Common Core-Aligned DDI Primer in Math: Designing and Leading DDI Using Assessment Guidance Documents

Agenda:

- Introduction

I. What makes a high quality assessment?
   - Sorting Task: Why Do We Assess?
   - Balance of Rigor
   - Students At All Levels

II. How can we analyze assessment data?
   - Looking at Student Work
   - Tracking Data
   - Leading a Data Meeting

July 9, 2014
Mathematics - Grade 6: Introduction

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.
Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Overview

Ratios and Proportional Relationships
• Understand ratio concepts and use ratio reasoning to solve problems.

The Number System
• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
• Compute fluently with multi-digit numbers and find common factors and multiples.
• Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations
• Apply and extend previous understandings of arithmetic to algebraic expressions.
• Reason about and solve one-variable equations and inequalities.
• Represent and analyze quantitative relationships between dependent and independent variables.

Geometry
• Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability
• Develop understanding of statistical variability.
• Summarize and describe distributions.

Ratios & Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.
1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

1 Expectations for unit rates in this grade are limited to non-complex fractions.
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because \(3/4\) of \(8/9\) is \(2/3\). (In general, \((a/b) \div (c/d) = ad/bc\).) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.
2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express \(36 + 8\) as \(4 \cdot (9 + 2)\).

Apply and extend previous understandings of numbers to the system of rational numbers.
5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., \(–(–3) = 3\), and that 0 is its own opposite.
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret \(–3 > –7\) as a statement that \(–3\) is located to the right of \(–7\) on a number line oriented from left to right.
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write \(–3^\circ C > –7^\circ C\) to express the fact that \(–3^\circ C\) is warmer than \(–7^\circ C\).
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of \(–30\) dollars, write \(|–30| = 30\) to describe the size of the debt in dollars.
   d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than \(–30\) dollars represents a debt greater than \(30\) dollars.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Expressions & Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.
1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5 – y.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s^3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = 1/2.
3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Reason about and solve one-variable equations and inequalities.
5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.
9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics & Probability

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Mathematics - Grade 7: Introduction

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Grade 7 Overview

**Ratios and Proportional Relationships**
- Analyze proportional relationships and use them to solve real-world and mathematical problems.

**The Number System**
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

**Expressions and Equations**
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

**Geometry**
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

**Statistics and Probability**
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

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**Ratios & Proportional Relationships 7.RP**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t = pn$.
   d. Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

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**The Number System 7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand $p + q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.

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**Expressions & Equations**

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p$, $q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.
Geometry

**Draw construct, and describe geometrical figures and describe the relationships between them.**

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics & Probability

**Use random sampling to draw inferences about a population.**

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

**Draw informal comparative inferences about two populations.**

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

**Investigate chance processes and develop, use, and evaluate probability models.**

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
Mathematics - Grade 8: Introduction

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions \((y/x = m \text{ or } y = mx)\) as special linear equations \((y = mx + b)\), understanding that the constant of proportionality \((m)\) is the slope, and the graphs are lines through the origin. They understand that the slope \((m)\) of a line is a constant rate of change, so that if the input or \(x\)-coordinate changes by an amount \(A\), the output or \(y\)-coordinate changes by the amount \(m \cdot A\). Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and \(y\)-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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4. Model with mathematics.
5. Use appropriate tools strategically.
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Grade 8 Overview

The Number System
• Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
• Work with radicals and integer exponents.
• Understand the connections between proportional relationships, lines, and linear equations.
• Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
• Define, evaluate, and compare functions.
• Use functions to model relationships between quantities.

Geometry
• Understand congruence and similarity using physical models, transparencies, or geometry software.
• Understand and apply the Pythagorean Theorem.
• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability
• Investigate patterns of association in bivariate data.

The Number System

8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.
1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., 2). For example, by truncating the decimal expansion of 2, show that 2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions & Equations

8.EE

Work with radicals and integer exponents.
1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 × 3^5 = 3^7 = 1/3^3 = 1/27.
2. Use square root and cube root symbols to represent solutions to equations of the form x^2 = p and x^3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that 2 is irrational.
3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Understand the connections between proportional relationships, lines, and linear equations.
5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).

Analyze and solve linear equations and pairs of simultaneous linear equations.
7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a \), \( a = a \), or \( a = b \) results (where \( a \) and \( b \) are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, \( 3x + 2y = 5 \) and \( 3x + 2y = 6 \) have no solution because \( 3x + 2y \) cannot simultaneously be 5 and 6.
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions

Define, evaluate, and compare functions.
1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function \( A = s^2 \) giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.
4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

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1Function notation is not required in Grade 8.
**Geometry**

**8.G**

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

1. Verify experimentally the properties of rotations, reflections, and translations:
   - **a.** Lines are taken to lines, and line segments to line segments of the same length.
   - **b.** Angles are taken to angles of the same measure.
   - **c.** Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

**Understand and apply the Pythagorean Theorem.**

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

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**Statistics & Probability**

8.SP

**Investigate patterns of association in bivariate data.**

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
Angel and Jayden were at track practice. The track is \( \frac{2}{5} \) kilometers around.

- Angel ran 1 lap in 2 minutes.
- Jayden ran 3 laps in 5 minutes.

a. How many minutes does it take Angel to run one kilometer? What about Jayden?
b. How far does Angel run in one minute? What about Jayden?
c. Who is running faster? Explain your reasoning.
Commentary

Parts (a) and (b) of the task ask students to find the unit rates that one can compute in this context. Part (b) does not specify whether the units should be laps or km, so answers can be expressed using either one.

The purpose of part (c) is to give students an opportunity to make use of the unit rates that they found in parts (a) and (b). While it is possible for students to solve part (c) in other ways, the solution shown represents the kind of reasoning with unit rates that 7th graders should be able to do. It is important to note that the answer can be determined using different unit rates as long as the reasoning behind it is correct.

Solutions

Solution: km per min and min per km

a. We can create a table that shows how far each person runs for a certain number of laps:

<table>
<thead>
<tr>
<th>Number of laps</th>
<th>Number of km</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{2}{5} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{4}{5} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{6}{5} )</td>
</tr>
</tbody>
</table>

We can see from the table that 1 km is exactly halfway between 2 and 3 laps. So it will take 2.5 laps to run 1 km.

Since it takes Angel 2 minutes to run 1 lap, she will take

\[
\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{2 \text{ minutes}}{1 \text{ lap}} = \frac{5 \text{ minutes}}{1 \text{ km}}.
\]

So it takes Angel 5 minutes to run 1 km.

