Lesson 4: Percent Increase and Decrease

Student Outcomes

- Students solve percent problems when one quantity is a certain percent more or less than another.
- Students solve percent problems involving a percent increase or decrease.

Lesson Notes

Students begin the lesson by reviewing the prerequisite understanding of percent. Following this are examples and exercises related to percent increase and decrease. Throughout the lesson, students should continue to relate 100% to the whole and identify the original whole each time they solve a percent increase or decrease problem. When students are working backward, a common mistake is to erroneously represent the whole as the amount after the increase or decrease, rather than the original amount. Be sure to address this common mistake during whole-group instruction.

Classwork

Opening Exercise (4 minutes)

Opening Exercise
Cassandra likes jewelry. She has 5 rings in her jewelry box.

a. In the box below, sketch Cassandra’s 5 rings.

b. Draw a double number line diagram relating the number of rings as a percent of the whole set of rings.

0 20 40 60 80 100 Percent (%)

0 1 2 3 4 5 Number of Rings

c. What percent is represented by the whole collection of rings? What percent of the collection does each ring represent?

100%, 20%
Discussion (2 minutes)

Whole-group discussion of the Opening Exercise ensues. Students’ understanding of Opening Exercise part (c) will be critical for their understanding of percent increase and decrease. A document camera may be used for a student to present work to the class, or a student may use the board to draw a double number line diagram to explain.

- How did you arrive at your answer for Opening Exercise part (c)?
  - I knew that there were 5 rings. I knew that the 5 rings represented the whole, or 100%. So, I divided 100% and the total number of rings into 5 pieces on each number line. Each piece (or ring) represents 20%.

Example 1 (4 minutes): Finding a Percent Increase

Let’s look at some additional information related to Cassandra’s ring collection.

Example 1
Cassandra’s aunt said she will buy Cassandra another ring for her birthday. If Cassandra gets the ring for her birthday, what will be the percent increase in her ring collection?

- Looking back at our answers to the Opening Exercise, what percent is represented by 1 ring? If Cassandra gets the ring for her birthday, by what percent did her ring collection increase?
  - 20% represents 1 ring, so her ring collection would increase by 20%.
- Compare the number of new rings to the original total:
  - \( \frac{1}{5} = \frac{20}{100} = 0.20 = 20\%
- Use an algebraic equation to model this situation. The quantity is represented by the number of new rings.

Quantity = Percent × Whole. Let \( p \) represent the unknown percent.

\[
1 = p \cdot 5 \\
\frac{1}{5} = p \\
\frac{1}{5} = \frac{20}{100} = 0.2 = 20\%
\]
Exercise 1 (3 minutes)

Students work independently to answer this question.

Exercise 1

a. Jon increased his trading card collection by 5 cards. He originally had 15 cards. What is the percent increase? Use the equation Quantity = Percent \times Whole to arrive at your answer, and then justify your answer using a numeric or visual model.

\[ \text{Quantity} = \text{Percent} \times \text{Whole} \]

\[ 5 = p \times 15 \]

\[ \frac{5}{15} = \frac{5}{15} \times \frac{1}{15} \]

\[ 5 = \frac{1}{3} = 0.333... \]

\[ 0.333... = \frac{33}{100} \times \frac{33}{100} = 33\% + \frac{1}{3} = 33\frac{1}{3}\% \]

b. Suppose instead of increasing the collection by 5 cards, John increased his 15-card collection by just 1 card. Will the percent increase be the same as when Cassandra's ring collection increased by 1 ring (in Example 1)? Why or why not? Explain.

No, it would not be the same because the part-to-whole relationship is different. Cassandra's additional ring compared to the original whole collection was 1 to 5, which is equivalent to 20 to 100, which is 20%. John's additional trading card compared to his original card collection is 1 to 15, which is less than 10%.

since \( \frac{1}{15} < \frac{1}{10} \) and \( \frac{1}{10} = 10\% \).

c. Based on your answer to part (b), how is displaying change as a percent useful?

