Lesson 26

Objective: Divide a unit fraction by a whole number.

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Development (30 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (12 minutes)

- Count by Fractions 5.NF.7 (5 minutes)
- Divide Whole Numbers by Fractions 5.NF.7 (4 minutes)
- Multiply Fractions 5.NF.4 (3 minutes)

Count by Fractions (5 minutes)

Note: This fluency activity reviews Lesson 21.

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  1  2  3  4  5  6  7  8  9  10  11  12
  1/4 1/2 3/4 1 1 1 1 1 1 1 1 1
  2/4 1 3/4 2 2 2 2 2 2 2 2 2
  3/4 1 1/4 1 1/4 1 1/4 1 1/4 1 1/4 1
  4  4  4  4  4  4  4  4  4  4  4  4
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T: Count by fourths to 12 fourths. (Write as students count.)
S: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, 6 fourths, 7 fourths, 8 fourths, 9 fourths, 10 fourths, 11 fourths, 12 fourths.
T: Let’s count by fourths again. This time, when we arrive at a whole number, say the whole number. (Write as students count.)
S: 1 fourth, 2 fourths, 3 fourths, 1 whole, 5 fourths, 6 fourths, 7 fourths, 2 wholes, 9 fourths, 10 fourths, 11 fourths, 3 wholes.
T: Let’s count by fourths again. This time, change improper fractions to mixed numbers.
S: 1 fourth, 2 fourths, 3 fourths, 1 whole, 1 and 1 fourth, 1 and 2 fourths, 1 and 3 fourths, 2 wholes, 2 and 1 fourth, 2 and 2 fourths, 2 and 3 fourths, 3 wholes.

T: Let’s count by fourths again. This time, simplify 2 fourths to 1 half. (Write as students count.)

S: 1 fourth, 1 half, 3 fourths, 1 whole, 1 and 1 fourth, 1 and 1 half, 1 and 3 fourths, 2 wholes, 2 and 1 fourth, 2 and 1 half, 2 and 3 fourths, 3 wholes.

Continue the process, counting by fifths to 15 fifths.

**Divide Whole Numbers by Fractions (4 minutes)**

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 25.

T: (Write $1 \div \frac{1}{2} = \_\_\_\_\_.\) Say the division problem.

S: $1 \div \frac{1}{2}$.

T: How many halves are in 1 whole?

S: 2.

T: (Write $1 \div \frac{1}{2} = 2$. Beneath it, write $2 \div \frac{1}{2}$.) How many halves are in 2 wholes?

S: 4.

T: (Write $2 \div \frac{1}{2} = 4$. Beneath it, write $3 \div \frac{1}{2}$.) How many halves are in 3 wholes?

S: 6.

T: (Write $3 \div \frac{1}{2} = 6$. Beneath it, write $8 \div \frac{1}{2}$.) On your personal white board, write the complete number sentence.

S: (Write $8 \div \frac{1}{2} = 16$.)

Continue with the following possible sequence: $1 \div \frac{1}{3}, 2 \div \frac{1}{3}, 5 \div \frac{1}{3}, 1 \div \frac{1}{4}, 2 \div \frac{1}{4}, 7 \div \frac{1}{4}, 3 \div \frac{1}{5}, 4 \div \frac{1}{6}$, and $7 \div \frac{1}{8}$.

**Multiply Fractions (3 minutes)**

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 13–16.

T: (Write $\frac{1}{2} \times \frac{1}{3} = \_\_\_\_\_.\) Say the multiplication number sentence with the answer.

S: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Continue this process with the following possible problems: $\frac{1}{2} \times \frac{1}{4}$ and $\frac{1}{2} \times \frac{1}{5}$.

T: (Write $\frac{1}{2} \times \frac{1}{9} = \_\_\_\_\_.\) On your personal white board, write the number sentence and the answer.
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S: \( \frac{1}{2} \times \frac{1}{9} = \frac{1}{18} \)

T: (Write \( \frac{1}{2} \times \frac{5}{9} = \___ \)) Say the multiplication sentence with the answer.

S: \( \frac{1}{2} \times \frac{5}{9} = \frac{5}{18} \)

Repeat this process with the following possible problems: \( \frac{1}{4} \times \frac{3}{3} \), \( \frac{4}{4} \times \frac{1}{5} \), \( \frac{1}{2} \times \frac{1}{9} \), and \( \frac{3}{4} \times \frac{3}{2} \).

T: (Write \( \frac{2}{3} \times \frac{2}{5} = \___ \)) Write the multiplication sentence with the answer.

S: (Write \( \frac{2}{3} \times \frac{2}{5} = \frac{4}{15} \))

Continue this process with the following possible problem: \( \frac{3}{4} \times \frac{3}{5} \).

T: (Write \( \frac{1}{4} \times \frac{3}{5} = \___ \)) On your personal white board, write the number sentence and the answer.

S: (Write \( \frac{1}{4} \times \frac{3}{5} = \frac{3}{20} \))

T: (Write \( \frac{3}{4} \times \frac{4}{3} = \___ \)) Try this problem.