Since it takes Jayden 5 minutes to run 3 laps, she runs 1 lap in \( \frac{5}{3} \) minutes. Thus, it takes Jayden

\[
\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{5 \text{ minutes}}{3 \text{ laps}} = \frac{5}{2} \cdot \frac{5}{3} \text{ minutes/km} = \frac{25}{6} \text{ minutes/km} = 4 \frac{1}{6} \text{ minutes/km}.
\]

So it takes Jayden 4 \( \frac{1}{6} \) minutes to run 1 km.

b. Angel runs 1 lap in 2 minutes so she runs \( \frac{1}{2} \) lap in 1 minute. Since 1 lap is \( \frac{2}{5} \) km, \( \frac{1}{2} \) lap is \( \frac{1}{5} \) km. So she also runs \( \frac{1}{5} \) km in one minute.
Since Jayden runs 1 lap in \( \frac{5}{3} \) minutes, she will run \( \frac{1}{3} \) laps in 1 minute. Since Jayden runs 1 km in \( \frac{25}{6} \) minutes, she will run \( \frac{6}{25} \) km in 1 minute.

c. Jayden runs the same distance in less time than Angel (alternatively, Jayden runs farther in the same time than Angel), so Jayden is running faster than Angel.
Travis was attempting to make muffins to take to a neighbor that had just moved in down the street. The recipe that he was working with required $\frac{3}{4}$ cup of sugar and $\frac{1}{8}$ cup of butter.

a. Travis accidentally put a whole cup of butter in the mix.

i. What is the ratio of sugar to butter in the original recipe? What amount of sugar does Travis need to put into the mix to have the same ratio of sugar to butter that the original recipe calls for?

ii. If Travis wants to keep the ratios the same as they are in the original recipe, how will the amounts of all the other ingredients for this new mixture compare to the amounts for a single batch of muffins?

iii. The original recipe called for $\frac{3}{8}$ cup of blueberries. What is the ratio of blueberries to butter in the recipe? How many cups of blueberries are needed in the new enlarged mixture?

b. This got Travis wondering how he could remedy similar mistakes if he were to dump in a single cup of some of the other ingredients. Assume he wants to keep the ratios the same.

i. How many cups of sugar are needed if a single cup of blueberries is used in the mix?

ii. How many cups of butter are needed if a single cup of sugar is used in the mix?

iii. How many cups of blueberries are needed for each cup of sugar?
Commentary

While the task as written does not explicitly use the term "unit rate," most of the work students will do amounts to finding unit rates. A recipe context works especially well since there are so many different pair-wise ratios to consider.

This task can be modified as needed; depending on the choice of numbers, students are likely to use different strategies which the teacher can then use to help students understand the connection between, for example, making a table and strategically scaling a ratio.

The choice of numbers in this task is already somewhat strategic: in part (a), the scale factor is a whole number and in part (b), the scale factors are fractions. Because of this difference, students will likely approach the parts of the task in different ways. The teacher can select and sequence a discussion of the different approaches to highlight the structure of the mathematics and allow for connections to proportional relationships.

This task was submitted by Travis Lemon for the first IMP task writing contest 2011/12/12-2011/12/18.

Solutions

Solution: Solution

a. i. The ratio of cups of sugar to cups of butter is \(\frac{3}{4} : \frac{1}{8}\). If we multiply both numbers in the ratio by 8, we get an equivalent ratio that involves 1 cup of butter.

\[
8 \times \frac{3}{4} = 6
\]

and

\[
8 \times \frac{1}{8} = 1
\]

In other words, \(\frac{3}{4} : \frac{1}{8}\) is equivalent to \(6 : 1\), and so six cups of sugar is needed if there is one cup of butter.

ii. In the previous part we saw that we have 8 times as much butter, so all the ingredients need to be increased by a factor of 8. That is, the quantity of each ingredient in the original recipe needs to be multiplied by 8 in order for all the ratios to be the same in the new mixture.

iii. The ratio of cups of blueberries to cups of butter is \(\frac{3}{8} : \frac{1}{8}\) in the original recipe, so Travis will need to add \(8 \times \frac{3}{8} = 3\) cups of blueberries to his new mixture.

b. i. The ratio of cups of sugar to cups of blueberries is \(\frac{3}{4} : \frac{3}{8}\). If we multiply both numbers in the ratio by \(\frac{8}{3}\), we get an equivalent ratio.
\[
\frac{8}{3} \times \frac{3}{4} = 2 \text{ and } \frac{8}{3} \times \frac{3}{8} = 1.
\]

Since \(\frac{3}{4} : \frac{3}{8}\) is equivalent to \(2 : 1\), two cups of sugar is needed if there is one cup of blueberries.

ii. The ratio of cups of butter to cups of sugar is \(\frac{1}{8} : \frac{3}{4}\). If we multiply both numbers in the ratio by \(\frac{4}{3}\), we get an equivalent ratio.

\[
\frac{4}{3} \times \frac{1}{8} = \frac{1}{6} \text{ and } \frac{4}{3} \times \frac{3}{4} = 1.
\]

In other words, \(\frac{1}{8} : \frac{3}{4}\) is equivalent to \(\frac{1}{6} : 1\), and \(\frac{1}{6}\) cup of butter is needed if there is one cup of sugar.

iii. The ratio of cups of blueberries to cups of sugar is \(\frac{3}{8} : \frac{3}{4}\). If we multiply both numbers in the ratio by \(\frac{4}{3}\), we get an equivalent ratio.

\[
\frac{4}{3} \times \frac{3}{8} = \frac{1}{2} \text{ and } \frac{4}{3} \times \frac{3}{4} = 1.
\]

Since \(\frac{3}{8} : \frac{3}{4}\) is equivalent to \(\frac{1}{2} : 1\), Travis would need \(\frac{1}{2}\) cup of blueberries if there is one cup of sugar.

Instructional Note: For part (b), I have encouraged students to think about unit fractions as an intermediate step to developing an understanding of how to multiply by fractions. With the emphasis on unit fractions in the CCSSM, I decided to use this approach this year and have found success. Students see the value of scaling to a unit fraction and then going from there.

For example, if a student realizes that \(\frac{5}{8}\) needs to become 1 to answer part (b.i), she can first take \(\frac{1}{3}\) of the amount to create a unit fraction of \(\frac{1}{8}\) and then multiply this by 8 to create 1.

The composite result of these calculations is equivalent to multiplying by \(\frac{8}{3}\). Students often find that the two calculations (taking \(\frac{1}{3}\) of the amount to create a unit fraction of \(\frac{1}{8}\) and then multiply this by 8) made independently are more mentally accessible, which makes them a nice intermediate step in understanding the composite calculation of multiplying by the reciprocal.

Solution: Using tables

a. i. The ratio of cups of sugar to cups of butter is \(\frac{3}{4} : \frac{1}{8}\). If we set up a table, we can successively double the amounts:

<table>
<thead>
<tr>
<th>cups of sugar</th>
<th>(\frac{3}{4})</th>
<th>(\frac{6}{4})</th>
<th>(\frac{12}{4})</th>
<th>(\frac{24}{4} = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cups of butter</td>
<td>(\frac{1}{8})</td>
<td>(\frac{2}{8})</td>
<td>(\frac{4}{8})</td>
<td>(\frac{8}{8} = 1)</td>
</tr>
</tbody>
</table>
So six cups of sugar is needed if there is one cup of butter.

ii. In the previous part, we had to double the quantities three times: \(2 \cdot 2 \cdot 2 = 8\). So Travis needs 8 times as much butter as the original recipe required. If we want to keep all the ingredients in the same ratio, Travis needs to multiply the amount of each ingredient by 8.

iii. The ratio of cups of blueberries to cups of butter is \(\frac{3}{8} : \frac{1}{8}\) in the original recipe, so Travis will need to add \(8 \cdot \frac{3}{8} = 3\) cups of blueberries.

b. i. It is much harder to solve this problem using a table because the scale factor is no longer a whole number. Students who solved the first part using a table may need guidance from their classmates or the teacher to see that multiplying both numbers in the ratio by the reciprocal of the amount of blueberries will give an equivalent ratio with 1 cup of blueberries. Here is where the teacher should highlight the importance of being able to find a unit rate.