Representing change as a percent helps us to understand how large the change is compared to the whole.

Discussion (4 minutes)

Ask the class for an example of a situation that involves a percent decrease, or use the sample given below, and conduct a brief whole-group discussion about the meaning of the percent decrease. Then, in a whole-group instructional setting, complete Example 2.

Provide each student (or pair of students) with a small piece of paper or index card to answer the following question. Read the question aloud.

Consider the following statement: “A sales representative is taking 10% off of your bill as an apology for any inconveniences.” Write down what you think this statement implies.

Collect the responses to the question, and scan for examples that look like the following:

- I will only pay 90% of my bill.
- 10% of my bill will be subtracted from the original total.

how does this example differ from the percent increase problems?

In percent increase problems, the final value or quantity is greater than the original value or quantity; therefore, it is greater than 100% of the original value or quantity. In this problem, the final value is less than the original value or quantity; therefore, it is less than 100% of the original value or quantity.
Let’s examine these statements more closely. What will they look like in equation form?

<table>
<thead>
<tr>
<th>&quot;I will only pay 90% of my bill.&quot;</th>
<th>&quot;10% of my bill will be subtracted from the original total.&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>The new bill is part of the original bill, so the original bill is the whole. new bill = 0.9(original bill)</td>
<td>The new bill is the part of the original bill left over after 10% has been removed, so the original bill is the whole. new bill = (original bill) − 0.1(original bill)</td>
</tr>
</tbody>
</table>

These expressions are equivalent. Can you show and explain why?

If students are not able to provide the reasoning, provide scaffolding questions to help them through the following progression: One example is, if you are not paying 10% of your total (100%) bill, what percent are you paying?

- Let $n$ represent the amount of money due on the new bill, and let $b$ represent the amount of money due on the original bill.
  
  $n = b − 0.1(b)$  
  $10\%$ of the original bill is subtracted from the original bill.

- $n = 1b − 0.1(b)$  
  Multiplicative identity property of 1

- $n = b(1 − 0.1)$  
  Distributive property

- $n = b(0.9)$

- $n = 0.9(b)$  
  Any order (commutative property of multiplication)

The new bill is 90% of the original bill.

Example 2 (3 minutes): Percent Decrease

Example 2: Percent Decrease

Ken said that he is going to reduce the number of calories that he eats during the day. Ken’s trainer asked him to start off small and reduce the number of calories by no more than 7%. Ken estimated and consumed 2,200 calories per day instead of his normal 2,500 calories per day until his next visit with the trainer. Did Ken reduce his calorie intake by 7%? Justify your answer.

- Using mental math and estimation, was Ken’s estimate close? Why or why not?
  - No. 10% of 2,500 is 250, and 5% of 2,200 is 115 because $5\% = \frac{1}{2}(10\%)$. So mentally, Ken should have reduced his calorie intake between 125 and 250 calories per day, but he reduced his calorie intake by 300 calories per day. 300 > 250, which is more than a 10% decrease; therefore, it is greater than a 7% decrease.

- How can we use an equation to determine whether Ken made a 7% decrease in his daily calories?
  - We can use the equation Quantity = Percent $\times$ Whole and substitute the values into the equation to see if it is a true statement.

Scaffolding:

- Provide examples of the words increase and decrease in real-world situations. Provide opportunities for learners struggling with the language to identify situations involving an increase or decrease, distinguishing between the two.
- Create two lists of words: one listing synonyms for increase and one listing synonyms for decrease, so students can recognize keywords in word problems.
*Note that either of the following approaches, (a) or (b), could be used per previous discussion.

### Exercise 2 (5 minutes)

Students complete the exercise with a learning partner. The teacher should move around the room providing support where needed. After 3 minutes have elapsed, select students to share their work with the class.