S: (Write \( \frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1 \))

Application Problem (8 minutes)

A race begins with \( 2 \frac{1}{2} \) miles through town, continues through the park for \( 2 \frac{1}{3} \) miles, and finishes at the track after the last \( \frac{1}{6} \) mile. A volunteer is stationed every quarter mile and at the finish line to pass out cups of water and cheer on the runners. How many volunteers are needed?

Note: This multi-step problem requires that students first add three fractions and then divide the sum by a fraction, which reinforces yesterday’s division of a whole number by a unit fraction. (How many \( \frac{1}{4} \) miles are in 5 miles?) It also reviews adding fractions with different denominators (Module 3).
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Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1
Nolan gives some pans of brownies to his 3 friends to share equally.

a. If he has 3 pans of brownies, how many pans of brownies will each friend receive?

b. If he has 1 pan of brownies, how many pans of brownies will each friend receive?

c. If he has $\frac{1}{2}$ pan of brownies, how many pans of brownies will each friend receive?

d. If he has $\frac{1}{3}$ pan of brownies, how many pans of brownies will each friend receive?

T: (Post Problem 1(a) on the board, and read it aloud with the students.) Work on your personal white board, and write a division sentence to solve this problem. Be prepared to share.

S: (Work.)

T: How many pans of brownies does Nolan have?
S: 3 pans.
T: The 3 pans of brownies are divided equally with how many friends?
S: 3 friends.
T: Say the division sentence with the answer.
S: $3 \div 3 = 1$.
T: Answer the question in a complete sentence.
S: Each friend will receive 1 pan of brownies.

T: (In the problem, erase 3 pans and replace it with 1 pan.) Imagine that Nolan has 1 pan of brownies. If he gave it to his 3 friends to share equally, what portion of the brownies will each friend receive? Write a division sentence to show how you know.

S: (Write $1 \div 3 = \frac{1}{3}$ pan.)

T: Nolan starts out with how many pans of brownies?
S: 1 pan.
T: The 1 pan of brownies is divided equally by how many friends?
S: 3 friends.
T: Say the division sentence with the answer.
S: $1 \div 3 = \frac{1}{3}$.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:
While the tape diagramming in the beginning of this lesson is presented as teacher-directed, it is equally acceptable to elicit each step of the diagram from the students through questioning. Many students benefit from verbalizing the next step in a diagram.
T: Let’s model that thinking with a tape diagram. I’ll draw a bar and shade it in representing 1 pan of brownies. Next, I’ll partition it equally with dotted lines into 3 units, and each unit is \( \frac{1}{3} \). (Draw a bar and cut it equally into three parts.) How many pans of brownies did each friend receive this time? Answer the question in a complete sentence.

S: Each friend will receive \( \frac{1}{3} \) pan of brownies. (Label \( \frac{1}{3} \) underneath one part.)

T: Let’s rewrite the problem as thirds. How many thirds are in 1 one?

S: 3 thirds.

T: (Write 3 thirds \( \div 3 = \_\_\_ \)) What is 3 thirds divided by 3?

S: 1 third. (Write 3 thirds \( \div 3 = 1 \) third.)

T: Another way to interpret this division expression would be to ask, “1 is 3 of what number?” And of course, we know that 3 thirds makes 1.

T: But just to be sure, let’s check our work. How do we check a division problem?

S: Multiply the answer and the divisor.

T: Check it now.

S: (Work and show \( \frac{1}{3} \times 3 = \frac{3}{3} = 1 \).)

T: (Replace 1 pan in the problem with \( \frac{1}{2} \) pan.) Now, imagine that he only has \( \frac{1}{2} \) pan. Still sharing them with 3 friends equally, how many pans of brownies will each friend receive?

T: Now that we have half of a pan instead of 1 whole pan to share, will each friend receive more or less than \( \frac{1}{3} \) pan? Turn and discuss.

S: Less than \( \frac{1}{3} \) pan. \( \Rightarrow \) We have less to share, but we are sharing with the same number of people. They will receive less. \( \Rightarrow \) Since we’re starting out with \( \frac{1}{2} \) pan, which is less than 1 whole pan, the answer should be less than \( \frac{1}{3} \) pan.
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T: (Draw a bar and cut it into 2 parts. Shade in 1 part.) How can we show how many people are sharing this $\frac{1}{2}$ pan of brownies? Turn and talk.

S: We can draw dotted lines to show the 3 equal parts that he cuts the half into. → We have to show the same size units, so I’ll cut the half that’s shaded into 3 parts and the other half into 3 parts, too.

T: (Partition the whole into 6 parts.) What fraction of the pan will each friend receive?

S: $\frac{1}{6}$. (Label $\frac{1}{6}$ underneath one part.)

T: (Write $\frac{1}{2} \div 3 = \frac{1}{6}$.) Let’s think again. 1 half is equal to how many sixths? Look at the tape diagram to help you.

S: 3 sixths.

T: So, what is 3 sixths divided by 3? (Write 3 sixths $\div 3 = \underline{\text{____.}}$)

S: 1 sixth. (Write 3 sixths $\div 3 = 1$ sixth.)

