Topic E

Multiplication of a Fraction by a Fraction

5.NBT.7, 5.NF.4a, 5.NF.6, 5.MD.1, 5.NF.4b

Focus Standards:

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF.4a Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product of \((a/b) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times \frac{4}{5} = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Instructional Days: 8

Coherence -Links from:
G4–M6 Decimal Fractions
G5–M2 Multi-Digit Whole Number and Decimal Fraction Operations

-Links to:
G6–M2 Arithmetic Operations Including Division of Fractions
G6–M4 Expressions and Equations

Topic E introduces students to multiplication of fractions by fractions—both in fraction and decimal form (5.NF.4a, 5.NBT.7). The topic starts with multiplying a unit fraction by a unit fraction and progresses to multiplying two non-unit fractions. Students use area models, rectangular arrays, and tape diagrams to model the multiplication. These familiar models help students draw parallels between whole number and fraction multiplication, as well as solve word problems. This intensive work with fractions positions students to extend their previous work with decimal-by-whole number multiplication to decimal-by-decimal multiplication. Just as students used unit form to multiply fractional units by wholes in Module 2 (e.g., \(3.5 \times 2 = 35\) tenths \(\times\) 2 ones = \(70\) tenths), they connect fraction-by-fraction multiplication to multiply fractional units-by-fractional units \((3.5 \times 0.2 = 35\) tenths \(\times\) 2 tenths = \(70\) hundredths).


\[ \frac{3}{4} \text{ of a foot} = \frac{3}{4} \times (1 \text{ foot}) \]

\[ = \frac{3}{4} \times (12 \text{ inches}) \]

\[ = 9 \text{ inches} \]

1 foot = 12 inches

\[ \frac{3}{4} \times 12 \]

Express 5 \( \frac{3}{4} \) feet as inches.

\[ 5 \frac{3}{4} \text{ feet} = (5 \times 12) \text{ inches} + \left( \frac{3}{4} \right) \times 12 \text{ inches} \]

\[ = 60 + 9 \text{ inches} \]

\[ = 69 \text{ inches} \]

Reasoning about decimal placement is an integral part of these lessons. Finding fractional parts of customary measurements and measurement conversion (5.MD.1) concludes Topic E. Students convert smaller units to fractions of a larger unit (e.g., 6 inches = \( \frac{1}{2} \) foot). The inclusion of customary units provides a meaningful context for many common fractions (e.g., \( \frac{1}{2} \) pint = 1 cup, \( \frac{1}{3} \) yard = 1 foot, \( \frac{1}{4} \) gallon = 1 quart). This topic, together with the fraction concepts and skills learned in Module 3, opens the door to a wide variety of application word problems (5.NF.6).

### A Teaching Sequence Toward Mastery of Multiplication of a Fraction by a Fraction

**Objective 1:** Multiply unit fractions by unit fractions.

(Lesson 13)

**Objective 2:** Multiply unit fractions by non-unit fractions.

(Lesson 14)

**Objective 3:** Multiply non-unit fractions by non-unit fractions.

(Lesson 15)

**Objective 4:** Solve word problems using tape diagrams and fraction-by-fraction multiplication.

(Lesson 16)

**Objective 5:** Relate decimal and fraction multiplication.

(Lessons 17–18)

**Objective 6:** Convert measures involving whole numbers, and solve multi-step word problems.

(Lesson 19)

**Objective 7:** Convert mixed unit measurements, and solve multi-step word problems.

(Lesson 20)