The ratio of cups of sugar to cups of blueberries is \(\frac{3}{4} : \frac{1}{8}\). If we multiply both numbers in the ratio by \(\frac{8}{3}\), we get an equivalent ratio.

\[
\frac{3}{4} \times \frac{8}{3} = 2 \quad \text{and} \quad \frac{3}{8} \times \frac{8}{3} = 1.
\]

So two cups of sugar is needed if there is one cup of blueberries.

ii. The ratio of cups of butter to cups of sugar is \(\frac{1}{8} : \frac{3}{4}\). If we multiply both numbers in the ratio by \(\frac{4}{3}\), we get an equivalent ratio.

\[
\frac{1}{8} \times \frac{4}{3} = \frac{1}{6} \quad \text{and} \quad \frac{3}{4} \times \frac{4}{3} = 1.
\]

So \(\frac{1}{6}\) cup of butter is needed if there is one cup of sugar.

iii. The ratio of cups of blueberries to cups of sugar is \(\frac{3}{8} : \frac{3}{4}\). If we multiply both numbers in the ratio by \(\frac{4}{3}\), we get an equivalent ratio.

\[
\frac{3}{8} \times \frac{4}{3} = \frac{1}{2} \quad \text{and} \quad \frac{3}{4} \times \frac{4}{3} = 1.
\]

So \(\frac{1}{2}\) cup of blueberries is needed if there is one cup of sugar.
Molly runs \( \frac{1}{3} \) of a mile in 4 minutes.

a. If Molly continues at the same speed, how long will it take her to run one mile?

b. Draw and label a picture showing why your answer to part (a) makes sense.
Commentary

This task asks students to solve a problem in a context involving constant speed. This task provides a transition from working with ratios involving whole numbers to ratios involving fractions. This problem can be thought of in several ways; in particular, this problem also provides an opportunity for students to work with the "How many in one group?" interpretation of division. To recall briefly, in a multiplication problem such as

\[ \frac{1}{3} \times ? = 4, \]

the \( \frac{1}{3} \) represents the fraction of a group which is equal to 4. In this problem, the \( \frac{1}{3} \) refers to \( \frac{1}{3} \) of a mile and the 4 tells us how many minutes it takes Molly to run \( \frac{1}{3} \) of a mile. So the \( ? \), the time it takes Molly to run one mile, can be found by performing the division problem \( 4 \div \frac{1}{3} \).

Alternatively, thinking in the language of ratios, the ratio

\[ \frac{1}{3} : 4 \]

is equivalent to the ratio

\[ 1 : ? \]

This is because Molly’s speed does not change throughout the run. Students should be familiar with dividing a whole number by a unit fraction from 5th grade (see 5.NF.7), so in some ways this task can be thought of as a natural extension of that standard. Since students are just learning about dividing fractions by fractions, a problem involving constant speed that can also be solved by dividing a fraction by a fraction would be especially challenging for sixth grade.

This problem is suitable for instruction or assessment. If used for instruction, it could be combined with the task "5.NF How many marbles?" which models the division problem \( 4 \div \frac{1}{3} \) from the how many in one group perspective. 6.RP Molly’s Run is both more abstract than "5.NF How many marbles?" and more closely related to ratios. The relative abstractness of the two problems is seen in the pictures: while "5.NF How many marbles?" has a very concrete picture of groups of marbles, the picture here is a number line.

Solutions

Solution: 1

a. If Molly runs \( \frac{1}{3} \) of a mile every four minutes, then \( 2 \times 4 \) minutes is \( \frac{2}{3} \) of this time.

Similarly \( 3 \times 4 \) minutes is \( \frac{3}{3} \) of Molly’s time to run a mile. Since \( \frac{3}{3} = 1 \), this means that it takes \( 3 \times 4 = 12 \) minutes to run one mile.

b. The following picture illustrates this reasoning:
since there are 3 groups of 4 minutes, the picture shows why

\[ 3 \times 4 \]

represents the solution to the problem. It is important to note that the constant speed Molly is running is shown in the picture by the equally spaced intervals of both time and distance: traveling at a constant speed means that equal distances are traveled in equal times.

Solution: 2

a. Alternatively, the question can be viewed as a division problem: we are given that 4 minutes is \( \frac{1}{3} \) of the amount of time it takes Molly to run one mile. If we let \( ? \) represent the time it takes her to run 1 mile, then we can write this symbolically as:

\[ \frac{1}{3} \times ? = 4 \]

This multiplication problem is equivalent to the following division problem:

\[ 4 \div \frac{1}{3} = ? \]

Since

\[ 4 \div \frac{1}{3} = 4 \times 3 = 12 \]

it will take Molly 12 minutes to run 1 mile at this speed.

b. Note that the same picture works for this way of viewing the problem:

We just need to interpret the picture slightly differently as \( \frac{1}{3} \) of what number is 4? In fact, this picture shows why dividing by \( \frac{1}{3} \) is the same as multiplying by 3.
A crew of highway workers paved $\frac{2}{15}$ mile in 20 minutes. If they work at the same rate, what portion of a mile will they pave in one hour?

A. $\frac{1}{150}$

B. $\frac{2}{45}$

C. $\frac{2}{5}$

D. $\frac{5}{2}$

Key: C  
Measured CCLS: 7.RP.1  

Commentary: The item measures 7.RP.1 because it requires computing unit rates associated with ratios of fractions, including quantities measured in like or different units.

Answer Choice A: $\frac{1}{150}$. This response reflects the number of miles of road that workers pave in one minute. The student may have divided the distance by time, but did not convert the time in minutes to time in hours. A student who selects this response may be able to find unit rates, but was not precise in their use of units or in providing an answer in the units demanded by the question.

$$\frac{2}{15} \times \frac{1}{20} = \frac{1}{150}$$

Answer Choice B: $\frac{2}{45}$. This response reflects the number of miles of road that workers pave multiplied by time in hours. The student may have converted 20 minutes to $\frac{1}{3}$ hour, but multiplied, instead of dividing, the fractions to determine the unit rate. A student who selects this response may not understand how to use division of fractions to find unit rates associated with fractions.

$$\frac{2}{15} \times \frac{1}{3} = \frac{2}{45}$$

Answer Choice C: $\frac{2}{5}$. The student correctly determined the number of miles of road that workers pave in one hour. The student who selects this response computed the unit rate, in miles per hour, associated with paving $\frac{2}{15}$ miles in 20 minutes, which is equivalent to $\frac{1}{3}$ hour.

$$\frac{2}{15} \times 3 = \frac{2}{5}$$

Similarly, students may have used proportional reasoning to determine that if $\frac{2}{15}$ of mile can be paved in 20 minutes, then in one hour or 60 minutes three times as much road could be paved.

$$\frac{2}{15} \times 3 = \frac{6}{15} = \frac{2}{5}$$
Answer Choice D: \( \frac{5}{2} \). This response reflects the number of hours it will take workers to pave one mile. The student may have converted 20 minutes to \( \frac{1}{3} \) hour and used division of fractions to determine the unit rate, but reversed the position of the dividend and divisor in the equation. A student who selects this response may not understand how to create the expression needed to find the unit rate in miles per hour when the numerator and denominator are both fractions.

\[
\frac{1}{3} \times \frac{15}{2} = \frac{5}{2}
\]