**Exercise 2**

Skylar is answering the following math problem:

*The value of an investment decreased by 10%. The original amount of the investment was $75.00. What is the current value of the investment?*

a. Skylar said 10% of $75.00 is $7.50, and since the investment decreased by that amount, you have to subtract $7.50 from $75.00 to arrive at the final answer of $67.50. Create one algebraic equation that can be used to arrive at the final answer of $67.50. Solve the equation to prove it results in an answer of $67.50. Be prepared to explain your thought process to the class.

Let \( F \) represent the final value of the investment.

The final value is 90% of the original investment, since 100% − 10% = 90%.

\[
F = \text{Percent} \times \text{Whole} \\
F = (0.90)(75) \\
F = 67.5
\]

*The final value of the investment is $67.50.*
b. Skylar wanted to show the proportional relationship between the dollar value of the original investment, \( x \), and its value after a 10% decrease, \( y \). He creates the table of values shown below. Does it model the relationship? Explain. Then, provide a correct equation for the relationship Skylar wants to model.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>7.5</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>400</td>
<td>40</td>
</tr>
</tbody>
</table>

No. The table only shows the proportional relationship between the amount of the investment and the amount of the decrease, which is 10% of the amount of the investment. To show the relationship between the value of the investment before and after the 10% decrease, he needs to subtract each value currently in the \( y \)-column from each value in the \( x \)-column so that the \( y \)-column shows the following values: 67.5, 90, 180, 270, and 360. The correct equation is \( y = x - 0.10x \), or \( y = 0.90x \).

Let’s talk about Skylar’s thought process. Skylar’s approach to finding the value of a $75.00 investment after a 10% decline was to find 10% of 75 and then subtract it from 75. He generalized this process and created a table of values to model a 10% decline in the value of an investment. Did his table of values represent his thought process? Why or why not?

- The table only demonstrates the first part of Skylar’s process. The values in the \( y \)-column are 10% of the original value, so Skylar would have to subtract in order to get the correct values.

Example 3 (4 minutes): Finding a Percent Increase or Decrease

Students understand from earlier lessons how to convert a fraction to a percent. A common error in finding a percent increase or decrease (given the before and after amounts) is that students do not correctly identify the quantity (or part) and the whole (the original amount). Example 3 may reveal students’ misunderstandings related to this common error, which will allow the teacher to pinpoint misconceptions and correct them early on.

Example 3: Finding a Percent Increase or Decrease

Justin earned 8 badges in Scouts as of the Scout Master’s last report. Justin wants to complete 2 more badges so that he will have a total of 10 badges earned before the Scout Master’s next report.

a. If Justin completes the additional 2 badges, what will be the percent increase in badges?

\[ \text{Quantity} = \text{Percent} \times \text{Whole} \]
\[ 2 = p \times 8 \]
\[ \frac{1}{8} = \frac{1}{8} \times \frac{1}{8} \]
\[ \frac{25}{100} = \frac{25}{100} = 25\% = p \]

There would be a 25% increase in the number of badges.
b. Express the 10 badges as a percent of the 8 badges.

8 badges is the whole, or 100%, and 2 badges represent 25% of the badges, so 10 badges represent 100% + 25% = 125% of the 8 badges.

Check:

\[
10 = p \cdot 8
\]

\[
10 \left(\frac{1}{8}\right) = p \left(\frac{1}{8}\right) (8)
\]

\[
10 \left(\frac{5}{4}\right) = \frac{125}{100} = 125\% = p
\]

c. Does 100% plus your answer in part (a) equal your answer in part (b)? Why or why not?

Yes. My answer makes sense because 8 badges are the whole or 100%, and 2 badges represent 25% of the badges, so 10 badges represent 100% + 25% = 125% of the 8 badges.

Examples 4–5 (9 minutes): Finding the Original Amount Given a Percent Increase or Decrease

Note that upcoming lessons will focus on finding the whole given a percent change, as students often are challenged by these problem types.

Example 4: Finding the Original Amount Given a Percent Increase or Decrease

The population of cats in a rural neighborhood has declined in the past year by roughly 30%. Residents hypothesize that this is due to wild coyotes preying on the cats. The current cat population in the neighborhood is estimated to be 12. Approximately how many cats were there originally?

