frosting will be placed on each dessert? (G6 M2 L1)

\[
\frac{2}{5} \text{ cup} \div 3 = \frac{2}{15}
\]

$\frac{2}{15}$ cup of frosting will be placed on each dessert.
3. Four baby socks can be made from \( \frac{1}{3} \) skein of yarn. How many baby socks can be made from a whole skein? (G5 M4 L32)

\[
\frac{4 \text{ baby socks}}{\frac{1}{3}} = 12
\]

12 baby socks can be made from a whole skein.

4. Three gallons of water fills \( \frac{1}{4} \) of the elephant's pail at the zoo. How much water does the pail hold? (G5 M4 L25)

\[
\frac{3 \text{ gallons}}{\frac{1}{4}} = 12
\]

The pail hold 12 gallons.
Directions
Step 1: Examine the models below and identify all the information that is given in the model.
Step 2: Individually, create at least one word problem that can be solved using each model.
Step 3: When signaled, compare your word problems with a partner.

1. (G4 M3 L12)

Marsha took three trips to see her daughter at college. Each trip was 1,167 miles.
Joe took one trip to see his daughter at college. His trip was 239 miles less than Marsha's three trips combined. How far was Joe's trip?

Joe's trip was 3,262 miles.

2. (G5 M4 L28)

Saisha put \(\frac{1}{4}\) of her money into savings. She withdrew \(\frac{1}{4}\) of that amount to buy a savings bond. What fraction of her total money was spent on the savings bond?

\(\frac{1}{4}\) of \(\frac{1}{4}\) = \(\frac{1}{16}\)

\(\frac{1}{16}\) of Saisha's total money was spent on the savings bond.
3. (G6 M2 L6)

Mrs. Fowler graded 25 tests on Monday. That was \(\frac{5}{8}\) of the total number of tests that she had to grade. How many tests did she have to grade?

- 5 units = 25
- 1 unit = \(25 \div 5 = 5\)
- 8 units = \(8 \times 5 = 40\)

Mrs. Fowler had 40 tests to grade.

4. Draw a model and have your partner write a word problem.

If Sierra spent 12 hours completing \(\frac{3}{4}\) of her homework, how many hours did it take to complete all of the homework?

- 3 units = 12
- 1 unit = \(12 \div 3 = 4\)
- 4 units = \(4 \times 4 = 16\)

It took 16 hours to complete all of the homework.
1. Jackie buys 21 pizzas for a party. She places 3 pizzas on each table. How many tables are there? (G3 M1 L14)

\[ 21 \div 3 = 7 \]

There are 7 tables.

2. Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make? (G3 M3 L19)

\[ 152 - 80 = 72 \]

\[ 72 \div 8 = 9 \]

Mia makes 9 bracelets.
3. Monique needs exactly 4 plates on each table for the banquet. If she has 312 plates, how many tables is she able to prepare? (G4 M3 L31)  

\[ \begin{array}{c}
312 \\
\div 4 \\
\hline \\
78 \\
\hline \\
28 \\
\hline \\
32 \\
\hline \\
32 \\
\hline \\
0 \\
\end{array} \]

? tables

Monique is able to prepare 78 tables.

4. The cost of a babysitting service on a cruise is $10 for the first hour and $12 for each additional hour. If the total cost of babysitting baby Aaron was $58, how many hours was Aaron at the sitter? (G7 M2 L17)

\[ \begin{array}{c}
58 \\
\div 10 \\
\hline \\
48 \\
\hline \\
12 \\
\hline \\
0 \\
\end{array} \]

1st hour

? additional hours

Aaron was at the sitter for 5 hours.
Set 3, Group B

1. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up $\frac{3}{4}$ of a mile, how many family members are needed to clean the adopted section? (G6 M2 L2)

![Diagram showing 6 miles of trail with $\frac{3}{4}$ of a mile marked.]

8 family members are needed to clean the adopted section.

2. George bought 12 pizzas for a party. If each person will eat $\frac{3}{8}$ of a pizza, how many people can George feed with 12 pizzas? (G6 M2 L2)

![Diagram showing 12 pizzas with $\frac{3}{8}$ of a pizza marked.]