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when computing a unit rate associated with fractions. Answer option C represents the correct process used to compute the unit rate, in miles per hour, associated with paving \( \frac{2}{15} \) miles in 20 minutes, which is equivalent to \( \frac{1}{3} \) hour.
1. It is a Saturday morning and Jeremy has discovered he has a leak coming from the water heater in his attic. Since plumbers charge extra to come out on weekends, Jeremy is planning to use buckets to catch the dripping water. He places a bucket under the drip and steps outside to walk the dog. In half an hour the bucket is 1/5 of the way full.

a. What is the rate at which the water is leaking?

b. Write an equation that represents the relationship between the number of buckets filled, $y$, in $x$ hours.

c. What is the longest that Jeremy can be away from the house before the bucket will overflow?
2. Farmers often plant crops in circular areas because one of the most efficient watering systems for crops provides water in a circular area. Passengers in airplanes often notice the distinct circular patterns as they fly over land used for farming. A photographer takes an aerial photo of a field on which a circular crop area has been planted. He prints the photo out and notes that 2 centimeters of length in the photo corresponds to 100 meters in actual length.

a. What is the scale factor of the photo?

b. If the dimensions of the entire photo are 25 cm by 20 cm, what are the actual dimensions of the rectangular land area in meters captured by the photo?

c. If the area of the circular area on the photo is $64\pi$ cm$^2$, what is the actual area of the circular crop area in square meters?
3. A store is having a sale to celebrate President’s Day. Every item in the store is advertised as one fifth off the original price. If an item is marked with a sale price of $140, what was its original price? Show your work.

4. Over the break, your uncle and aunt ask you to help them cement the foundation of their newly purchased land and give you a top-view blueprint of the area and proposed layout. A small legend on the corner states that 4 inches of the length corresponds to an actual length of 52 feet.

   Property Line
   
   Deck
   
   Foundation Line
   
   Driveway

   a. What is the scale factor?
b. If the dimensions of the foundation on the blueprint are 11 inches by 13 inches, what are the actual dimensions in feet?

c. You're asked to go buy bags of dry cement and know that one bag covers 350 square feet. How many bags do you need to buy to finish this project?

d. After the first 15 minutes of laying down the cement, you had used $\frac{1}{5}$ of the bag. What is the rate you are laying cement in bags per hour? What is the unit rate?
e. Write an equation that represents the relationship between the number of bags used, \( y \), in \( x \) hours.

f. Your uncle is able to work faster than you. He uses 3 bags for every 2 bags you use. Is the relationship proportional? Explain your reasoning using a graph on a coordinate plane.

g. What does (0,0) represent in terms of the situation being described by the graph created in part (f)?
h. Using a graph, show how many bags you would have used if your uncle used 18 bags.
### A Progression Toward Mastery

<table>
<thead>
<tr>
<th>Assessment Task Item</th>
<th>STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.</th>
<th>STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.</th>
<th>STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.</th>
<th>STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 a</strong> 7.RP.1</td>
<td>Student answered rate incorrectly and showed no or very limited calculations.</td>
<td>Student set the problem up incorrectly resulting in an incorrect rate.</td>
<td>Student set the problem up correctly but made minor mistakes in the calculation.</td>
<td>Student correctly stated the rate as $\frac{2}{5}$ buckets per hour with correct problem set up and calculations.</td>
</tr>
<tr>
<td><strong>1 b</strong> 7.RP.1 7.RP.2c 7.EE.4a</td>
<td>Student was unable to write an equation OR wrote an equation that was not in the form $y = kx$ or even $x = ky$ for any value k.</td>
<td>Student wrote an incorrect equation, such as $y = \frac{5}{2}x$ or $x = \frac{2}{5}y$, AND/OR used an incorrect value of unit rate from part (a) to write an their equation in the form $y = kx$.</td>
<td>Student created an equation using the constant of proportionality, but wrote the equation in the form $x = \frac{5}{2}y$ OR some other equivalent equation.</td>
<td>Student correctly answered $y = \frac{2}{5}x$.</td>
</tr>
<tr>
<td><strong>1 c</strong> 7.RP.1 7.RP.2c 7.EE.4a</td>
<td>Student answer is incorrect. Little or no evidence of reasoning is given.</td>
<td>Student answer is incorrect, but shows some evidence of reasoning and usage of an equation for the proportional relationship (though the equation itself may be incorrect).</td>
<td>Student correctly answers 2.5 hours but with minor errors in the use of and calculations based on the equation $y = \frac{2}{5}x$.</td>
<td>Student correctly answers 2.5 hours with correct use of AND calculations based on the equation $y = \frac{2}{5}x$.</td>
</tr>
<tr>
<td><strong>2 a</strong> 7.G.1</td>
<td>Student is unable to answer OR the answer gives no evidence of understanding the fundamental concept of scale factor as a ratio comparison of corresponding lengths between the image and the actual object.</td>
<td>Student incorrectly answers the scale factor to be 2:100, 1:50, OR 1/50. The answer expresses scale factor as a comparison of corresponding lengths, but does not show evidence of choosing the same measurement unit</td>
<td>Student correctly answers the scale factor to be 1:5000 OR 1/5000, but has a minor error in calculations or notation. For example, student writes 1/5000 cm.</td>
<td>Student correctly answers the scale factor to be 1:5000 OR 1/5000 with correct calculations and notation.</td>
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<tr>
<td><strong>36</strong></td>
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<td></td>
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<tr>
<td></td>
<td>to make the comparison.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Student answers incorrectly and gives little or no evidence of understanding scale factor.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.G.1</td>
<td>Student shows some evidence of reasoning, but makes one or more calculation errors thereby providing an incorrect answer.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Student correctly answers the actual dimensions as $1,250 , \text{m} \times 1,000 , \text{m}$, but does not show work to support their answer.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Student correctly answers the actual dimensions as $1,250 , \text{m} \times 1,000 , \text{m}$ with correct calculations.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Student answers incorrectly and gives little or no evidence of understanding scale factor.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.G.1</td>
<td>Student shows some evidence of reasoning, but makes one or more calculation errors thereby providing an incorrect answer.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Student correctly answers the actual area as $160,000\pi , \text{m}^2$, but does not show work to support their answer.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Student correctly answers the actual area as $160,000\pi , \text{m}^2$ with correct calculations.</td>
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</tr>
<tr>
<td>3</td>
<td>Student answers incorrectly with no or little evidence of understanding scale factor shown.</td>
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<td></td>
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</tr>
<tr>
<td>7.RP.3</td>
<td>Student answers the original price incorrectly, but provides some evidence of reasoning.</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Student shows solid evidence of reasoning, but makes minor errors in calculations or representations. The answer may or may not be accurate.</td>
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</tr>
<tr>
<td></td>
<td>Student correctly answers the original price as $175; student’s work demonstrates solid reasoning AND calculations were made without error.</td>
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</tr>
<tr>
<td>4</td>
<td>Student answered rate incorrectly and showed no or very limited calculations.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7.RP.1 7.RP.2</td>
<td>Student set the problem up incorrectly resulting in an incorrect rate.</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Student set the problem up correctly, but made minor mistakes in the calculation.</td>
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</tr>
<tr>
<td></td>
<td>Student correctly stated the rate as $4/5 , \text{bags per hour}$ AND identified the unit rate as $4/5$ with correct problem setup and calculations.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.RP.2c 7.EE.4a</td>
<td>7.RP.2</td>
<td>7.RP.2d</td>
<td>7.RP.2</td>
</tr>
<tr>
<td>---</td>
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</tr>
<tr>
<td>e</td>
<td>Student was unable to write an equation or wrote an equation that was not in the form ( y = kx ) or even ( x = ky ) for any value ( k ).</td>
<td>Student wrote an incorrect equation, such as ( y = \frac{5}{2}x ), or ( x = \frac{4}{5}y ), AND/OR used an incorrect value of unit rate from part (d) to write an their equation in the form ( y = kx ).</td>
<td>Student created an equation using the constant of proportionality, but wrote the equation in the form ( x = \frac{5}{4}y ) or some other equivalent equation.</td>
<td>Student correctly answered ( y = \frac{2}{5}x ).</td>
</tr>
<tr>
<td>f</td>
<td>Student may or may not have answered that the relationship was proportional. Student was unable to provide a complete graph. Student was unable to relate the proportional relationship to the graph.</td>
<td>Student may or may not have answered that the relationship was proportional. Student provided a graph with mistakes (i.e., unlabeled axis, incorrect points). Student provided a limited expression of reasoning.</td>
<td>Student correctly answered that the relationship was proportional. Student labeled the axis AND plotted points with minor error. Student explanation was slightly incomplete.</td>
<td>Student correctly answered that the relationship was proportional. Student correctly labeled the axis AND plotted the graph on the coordinate plane. Student reasoned that the proportional relationship was due to the graph being straight and going through the origin.</td>
</tr>
<tr>
<td>g</td>
<td>Student was unable to describe the situation correctly.</td>
<td>Student was able to explain that the zero was the amount of bags used by either him/her or the uncle, but unable to describe the relationship.</td>
<td>Student describes the relationship correctly, but with minor error.</td>
<td>Student correctly explains that ((0,0)) represents when she/he used zero bags, the uncle doesn't use any bags.</td>
</tr>
<tr>
<td>h</td>
<td>Student answers incorrectly and shows no or little understanding of analyzing graphs.</td>
<td>Student answers incorrectly, but shows some understanding of analyzing graphs.</td>
<td>Student correctly answers 12 bags, but does not identify the point on the graph clearly.</td>
<td>Student correctly answers 12 bags by identifying the point on the graph.</td>
</tr>
</tbody>
</table>
New York State Testing Program
Common Core Mathematics Test