- Do we know the part or the whole?
  - We know the part (how many cats are left), but we do not know the original whole.

- Is this a percent increase or decrease problem? How do you know?
  - Percent decrease because the word “declined” means decreased.

- If there was about a 30% decline in the cat population, then what percent of cats remain?
  - 100% – 30% = 70%, so about 70% of the cats remain.

- How do we write an equation to model this situation?
  - 12 cats represent the quantity that is about 70% of the original amount of cats. We are trying to find the whole, which equals the original number of cats. So, using Quantity = Percent × Whole and substituting the known values into the equation, we have 12 = 70% · W, where W represents the original number of cats.

\[
\text{Quantity} = \text{Percent} \times \text{Whole}
\]

\[
12 = \left(\frac{7}{10}\right) \cdot W
\]

\[
(12) \left(\frac{10}{7}\right) = \left(\frac{7}{10}\right) \left(\frac{10}{7}\right) \cdot W
\]

\[
\frac{120}{7} = W
\]

\[
17.1 \approx 17 = W
\]

There must have been 17 cats originally.
Let’s relate our algebraic work to a visual model.

<table>
<thead>
<tr>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
<th>$\frac{12}{7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
</tr>
<tr>
<td>70% of the whole equals $12$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What quantity represents 100% of the cats?

To find the original amount of cats or the whole (100% of the cats), we need to add three more twelve-sevenths to $12$.

$$12 + 3 \left( \frac{12}{7} \right) = \frac{84}{7} + \frac{36}{7} = \frac{120}{7} \approx 17$$

The decrease was given as approximately 30%, so there must have been 17 cats originally.

Example 5

Lu’s math score on her achievement test in seventh grade was a 650. Her math teacher told her that her test level went up by 25% from her sixth grade test score level. What was Lu’s test score level in sixth grade?

Does this represent a percent increase or decrease? How do you know?

- **Percent increase because the word “up” means increase.**

Using the equation Quantity = Percent \times Whole, what information do we know?

- We know Lu’s test score level in seventh grade after the change, which is the quantity, and we know the percent. But we do not know the whole (her test score level from sixth grade).

If Lu’s sixth grade test score level represents the whole, then what percent represents the seventh grade level?

- 100% + 25% = 125%

How do we write an equation to model this situation? Let $W$ represent Lu’s test score in sixth grade.

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$

$$650 = 125\% \times W$$

$$650 = 1.25W$$

$$650 ( \frac{1}{1.25} ) = 1.25 ( \frac{1}{1.25} ) W$$

$$\frac{650}{1.25} = \frac{65,000}{125} = 520 = W$$

Lu’s sixth grade test score level was 520.

Closing (2 minutes)

- How does the context of a problem determine whether there is percent increase or decrease?

  - We can look for keywords in the problem to determine if there is a percent increase or a percent decrease.
Using the equation Quantity = Percent × Whole, what does the whole represent in a percent increase or decrease problem? What does the quantity represent?

- **The whole represents the original amount, and the quantity represents the amount of change or the amount after the change.**

For each phrase, identify the whole unit.

Read each phrase aloud to the class, and ask for student responses.

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Whole Unit (100%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Mary has 20% more money than John.”</td>
<td>John’s money</td>
</tr>
<tr>
<td>“Anne has 15% less money than John.”</td>
<td>John’s money</td>
</tr>
<tr>
<td>“What percent more (money) does Anne have than Bill?”</td>
<td>Bill’s money</td>
</tr>
<tr>
<td>“What percent less (money) does Bill have than Anne?”</td>
<td>Anne’s money</td>
</tr>
</tbody>
</table>

Lesson Summary

- Within each problem, there are keywords that determine if the problem represents a percent increase or a percent decrease.
- Equations can be used to solve percent problems using the basic equation
  \[ \text{Quantity} = \text{Percent} \times \text{Whole}. \]
- Quantity in the percent formula is the amount of change (increase or decrease) or the amount after the change.
- Whole in the percent formula represents the original amount.