$12 \div \frac{3}{8} = \frac{96}{3} = 32$

George can feed 32 people with 12 pizzas.

3. A race begins with $2 \frac{1}{2}$ miles through town, continues through the park for $2 \frac{1}{3}$ miles, and finishes at the track after the last $\frac{1}{6}$ mile. A volunteer is stationed every quarter mile and at the finish line to pass out cups of water and cheer on the runners. How many volunteers are needed? (G5 M4 L26)

![Diagram showing a race course with distances marked.]

$2 \frac{1}{2} + 2 \frac{1}{3} + \frac{1}{6} = 5$

$5 \div \frac{1}{4} = 20$

20 volunteers are needed.
**Larger Unknown**

An eraser is 2 inches long. A pencil is 4 times as long as the eraser. How long is the pencil?

\[4 \times 2 = 8\]

The pencil is 8 inches long.

**Smaller Unknown**

A pencil is 8 inches long and is 4 times as long as an eraser. How long is the eraser?

\[4 \times n = 8\]
\[\frac{8}{4} = n\]
\[n = 2\]

The eraser is 2 inches long.

**Smaller Unknown A < 1**

A pencil is 8 inches long. An eraser is \(\frac{1}{4}\) as long as the pencil. How long is the eraser?

\[\frac{1}{4} \times 8 = \frac{8}{4} = 2\]

The eraser is 2 inches long.

**Larger Unknown A < 1**

An eraser is 2 inches long and that is \(\frac{1}{4}\) the length of a pencil. How long is the pencil?

\[4 \times 2 = 8\]

The pencil is 8 inches long.
Set 4, Group A

1. Andre bought a stamp to mail a letter that cost 46 cents. He also mailed a package that cost 5 times as much as a stamp. How much did it cost to mail the package and the letter? (G4 M3 L8) Compare | larger unknown

\[
\begin{align*}
\text{Stamp} & \quad 46 \\
\text{package} & \quad 46 \quad 46 \quad 46 \quad 46 \\
\end{align*}
\]

\[
\begin{align*}
? & \quad \frac{46}{5} + \frac{230}{46} \\
& \quad \frac{276}{230} \\
& \quad \frac{276}{276} \\
& \quad \frac{276}{276} \\
276 \text{ cents} & = \$2.76
\end{align*}
\]

- or -

\[
\begin{align*}
\frac{46}{276} & \quad \frac{6}{276} \\
6 \text{ units of 46} & = 6 \times 46 = 276
\end{align*}
\]

It cost 276 cents to mail the package and letter.

2. Jayden has 347 marbles. Elvis has 4 times as many as Jayden. Presley has 799 fewer than Elvis. How many marbles does Presley have? (G4 M3 L12) Compare | larger unknown

\[
\begin{align*}
\text{Jayden} & \quad 347 \\
\text{Elvis} & \quad 347 \quad 347 \quad 347 \quad 347 \\
\text{Presley} & \quad m = 799
\end{align*}
\]

\[
\begin{align*}
1 \text{ unit} & = 347 \\
4 \text{ units} & = 4 \times 347
\end{align*}
\]

\[
\begin{align*}
\frac{347}{1388} & = \frac{347}{589} \\
4 \times 347 & = 1388 \\
m & = 589
\end{align*}
\]

Presley has 589 marbles.

3. Carol made punch. She used \(12\frac{3}{8}\) cups of juice and then added three times as much ginger ale. Then she added 1 cup of lemonade. How many cups of punch did her recipe make? (G4 M5 L39) Compare | larger unknown \to Put together | Total unknown

\[
\begin{align*}
\text{J} & \quad 12 \quad \frac{3}{8} \\
\text{GA} & \quad 12 \quad \frac{3}{8} \quad 12 \quad \frac{3}{8} \quad 12 \quad \frac{3}{8} \\
\text{L} & \quad 1
\end{align*}
\]

\[
\begin{align*}
4 \times 12\frac{3}{8} + 1 & = (4 \times 12) + (4 \times \frac{3}{8}) + 1 \\
& = 48 + \frac{12}{8} + 1 \\
& = 49 + \frac{4}{8} + 1 \\
& = 49 + \frac{4}{8} + \frac{8}{8} + \frac{1}{8} \\
& = 49\frac{4}{8} + 1 = 50\frac{4}{8} = 50\frac{1}{2}
\end{align*}
\]