Performance Level Descriptions

Grade 7

August 2013
New York State Testing Program
Common Core Mathematics Test

Performance Level Descriptions

GRADE 7

Policy-Level Performance Level Definitions
For each grade, there are students performing along a proficiency continuum with regard to the skills and knowledge necessary to meet the demands of grade-specific Common Core Standards for Mathematics. There are students who are above proficiency, students who are proficient, students who are not quite proficient, and students who are well below proficient at each grade level. New York State assessments are designed to classify students into one of four proficiency categories; these proficiency categories are defined as:

**NYS Level 4**
Students performing at this level excel in standards for their grade. They demonstrate knowledge, skills, and practices embodied by the New York State P-12 Common Core Learning Standards for Mathematics that are considered more than sufficient for the expectations at this grade.

**NYS Level 3**
Students performing at this level are proficient in standards for their grade. They demonstrate knowledge, skills, and practices embodied by the New York State P-12 Common Core Learning Standards for Mathematics that are considered sufficient for the expectations at this grade.

**NYS Level 2**
Students performing at this level are below proficient in standards for their grade. They demonstrate knowledge, skills, and practices embodied by the New York State P-12 Common Core Learning Standards for Mathematics that are considered partial but insufficient for the expectations at this grade.

**NYS Level 1**
Students performing at this level are well below proficient in standards for their grade. They demonstrate limited knowledge, skills, and practices embodied by the New York State P-12 Common Core Learning Standards for Mathematics that are considered insufficient for the expectations at this grade.
**Performance Level Descriptions**

Performance Level Descriptions (PLDs) describe the range of knowledge and skills students should demonstrate at a given performance level.

**How were the PLDs developed?**

The New York State Education Department (NYSED) convened the state’s English Language Arts (ELA) and Math Content Advisory Panels (CAPs) to develop the initial draft PLDs for grades 3 - 8. The CAPs are classroom teachers from elementary, middle and high school, school and district administrators, English Language Learner (ELL) and students with disabilities (SWD) specialists, and higher education faculty members from across the state. The draft PLDs from the CAPs then went through additional rounds of review and edit from a number of content experts and assessment experts under NYSED supervision.

In developing PLDs, the CAPs considered policy-level definitions of the performance levels (see above) and the expectations for each grade level in the Common Core State Standards. Drafting PLDs began with Level 3, the proficiency level, to determine the content knowledge and skill necessary at a given grade level and content standard to be considered proficient according to the rigor and demand of the Common Core. CAP members then drafted PLDs at Levels 4 (excel) and 2 (partial but insufficient for proficiency). Finally, Level 1 PLDs describe a wide range of students, including both those who are just below meeting the requirements for Level 2 and those who attempted but did not answer any questions correctly.

Because of the range of students covered in Level 1, these PLDs were developed last and will be released in the next version of this document. The next version of this document, to be released in September 2013, will include Level 1 PLDs for the Major Clusters and PLDs for the Supporting and Additional Clusters for all levels.

**How are the PLDs used in Assessment?**

PLDs are essential in setting standards for the New York State Grades 3-8 assessments. Standard setting panelists use PLDs to determine the threshold expectations for students to demonstrate the knowledge and skills necessary to attain just barely a Level 2, Level 3, or Level 4 on the assessment. These discussions then influence the panelists in establishing the cut scores on the assessment. PLDs are also used to inform item development, as each test needs questions that distinguish performance all along the continuum.

**How can the PLDs be used in Instruction?**

PLDs help communicate to students, families, educators and the public the specific knowledge and skills expected of students to demonstrate proficiency and can serve a number of purposes in classroom instruction. They are the foundation of rich discussion around what students need to do to perform at higher levels and to explain the progression of learning within a subject and grade level. We encourage the use of the PLDs for a variety of purposes, such as differentiating instruction to maximize individual student outcomes, creating classroom assessments and rubrics to help in identifying target performance levels for individual or groups of students, and tracking student growth along the proficiency continuum as described by the PLDs.
Grade 7 Mathematics Performance Level Descriptions