Exit Ticket (5 minutes)
Lesson 4: Percent Increase and Decrease

Exit Ticket

Erin wants to raise her math grade to a 95 to improve her chances of winning a math scholarship. Her math average for the last marking period was an 81. Erin decides she must raise her math average by 15% to meet her goal. Do you agree? Why or why not? Support your written answer by showing your math work.
Exit Ticket Sample Solutions

Erin wants to raise her math average to a 95 to improve her chances of winning a math scholarship. Her math average for the last marking period was an 81. Erin decides she must raise her math average by 15% to meet her goal. Do you agree? Why or why not? Support your written answer by showing your math work.

No, I do not agree. 15% of 81 is 12.15. 81 + 12.15 = 93.15, which is less than 95. I arrived at my answer using the equation below to find 15% of 81.

**Quantity = Percent x Whole**

Let $G$ stand for the number of points Erin’s grade will increase by after a 15% increase from 81. The whole is 81, and the percent is 15%. First, I need to find 15% of 81 to arrive at the number of points represented by a 15% increase. Then, I will add that to 81 to see if it equals 95, which is Erin’s goal.

\[
G = 0.15 \times 81
\]
\[
G = 12.15
\]

Adding the points onto her average: 81.00 + 12.15 = 93.15

Comparing it to her goal: 93.15 < 95.

Problem Set Sample Solutions

1. A store advertises 15% off an item that regularly sells for $300.
   a. What is the sale price of the item?
      \[(0.85)300 = 255; \text{ the sale price is }$255.\]
   b. How is a 15% discount similar to a 15% decrease? Explain.
      In both cases, you are subtracting 15% of the whole from the whole, or finding 85% of the whole.
   c. If 8% sales tax is charged on the sale price, what is the total with tax?
      \[(1.08)(255) = 275.40; \text{ the total with tax is }$275.40.\]
   d. How is 8% sales tax like an 8% increase? Explain.
      In both cases, you are adding 8% of the whole to the whole, or finding 108% of the whole.

2. An item that was selling for $72.00 is reduced to $60.00. Find the percent decrease in price. Round your answer to the nearest tenth.
   The whole is 72. 72 – 60 = 12. 12 is the part. Using **Quantity = Percent x Whole**, I get
   \[12 = p \times 72, \text{ where } p \text{ represents the unknown percent, and working backward, I arrive at } \frac{12}{72} = \frac{1}{6} = 0.16 = p.\]
   So, it is about a 16.7% decrease.
Lesson 4: Percent Increase and Decrease

3. A baseball team had 80 players show up for tryouts last year and this year had 96 players show up for tryouts. Find the percent increase in players from last year to this year.

The number of players that showed up last year is the whole; 16 players are the quantity of change since 96 – 80 = 16.

Quantity = Percent \times Whole. \ Let p represent the unknown percent.

\[
16 = p(80)
\]
\[
p = 0.2
\]
\[
0.2 = \frac{20}{100} = 20\%
\]

The number of players this year was a 20% increase from last year.

4. At a student council meeting, there was a total of 60 students present. Of those students, 35 were female.

a. By what percent is the number of females greater than the number of males?

The number of males (60 – 35 = 25) at the meeting is the whole. The part (quantity) can be represented by the number of females (35) or how many more females there are than the number of males.

Quantity = Percent \times Whole

\[
35 = p(25)
\]
\[
p = 1.4
\]
\[
1.4 = 140\%, \ which \ is \ 40\% \ more \ than \ 100\%. \ Therefore, \ there \ were \ 40\% \ more \ females \ than \ males \ at \ the \ student \ council \ meeting.
\]

b. By what percent is the number of males less than the number of females?

The number of females (35) at the meeting is the whole. The part (quantity) can be represented by the number of males, or the number less of males than females (10).