Carol's recipe made 50\(\frac{1}{2}\) cups of punch.
4. In one month, Charlie read 814 pages. In the same month, his mom read 4 times as many pages as Charlie, and that was 143 pages more than Charlie's dad read. What was the total number of pages read by Charlie and his parents? (G4 M3 L13)

\[
\begin{align*}
C &= 814 \\
M &= 814 + 814 + 814 + 814 \\
D &= 814 + 814 + 814 + 814 \\
1 \text{ unit} &= 814 \\
9 \text{ units} &= 9 \times 814 \\
\text{Total} &= 7326 \\
\text{Unknown} &= 7183
\end{align*}
\]
Charlie and his parents read 7,183 pages.

5. Shelby is 7 times as old as Bonnie. If in 5 years the sum of Bonnie's and Shelby's ages is 98, find Bonnie's present age. (G7 M3 L9)

\[
\begin{align*}
\text{Bonnie} &= 5 \\
\text{Shelby} &= 5 \\
8b + 10 &= 98 \\
8b &= 88 \\
b &= 88 \div 8 = 11
\end{align*}
\]
Bonnie's present age is 11.

6. 16 years from now, Pia's age will be twice her age 12 years ago. Find her present age. (G9 M1 L25)

\[
\begin{align*}
P &= 16 \\
P + 16 &= (P - 12) + (P - 12) \\
P + 16 &= 2P - 24 \\
40 &= P
\end{align*}
\]
Pia is 40 years old.
1. Jasmine took $\frac{2}{3}$ as much time to take a math test as Paula. If Paula took 2 hours to take the test, how long did it take Jasmine to take the test? Express your answer in minutes. (G5 M5 L23)

\[2 \text{ hours} = 120 \text{ minutes} \quad 2 \text{ hours} = 120 \text{ minutes}\]

\[3 \text{ units} = 120 \text{ minutes} \quad 1 \text{ unit} = 120 \div 3 = 40 \text{ minutes} \quad 2 \text{ units} = 80 \text{ minutes}\]

It took Jasmine 80 minutes to take the test.

2. Carli has 90 apps on her phone. Braylen has $\frac{1}{2}$ the amount of apps as Theiss. If Carli has 3 times the amount of apps as Theiss, how many apps does Braylen have? Let b represent the number of Braylen's apps and t represent the number of Theiss' apps. (G6 M4 L28)

\[\begin{array}{c}
\text{C} \\
\hline
90 \\
\text{T} \\
\hline
\text{B} \\
\hline
b
\end{array}\]

\[6 \text{ units} = 90 \quad 1 \text{ unit} = 90 \div 6 = 15\]

Braylen has 15 apps.
1. Jamie drank 4 times as much juice as Brodie. Jamie drank 280 mL of juice. How much juice did Brodie drink? (G4 M3 L26)

![Diagram]

\[
\begin{align*}
4 \text{ units} &= 280 \\
1 \text{ unit} &= \frac{280}{4} = 28 \text{ tens} \\
&= 70
\end{align*}
\]

Brodie drank 70 mL of juice.

2. The Hometown Hotel has a total of 480 guest rooms. That is 6 times as many rooms as the Travelers Hotel down the street. How many rooms are there in the Travelers Hotel? (G4 M3 L26)

![Diagram]

\[
\begin{align*}
6 \text{ units} &= 480 \\
1 \text{ unit} &= \frac{480}{6} = 80
\end{align*}
\]

There are 80 rooms in the Travelers Hotel.
3. The top surface of a desk has a length of 5.6 feet. The length is 4 times its width. What is the perimeter of the desk? (derived from G5 M2 L1)

\[
\begin{align*}
4 \text{ units} &= 5.6 \\
1 \text{ unit} &= 5.6 \div 4 \\
1 \text{ unit} &= 1.4
\end{align*}
\]

\[
\begin{array}{c}
5.6 \\
4 \\
1.4
\end{array}
\]

\[
\begin{array}{c}
-4 \\
16 \\
-16 \\
0
\end{array}
\]

The perimeter of the desk is 14 feet.