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Performance Level 4</th>
<th>Performance Level 3</th>
<th>Performance Level 2</th>
<th>Performance Level 1*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students analyze proportional relationships and use them to solve real-world and mathematical problems. (7.RP.1-3)</td>
<td>Compute unit rates associated with ratios of fractions and mixed numbers measured in like or different units.</td>
<td>Compute unit rates associated with ratios of fractions and mixed numbers measured in like or different units.</td>
<td>Compute unit rates associated with ratios of fractions and mixed numbers measured in like or different units.</td>
<td>Compute unit rates associated with ratios consisting of at least one fraction measured in like or different units.</td>
</tr>
<tr>
<td></td>
<td>Explain whether two quantities are in a proportional relationship using multiple representations.</td>
<td>Determine whether two quantities are in a proportional relationship in a given graph, table, or verbal description.</td>
<td>Identify the constant of proportionality (unit rate) in equations, graphs, tables, diagrams, and verbal descriptions.</td>
<td>Identify the constant of proportionality (unit rate) in equations, graphs, tables, diagrams, and verbal descriptions.</td>
</tr>
<tr>
<td></td>
<td>Interpret any point ((0, 0)) and ((1, r)), where (r) is the unit rate, on the graph of a proportional relationship in terms of the situation.</td>
<td>Interpret the points ((0, 0)) and ((1, r)), where (r) is the unit rate, on the graph of a proportional relationship in terms of the situation.</td>
<td>Identify the points representing the initial value and the unit rate on the graph of a proportional relationship in terms of the situation.</td>
<td>Identify the points representing the initial value and the unit rate on the graph of a proportional relationship in terms of the situation.</td>
</tr>
<tr>
<td></td>
<td>Determine when it is appropriate to use unit rate, and recognize when it has its limitations.</td>
<td>Represent proportional relationships by equations.</td>
<td>Use proportional relationships to solve simple mathematical problems involving ratio or percent.</td>
<td>Use proportional relationships to solve simple mathematical problems involving ratio or percent.</td>
</tr>
<tr>
<td></td>
<td>Analyze and use proportional relationships to solve multi-step real-world and mathematical problems involving ratio and/or percent.</td>
<td>Use proportional relationships to solve multi-step mathematical problems involving ratio and/or percent.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Level 1 PLDs describe students who did not demonstrate sufficient evidence to be classified into Level 2; Level 1 contains the widest range of performance on the test: from the lowest-scoring students, including those students who attempted but got no answers correct, to those students who are on the cusp but just below Level 2. The next version of this document, to be released in September 2013, will include Level 1 PLDs for the Major Clusters and PLDs for the Supporting and Additional Clusters for all levels.
## Grade 7 Mathematics Performance Level Descriptions

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</tr>
</thead>
<tbody>
<tr>
<td>Students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (7.NS.1-3)</td>
<td>Add and subtract rational numbers using properties of operations.</td>
<td>Add and subtract rational numbers using properties of operations.</td>
<td>Add and subtract integers using properties of operations.</td>
<td>Add and subtract integers using properties of operations.</td>
</tr>
<tr>
<td></td>
<td>Represent addition and subtraction of rational numbers on a horizontal or vertical number line.</td>
<td>Represent addition and subtraction of rational numbers on a horizontal or vertical number line.</td>
<td>Represent addition and subtraction of integers on a horizontal or vertical number line.</td>
<td>Represent addition and subtraction of integers on a horizontal or vertical number line.</td>
</tr>
<tr>
<td></td>
<td>Identify situations in which opposite quantities combine to make zero.</td>
<td>Identify situations in which opposite quantities combine to make zero.</td>
<td>Identify situations in which opposite quantities combine to make zero.</td>
<td>Identify situations in which opposite quantities combine to make zero.</td>
</tr>
<tr>
<td></td>
<td>Use directional distance and absolute value to describe sums of rational numbers.</td>
<td>Multiply and divide rational numbers using properties of operations.</td>
<td>Multiply and divide integers using properties of operations.</td>
<td>Multiply and divide integers using properties of operations.</td>
</tr>
<tr>
<td></td>
<td>Convert a rational number to a decimal using long division.</td>
<td>Solve two-step real-world and mathematical problems involving four operations with rational numbers, justifying the steps taken using the properties of operations.</td>
<td>Solve two-step real-world and mathematical problems involving four operations with integers.</td>
<td>Solve two-step mathematical problems involving four operations with integers.</td>
</tr>
</tbody>
</table>

* Level 1 PLDs describe students who did not demonstrate sufficient evidence to be classified into Level 2; Level 1 contains the widest range of performance on the test: from the lowest-scoring students, including those students who attempted but got no answers correct, to those students who are on the cusp but just below Level 2. The next version of this document, to be released in September 2013, will include Level 1 PLDs for the Major Clusters and PLDs for the Supporting and Additional Clusters for all levels.
### Grade 7 Mathematics Performance Level Descriptions

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<th>Performance Level 2</th>
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</thead>
<tbody>
<tr>
<td>Students use properties of operations to generate equivalent expressions. (7.EE.1,2)</td>
<td>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions.</td>
<td>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions.</td>
<td>Apply properties of operations used as strategies to add and subtract linear expressions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Describe the relationship between equivalent quantities expressed algebraically in different forms in a problem context, and explain their equivalence in light of the context of the problem.</td>
<td>Rewrite an expression in different forms.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Level 1 PLDs describe students who did not demonstrate sufficient evidence to be classified into Level 2; Level 1 contains the widest range of performance on the test: from the lowest-scoring students, including those students who attempted but got no answers correct, to those students who are on the cusp but just below Level 2. The next version of this document, to be released in September 2013, will include Level 1 PLDs for the Major Clusters and PLDs for the Supporting and Additional Clusters for all levels.
## Grade 7 Mathematics Performance Level Descriptions

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<th>Performance Level 1*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students solve real-life and mathematical problems using numerical and algebraic expressions and equations. (7.EE.3,4)</td>
<td>Solve multi-step real-world and mathematical problems with rational numbers using the properties of operations. Solve word problems leading to linear equations of the form $px + q = r$ and $p(x + q) = r$ with rational coefficients. Explain the relationship between the steps used to solve a given equation in the form of $px + q = r$ or $p(x + q) = r$ using an algebraic solution and an arithmetic solution. Solve word problems leading to linear inequalities of the form $px + q &gt; r$ and $px + q &lt; r$ with rational coefficients, and then graph and interpret solution sets. Explain whether a solution to a given problem is reasonable.</td>
<td>Solve multi-step real-world and mathematical problems with rational numbers using the properties of operations. Solve word problems leading to linear equations of the form $px + q = r$ and $p(x + q) = r$ with rational coefficients.</td>
<td>Solve two-step mathematical problems with rational numbers using the properties of operations. Solve linear equations of the form $px + q = r$ and $p(x + q) = r$ with rational coefficients.</td>
<td>Solve linear inequalities of the form $px + q &gt; r$ and $px + q &lt; r$ with rational coefficients.</td>
</tr>
</tbody>
</table>

* Level 1 PLDs describe students who did not demonstrate sufficient evidence to be classified into Level 2; Level 1 contains the widest range of performance on the test: from the lowest-scoring students, including those students who attempted but got no answers correct, to those students who are on the cusp but just below Level 2. The next version of this document, to be released in September 2013, will include Level 1 PLDs for the Major Clusters and PLDs for the Supporting and Additional Clusters for all levels.
To evaluate each grade/course’s assessments for alignment with the Common Core State Standards (CCSS), analyze the assessments against the non-negotiable criteria on the following pages. Each grade/course’s assessments and item banks must meet all of the non-negotiable criteria and associated metrics to align with the CCSSM.

BEFORE YOU BEGIN

ALIGNMENT TO THE COMMON CORE STATE STANDARDS

Evaluators of assessments should understand that at the heart of the Common Core State Standards there are substantial shifts in mathematics that require the following:

1) Focus strongly where the Standards focus
2) Coherence: Think across grades and link to major topics within grade
3) Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Evaluators of assessments must be well versed in the CCSS for the grade level of the materials in question, including understanding the major work of the grade\(^1\) vs. the supporting and additional work, how the content fits into the progressions in the Standards, and the expectations of the Standards with respect to conceptual understanding, procedural skill and fluency, and application. It is also recommended that evaluators refer to the Spring 2013 K–8 Publishers’ Criteria for Mathematics and the Spring 2013 High School Publishers’ Criteria for the Common Core State Standards for Mathematics while using this tool (achievethecore.org/publisherscriteria).

ORGANIZATION

SECTION I: NON-NEGOTIABLE ALIGNMENT CRITERIA

All grade or course assessments must meet all of the non-negotiable criteria at each grade/course level to be aligned to CCSS.