Quantity = Percent \times Whole

\[
10 = p(35)
\]
\[
p = 0.29
\]
\[
0.29 = 29\%
\]

The number of males at the meeting is approximately 29% less than the number of females.

c. Why is the percent increase and percent decrease in parts (a) and (b) different?

The difference in the number of males and females is the same in each case, but the whole quantities in parts (a) and (b) are different.
5. Once each day, Darlene writes in her personal diary and records whether the sun is shining or not. When she looked back though her diary, she found that over a period of 600 days, the sun was shining 60% of the time. She kept recording for another 200 days and then found that the total number of sunny days dropped to 50%. How many of the final 200 days were sunny days?

To find the number of sunny days in the first 600 days, the total number of days is the whole.

\[
\text{Quantity} = \text{Percent} \times \text{Whole}. \text{ Let } s \text{ represent the number of sunny days.}
\]

\[
\begin{align*}
0.6(600) &= s \\
360 &= s
\end{align*}
\]

There were 360 sunny days in the first 600 days.

The total number of days that Darlene observed was 800 days because 600 + 200 = 800.

\[
\begin{align*}
0.5(800) &= d \\
400 &= d
\end{align*}
\]

There was a total of 400 sunny days out of the 800 days.

The number of sunny days in the final 200 days is the difference of 400 days and 360 days.

\[
400 - 360 = 40, \text{ so there were 40 sunny days of the last 200 days.}
\]

6. Henry is considering purchasing a mountain bike. He likes two bikes: One costs $500, and the other costs $600. He tells his dad that the bike that is more expensive is 20% more than the cost of the other bike. Is he correct? Justify your answer.

Yes. Quantity = Percent × Whole. After substituting in the values of the bikes and percent, I arrive at the following equation: 600 = 1.2(500), which is a true equation.

7. State two numbers such that the lesser number is 25% less than the greater number.

Answers will vary. One solution is as follows: Greater number is 100; lesser number is 75.

8. State two numbers such that the greater number is 75% more than the lesser number.

Answers will vary. One solution is as follows: Greater number is 175; lesser number is 100.

9. Explain the difference in your thought process for Problems 7 and 8. Can you use the same numbers for each problem? Why or why not?

No. The whole is different in each problem. In Problem 7, the greater number is the whole. In Problem 8, the lesser number is the whole.

10. In each of the following expressions, \(c\) represents the original cost of an item.

i. \(0.9c\)

ii. \(0.1c\)

iii. \(c - 0.1c\)

a. Circle the expression(s) that represents 10% of the original cost. If more than one answer is correct, explain why the expressions you chose are equivalent.
Lesson 4: Percent Increase and Decrease

Date: 11/19/14

b. Put a box around the expression(s) that represents the final cost of the item after a 10% decrease. If more than one is correct, explain why the expressions you chose are equivalent.

\[ c - 0.10c \]
\[ 1c - 0.10c \quad \text{Multiplicative identity property of 1} \]
\[ (1 - 0.10)c \quad \text{Distributive property (writing a sum or difference as a product)} \]
\[ 0.90c \]

Therefore, \( c - 0.10c = 0.90c \).

c. Create a word problem involving a percent decrease so that the answer can be represented by expression (ii).

Answers will vary. The store’s cashier told me I would get a 10% discount on my purchase. How can I find the amount of the 10% discount?

d. Create a word problem involving a percent decrease so that the answer can be represented by expression (i).

Answers will vary. An item is on sale for 10% off. If the original price of the item is \( c \), what is the final price after the 10% discount?

e. Tyler wants to know if it matters if he represents a situation involving a 25% decrease as \( 0.25x \) or \((1 - 0.25)x\). In the space below, write an explanation that would help Tyler understand how the context of a word problem often determines how to represent the situation.

If the word problem asks you to find the amount of the 25% decrease, then \( 0.25x \) would represent it. If the problem asks you to find the value after a 25% decrease, then \((1 - 0.25)x\) would be a correct representation.