4. Sylvia weighed 8 pounds when she was born. By her first birthday, her weight had tripled. By her second birthday, she had gained 12 more pounds. At that time, Sylvia’s father weighed 5 times as much as she did. What was Sylvia and her dad’s combined weight? (G4 M3 L13)

Birth: 8

Age 1: 8 8 8

\[36 \times \frac{36}{216} \]

Age 2: 8 8 8 12

Father

\[1 \text{ unit} = 36 \]

6 units = 6 \times 36

6 units = 216

Sylvia and her dad’s combined weight was 216 pounds.
5. Marissa has twice as much money as Frank. Christina has $20 more than Marissa. If Christina has $100, how much money does Frank have? Let \( f \) represent the amount of money Frank has in dollars and \( m \) represent the amount of money Marissa has in dollars. (G6 M4 L28)

\[
\begin{align*}
F & \quad \boxed{f} \\
M & \quad \boxed{f} \quad \boxed{f} \\
C & \quad \boxed{f} \quad \boxed{f} \quad 20 \\
& \quad \underline{100}
\end{align*}
\]

\[2f + 20 = 100\]
\[2f = 80\]
\[f = 80 \div 2 = 40\]

Frank has $40.

6. One angle is 5 less than 3 times the size of the other angle. Together, they have a sum of 143°. What are the sizes of the angles? (G8 M4 L5)

\[
\begin{align*}
\text{angle 1} & \quad a \\
\text{angle 2} & \quad 3a - 5 \\
& \quad 143
\end{align*}
\]

\[
\begin{align*}
a + 3a - 5 & = 143 \\
4a - 5 & = 143 \\
4a & = 148 \\
a & = 148 \div 4 \\
a & = 37
\end{align*}
\]

\[
\begin{align*}
3a - 5 & = 111 - 5 = 106 \\
x & = \frac{37 \times 3}{8} \\
& = \frac{111}{8}
\end{align*}
\]

The sizes of the angles are 37° and 106°.
1. Molly uses 9 cups of flour to bake bread. If this is $\frac{3}{4}$ of the total amount of flour she started with, what was the original amount of flour? (G6 M2 L2)

\[ \frac{9}{3} \text{ units} = 9 \]
\[ 1 \text{ unit} = 9 ÷ 3 = 3 \]
\[ 4 \text{ units} = 3 \times 4 = 12 \]

The original amount of flour was 12 cups.

2. Margo is freezing 8 cups of strawberries. If this is $\frac{2}{3}$ of the total strawberries that were picked, how many cups of strawberries did Margo pick? (G6 M2 L2)

\[ \frac{8}{2} \text{ units} = 8 \]
\[ 1 \text{ unit} = 8 ÷ 2 = 4 \]
\[ 3 \text{ units} = 3 \times 4 = 12 \]

Margo picked 12 cups of strawberries.
3. The measurement of an angle is \( \frac{2}{3} \) the measurement of its supplement. Find the measurement of the angle. (G7 M6 L4)

\[
\begin{align*}
\text{Angle 2} & \quad 180 \\
\text{Angle 1} & \quad \frac{36}{5} \quad \frac{180}{5} = 36 \\
5 \text{ units} & = 180 \\
1 \text{ unit} & = 180 \div 5 = 36 \\
2 \text{ units} & = 2 \times 36 = 72
\end{align*}
\]

The measurement of the angle is 72°.

4. The total age of a woman and her son is 51 years. Three years ago, the woman was eight times as old as her son. How old is her son now? (G9 M1 L25)

\[
\begin{align*}
\text{Age now:} & \quad a + 3 \\
\text{Son:} & \quad 3 \\
\text{Woman:} & \quad 3 \quad 51 \\
9a + 6 & = 51 \\
9a & = 45 \\
a & = 5 \\
a + 3 & = \text{age now} \\
5 + 3 & = 8
\end{align*}
\]

The woman’s son is 8 now.
Directions
Step 1  Examine the models below and identify all the information that is given in the model.
Step 2  Individually, create at least one comparison word problem that can be solved using each model.
Step 3  Compare your word problems with a partner.