SECTION 2: INDICATORS OF QUALITY.

Indicators of quality are scored differently from the non-negotiable criteria; a higher score in Section 2 indicates that assessments are more closely aligned.

REVIEW

Evaluator:__________________________ Assessments:_______________ Grade:___________ Date:________________

\(^1\) For more on the major work of each grade, see achievethecore.org/emphases.
SECTION I

| Non-Negotiable 1. FOCUS ON MAJOR WORK: The large majority of points in each grade K–8 are devoted to the major work of the grade, and the majority of points in each High School course are devoted to widely applicable prerequisites.² | For grades K–8, each grade/course’s assessments meet or exceed the following score-point distributions for the major work of the grade.

- 85% of the total points in grades K–2 align exclusively to the major work of the grade.
- 75% of the total points in grades 3–5 align exclusively to the major work of the grade.
- 65% of the total points in grades 6–8 align exclusively to the major work of the grade.

For high school, aligned assessments or sets of assessments meet or exceed the following score-point distribution:

- 50% of the total points in high school align to widely applicable prerequisites for postsecondary work. |

<table>
<thead>
<tr>
<th>Non-Negotiable 1. FOCUS ON MAJOR WORK To be aligned to the CCSSM, each grade/course’s assessments should meet or exceed the score-point distributions in the metrics.</th>
<th>Meet (Y/N)</th>
<th>Justification / Comments</th>
</tr>
</thead>
</table>

2 Refer also to criterion #1 in the K–8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013) and criterion #1 in the High School Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).
<table>
<thead>
<tr>
<th>SECTION I</th>
<th>METRICS</th>
</tr>
</thead>
</table>
| **Non-Negotiable 2. FOCUS IN K–8:** No item assesses topics directly or indirectly before they are introduced in the CCSSM.\(^3\) | 100% of items on an assessment do not assess knowledge of topics before the grade level they are introduced in the CCSSM. Commonly misaligned topics include, but are not limited to:  
- **Probability**, including chance, likely outcomes, probability models. (Introduced in the CCSSM in grade 7)  
- **Statistical distributions**, including center, variation, clumping, outliers, mean, median, mode, range, quartiles; and **statistical association or trends**, including two-way tables, bivariate measurement data, scatter plots, trend line, line of best fit, correlation. (Introduced in the CCSSM in grades 6–8; see CCSSM for specific expectations by grade level.)  
- **Similarity, congruence, or geometric transformations.** (Introduced in the CCSSM in grade 8)  
- **Symmetry** of shapes, including line/reflection symmetry, rotational symmetry. (Introduced in the CCSSM in grade 4) |

**Non-Negotiable 2. FOCUS IN K–8:** To be aligned to the CCSSM, each grade/course’s assessments do not assess topics directly or indirectly before they are introduced in the CCSSM. | Meet (Y/N) | Justification / Comments |

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\(^3\) Refer also to criterion #2 in the K–8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).
### SECTION I

**Non-Negotiable 3. RIGOR AND BALANCE:** Each grade/course’s assessments reflect the balances in the Standards and help students meet the Standards’ rigorous expectations by helping students develop conceptual understanding, procedural skill and fluency, and application.\(^4\)

*This criterion applies to fixed form or CAT assessments, whether summative assessments or a set of interim/benchmark assessments. Item banks also should reflect the proportions in the metrics.*

**For Conceptual Understanding:**
- **K–High School:** At least 20% of the total score-points on the assessment(s) for each grade or course explicitly require students to demonstrate conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.

**For Procedural Skill and Fluency:**
- **K–6:** At least 20% of the score-points on the assessment(s) for each grade explicitly assess procedural skill and fluency requirements in the Standards.
- **7–8 and High School:** At least 20% of the score-points on the assessment(s) for each grade or course explicitly assess procedural skill and fluency.

**For Applications**
- **K–5:** At least 20% of the total score-points on the assessment(s) for each grade explicitly assess solving single- or multi-step word problems.
- **6–8:** At least 25% of the total score points on the assessment(s) for each grade explicitly assess solving single- and multi-step word problems and simple models.
- **High School:** At least 30% of the total score-points on the assessment(s) for each high school course explicitly assess single- and multi-step word problems, simple models, and substantial modeling/application problems.

<table>
<thead>
<tr>
<th>METRICS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-Negotiable 3. RIGOR AND BALANCE</strong></td>
</tr>
<tr>
<td>To be aligned to the CCSSM, each grade/course’s assessments meet or exceed the percentages in the metrics.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Meet (Y/N)</th>
<th>Justification / Comments</th>
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\(^4\) Refer also to criterion #4 in the K-8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013) and criterion #2 in the High School Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).
### SECTION I

<table>
<thead>
<tr>
<th>METRICS</th>
</tr>
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</table>
| **Non-Negotiable 4. PRACTICE-CONTENT CONNECTIONS**: Each grade/course’s assessments include items that meaningfully connect the Standards for Mathematical Content and Standards for Mathematical Practice. However, not all items need to align to a Standard for Mathematical Practice. And there is no requirement to have an equal balance among the Standards for Mathematical Practice in any set of items or test forms.  

This criterion applies to fixed form or CAT assessments, whether summative assessments or a set of interim/benchmark assessments. Item banks also should reflect the metrics. |

All assessments or sets of assessments include accompanying analysis, aimed at evaluators, which shows how the Standards for Mathematical Practice are meaningfully connected to the Standards for Mathematical Content assessed. Practice demands are grade-appropriate, beginning in an elementary way in grades K–5 and showing an arc of growing sophistication across the grades. |

<table>
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<th>Justification / Comments</th>
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</thead>
</table>

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5 Refer also to criterion #7 in the K-8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013) and criteria #5 High School Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).
## SECTION I

**Non-Negotiable 5. ALIGNMENT OF TEST ITEMS:** Test items elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted standard(s), adhering to the full intent of the CCSSM.

This criterion applies to fixed form or CAT assessments, whether summative assessments or a set of interim/benchmark assessments. All items and/or sets of items should reflect the metric.

**Metrics for Non-Negotiable 5:**

100% of items and/or sets of items exhibit alignment to the full intent of the CCSSM for that grade or course:

- Directly reflecting the language of individual standards.
  - For example, 6.EE.3 puts the emphasis on applying properties of operations and generating equivalent expressions, not just mechanically simplifying.
  - Most items aligned to a single standard should assess the central concern of the standard in question.
- Reflecting the progressions in the Standards.
  - For example, multiplication and division items in grade 3 emphasize equal groups, with no rate problems (grade 6 in CCSS).
- Assessing all levels of the content hierarchy.
  - For example, by including some items that assess clusters.
- Using the number system appropriate to the grade level.
  - For example, in grade 3 there are some items involving fractions greater than 1; in the middle grades, arithmetic and algebra use the rational number system, not just the integers.

<table>
<thead>
<tr>
<th>Meet (Y/N)</th>
<th>Justification / Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

Each grade/course's assessments must meet all five of the non-negotiable criteria to be aligned to the CCSS and to continue to the evaluation in Section II.

**# Criteria Met:**

---

6 Refer also to the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics (Spring 2013) and the High School Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).

SECTION II: INDICATORS OF QUALITY

Each grade/course’s assessments must meet all five of the non-negotiable criteria to be aligned to the CCSS and to continue to the evaluation in Section II. Section 2 includes indicators of quality. Indicators of quality are scored differently from the non-negotiable criteria; a higher score in Section 2 indicates that assessments are more closely aligned.

Consider this guidance when evaluating:
- **2** – (meets criteria): A score of 2 means that the assessments meet the full intention of the criterion in a grade/course.
- **1** – (partially meets criteria): A score of 1 means that the assessments meet the criterion in many aspects but not the full intent of the criterion.
- **0** – (does not meet criteria): A score of 0 means that the materials do not meet many aspects of the criterion.

<table>
<thead>
<tr>
<th>SECTION II INDICATORS OF QUALITY</th>
<th>SCORE</th>
<th>JUSTIFICATION/NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Assessing Supporting Content.</td>
<td>2</td>
<td></td>
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<tr>
<td>Assessment of supporting content enhances focus and coherence simultaneously by engaging students in the major work of the grade or course.</td>
<td>1</td>
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<tr>
<td>2. Addressing Every Standard for Mathematical Practice.</td>
<td>2</td>
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<tr>
<td>Every Standard for Mathematical Practice is represented on the assessment(s) for each grade or course.</td>
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<tr>
<td>3. Expressing Mathematical Reasoning.</td>
<td>2</td>
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<tr>
<td>There are sufficiently many points on the assessment(s) for each grade or course that explicitly assess expressing and/or communicating mathematical reasoning.</td>
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<tr>
<td>4. Constructing Forms Without Cueing Solution Processes.</td>
<td>2</td>
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</tr>
<tr>
<td>Item sequences do not cue the student to use a certain solution process during problem solving and assessments include problems requiring different types of solution processes within the same section.</td>
<td>1</td>
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<tr>
<td>5. Calling for Variety in Student Work.</td>
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<tr>
<td>Items require a variety in what students produce. For example, items require students to produce answers and solutions, but also, in a grade-appropriate way, arguments and explanations, diagrams, mathematical models, etc.</td>
<td>1</td>
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<tr>
<td>6. Quality Materials.</td>
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<tr>
<td>The assessment items, answer keys, and documentation are free from mathematical errors.</td>
<td>1</td>
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ADD UP TOTAL POINTS EARNED

Total________

Notes/Justification:

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8 Refer also to criterion #3 in the K-8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013).
9 Refer also to criterion #9 in the K-8 Publishers’ Criteria for the Common Core State Standards for Mathematics (Spring 2013) and criteria #7 High School Publishers’ Criteria for the CCSSM (Spring 2013).
Ms. Alonzo’s Class

Equivalent Fractions Test

Name:____________________________________

1. Shade in 1/6 of the rectangle. Explain why you shaded it that way.

_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________

2. Shade in ½ of the rectangle. Explain why you shaded it that way.

_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________
3.

a. The rectangle below has length 1. What fraction does the shaded part represent?

b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?

3. c. Are the fractions above equivalent? Explain why or why not.

_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________
_____________________________________________________________________________________

4.

Gerald and Eliza each have a garden, with the same fraction growing turnips. If Gerald’s turnip section is shaded in, shade in a possible fraction of Eliza’s garden where she grows turnips.
Ms. Alonzo’s Class

Equivalent Fractions Test

Name: **Caitlin**

1. Shade in $\frac{1}{6}$ of the rectangle. Explain why you shaded it that way.

2. Shade in $\frac{2}{3}$ of the rectangle. Explain why you shaded it that way.
3.

a. The rectangle below has length 1. What fraction does the shaded part represent?

\[
\frac{3}{4}
\]

b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?

\[
\frac{9}{12}
\]

c. Are the fractions above equivalent? Explain why or why not.

Yes because \(3 \times 3 = 9\)

4.

Gerald and Eliza each have a garden, with the same fraction growing turnips. If Gerald’s turnip section is shaded in, shade in a possible fraction of Eliza’s garden where she grows turnips.
Ms. Alonzo's Class

Equivalent Fractions Test

Name: Truman

1. Shade in 1/6 of the rectangle. Explain why you shaded it that way.

The whole is six and I shaded 1

2. Shade in 2/3 of the rectangle. Explain why you shaded it that way.

1 shaded 2 which is 1/2
3.

a. The rectangle below has length 1. What fraction does the shaded part represent?

\[
\frac{3}{4}
\]

b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?

\[
\frac{3}{4}
\]

c. Are the fractions above equivalent? Explain why or why not.

Yes because \( \frac{3}{3} = \frac{3}{3} \)

4.

Gerald and Eliza each have a garden, with the same fraction growing turnips. If Gerald’s turnip section is shaded in, shade in a possible fraction of Eliza’s garden where she grows turnips.

Gerald

\[
\begin{array}{c}
\text{Gerald} \\
\end{array}
\]

Eliza
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</thead>
</table>

**Ms. Alonzo's Class**

Equivalent Fractions 3a 3b 3c 4
DATA-DRIVEN ANALYSIS MEETINGS:
Leading Effective 1-on-1 Meetings around Interim Assessment Results

PRE-CURSORS FOR EFFECTIVE ANALYSIS MEETINGS:

Before Giving Interim Assessment:
- **6 WEEKS PRIOR**: Teachers review assessment and plan towards the rigor of those assessments
- **A FEW WEEKS PRIOR**: Teacher predict performance on each assessment question:
  - a) confident they’ll get it right; b) not sure; c) no way they’ll get it right
- **PD (timing flexible)**: Teachers receive model of how to do assessment analysis and complete action plan, and they see model of effective and ineffective analysis meetings

Immediately Following Interim Assessment Administration:
- **TEACHER ANALYSIS**: Teachers do analysis of results prior to meeting, trying to answer fundamental question: why did the students not learn it?
- **TEACHER ACTION PLAN**: Teachers complete action plan
- **LEADER PREPARATION**: Leader analyzes teacher results, analysis and action plan:
  - Determine end goal for teach standard: explicit action step(s) based on deep analysis
  - Prepare questions to get the teacher to that action step
- **CONTENT EXPERTISE**: If the teacher or leader lacks deep content knowledge:
  - If Leader is lacking: identify expert within/outside of school to call on for extra support
  - If Teacher is lacking: leader should be ready to coach them about effective techniques

CONVERSATION STARTERS & RE-DIRECTORS DURING ANALYSIS MEETINGS:

Starters:
- “Congratulations on the improvement on _____ from last time!”
- “So…what’s the data telling you?”

Re-Directors & Data-Focusing Comments:
- “Let’s look at question ___. What do you think the students are doing wrong here?”
- “What did the students need to be able to do to make that question right? How was this more than what they are able to do with you in class?”
- “What’s so interesting is that they did really well on question #__ but struggled question #__ on the same standard. Why do you think that is?”

Making it Actionable:
- “What should students do when they hit this struggle the next time?”
- “Where will you do this [action step] in your upcoming lessons?”
- [When new analysis/action is proposed during the meeting] “Let’s summarize the action steps.” [Write them into action plan or future lesson plans.]
- “Let’s go back to your action plan and add these new actions.”

KEY PRINCIPLES FOR LEADING ANALYSIS MEETINGS:

- Let the data do the talking
- Let the teacher do the talking (or push them to!)
- Always go back to the test to specific questions
- Don’t fight the battles on ideological lines (in the larger picture, you’ll lose)
- Know the data yourself to lead an analysis meeting effectively
- Make explicit, detailed action steps & ensure that they happen in the classroom