1. (G4 M5 L39)

Natasha drew a segment that was \(5 \frac{3}{16}\) inches long. Maya drew a segment that was 4 times as long as Natasha's. How much longer was Maya's segment than Natasha's?

\[
\text{1 unit} = 5 \frac{3}{16}
\]
\[
\text{3 units} = 3 \times 5 \frac{3}{16} = (3 \times 5) + (3 \times \frac{3}{16}) = 15 + \frac{9}{16} = 15 \frac{9}{16}
\]

Maya's line was \(15 \frac{9}{16}\) inches longer than Natasha's.

2. (G7 M6 L1)

Angle B is three times the measure of its complement, Angle A. What are the measures of Angle A and Angle B?

\[
4 \text{ units} = 90
\]
\[
1 \text{ unit} = \frac{90}{4}
\]

\[
\text{Angle } A = 22 \frac{1}{2}^\circ
\]
\[
\text{Angle } B = 67 \frac{1}{2}^\circ
\]
Set 6, Group A

1. Josie took a multiple-choice end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number she got correct is 2:9. If Josie missed 8 problems, how many problems did she get correct? (G6 M1 L3)

\[
\begin{align*}
\text{Incorrect} & : 8 \text{ problems} \\
\text{Correct} & : 2 \text{ units} \times 4 \text{ problems/1 unit} = 8 \text{ problems}
\end{align*}
\]

Josie got 36 problems correct.

2. The ratio of Isabella's money to Shane's money is 3:11. If Isabella has $33, how much money do Shane and Isabella have together? Use diagrams to illustrate your answer. (G6 M1 L4)

\[
\begin{align*}
3 \text{ units} &= 33 \\
1 \text{ unit} &= \frac{33}{3} = 11 \\
14 \text{ units} &= 14 \times 11 = (14 \times 10) + (14 \times 1) = 140 + 14 = 154
\end{align*}
\]

Shane and Isabella have $154 together.

3. The measures of two angles have a sum of 180 degrees. The measures of the angles are in a ratio of 5:1. Determine the measures of both angles. (G6 M4 L30)

\[
\begin{align*}
6 \text{ units} &= 180^\circ \\
1 \text{ unit} &= \frac{180^\circ}{6} = 30^\circ
\end{align*}
\]

The measures of both angles are 30° and 150°.
4. On Saturday night, the ratio of the number of occupied rooms to the number of unoccupied rooms at a hotel is 2:5. On Sunday night, the ratio of the number of occupied rooms to the number of unoccupied rooms is 6:1. If the hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night? (derived from G6 M1 L6)

The hotel has 360 unoccupied rooms on Saturday night.

5. In a school choir, \( \frac{2}{3} \) of the members were girls. At the end of the year, 3 boys left the choir, and the ratio of boys to girls became 3:4. How many boys remained in the choir? (G9 M1 L25)

9 boys remained in the choir.
Directions
Step 1  Examine the models below and identify all the information that is given in the model.
Step 2  Individually, create at least one ratio word problem that can be solved using each model.
Step 3  Compare your word problems with a partner.

1. (G6 M1 L3)

Mason and Laney tallied their homework time for the week. Mason spent 4 hours on homework. If the ratio of the time Mason spent on homework to the time Laney spent on homework was 2:3, how long did Laney spend on homework?

2 units = 4
1 unit = \frac{4}{2} = 2
3 units = 2 \times 3 = 6

Laney spent 6 hours on homework.

2. (G6 M1 L5)

The boys and girls in Algebra had a math competition. The ratio of the points earned by the boys to the points earned by the girls was 2:7. The girls earned 250 more points than the boys. How many points did the boys earn? How many points did the girls earn?

The boys earned 100 points. The girls earned 350 points.