T: What other question could we ask from this division sentence?

S: $\frac{1}{2}$ is 3 of what number?

T: Also, 3 of what number makes half?

S: Three 1 sixths makes half.

T: Check your work, and then answer the question in a complete sentence.

S: Each friend will receive $\frac{1}{6}$ pan of brownies.

T: (Erase the $\frac{1}{2}$ in the problem, and replace it with $\frac{1}{3}$.) What if Nolan only has a third of a pan of brownies that he lets 3 friends share equally? How much of a pan of brownies will each friend get? Work with a partner to solve it.

S: (Work.)

T: Answer the question in a complete sentence.

S: Each friend will receive $\frac{1}{9}$ pan of brownies.

T: (Point to all the previous division sentences: $3 \div 3 = 1$, $1 \div 3 = \frac{1}{3}$, $\frac{1}{2} \div 3 = \frac{1}{6}$, and $\frac{1}{3} \div 3 = \frac{1}{9}$.) Compare our division sentences. What do you notice about the quotients? Turn and talk.

S: The answer is becoming smaller and smaller because Nolan kept giving his friends a smaller and smaller part of a pan to share. → The original whole is becoming smaller from 3 to 1, to $\frac{1}{2}$, to $\frac{1}{3}$, and the 3 people sharing the brownies stayed the same. That’s why the answer is becoming smaller.
Problem 2

\[ \frac{1}{5} \div 2 \]

T: (Post Problem 2 on the board.) Work independently to solve this problem on your personal white board. Draw a tape diagram to show your thinking.

S: (Work.)

T: What’s the answer?
S: \( \frac{1}{10} \)
T: How many tenths are in 1 fifth?
S: 2 tenths.
T: (Write 2 tenths \( \div 2 = \_\_\_) \) What’s 2 tenths divided by 2?
S: 1 tenth. (Write 2 tenths \( \div 2 = 1 \) tenth.)
T: Let’s try another way: (Write \( \frac{1}{5} = 2 \times \_\_\_\_\_. \) ) Fill in the missing factor.
S: 1 tenth.
T: Let’s check our work aloud together. What is the quotient?
S: 1 tenth.
T: The divisor?
S: 2.
T: Let’s multiply the quotient by the divisor. What is 1 tenth times 2?
S: 2 tenths.
T: Is 2 tenths the same unit as our original whole?
S: No.
T: Did we make a mistake?
S: No, 2 tenths is just another way to say 1 fifth.
T: Say 2 tenths in its simplest form.
S: 1 fifth.
Problem 3

If Melanie pours \( \frac{1}{2} \) liter of water into 4 bottles, putting an equal amount in each, how many liters of water will be in each bottle?

T: (Post Problem 3 on the board and read it with the class.) How many liters of water does Melanie have?

\[ \frac{1}{2} \div 4 = \frac{1}{8} \]

S: Half a liter.

T: Half a liter is being poured into how many bottles?

S: 4 bottles.

T: How do you solve this problem? Turn and discuss.

S: We have to divide. \( \rightarrow \) The division expression is \( \frac{1}{2} \div 4 \). \( \rightarrow \) I need to divide the dividend 1 half by the divisor 4. \( \rightarrow \) I can draw 1 half and cut it into 4 equal parts. \( \rightarrow \) I can think of this as \( \frac{1}{2} \div 4 = \frac{1}{8} \times \_ \).

T: On your personal white board, draw a tape diagram and solve this problem independently.

S: (Work.)

T: Say the division sentence and the answer.

S: \( \frac{1}{2} \div 4 = \frac{1}{8} \). (Write \( \frac{1}{2} \div 4 = \frac{1}{8} \).)

T: Now, say the division sentence using eighths and unit form.

S: 4 eighths \( \div 4 = 1 \) eighth.

T: Show me your checking solution.

S: (Work and show \( \frac{1}{8} \times 4 = \frac{4}{8} = \frac{1}{2} \).)

T: If you used a multiplication sentence with a missing factor, say it now.

S: \( \frac{1}{2} = 4 \times \frac{1}{8} \).

T: No matter your strategy, we all got the same result. Answer the question in a complete sentence.

S: Each bottle will have \( \frac{1}{8} \) liter of water.
Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Divide a unit fraction by a whole number.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, what is the relationship between (a) and (b), (c) and (d), and (b) and (d)?
- Why is the quotient of Problem 1(c) greater than that of Problem 1(d)? Is it reasonable? Explain your thinking to your partner.
- In Problem 2, what is the relationship between (c) and (d) and (b) and (f)?
- Compare your drawing for Problem 3 with that of a partner. How is it the same as or different from your partner’s?
- How did you solve Problem 5? Share your solution with and explain your strategy to a partner.
While the invert and multiply strategy is not explicitly taught (nor should it be while students grapple with these abstract concepts of division), discussing various ways of thinking about division in general can be fruitful. A discussion might proceed as follows:

T: Is dividing something by 2 the same as taking 1 half of it? For example, is \( \frac{4}{2} = 4 \times \frac{1}{2} \) ? (Write this on the board and allow some quiet time for thinking.) Can you think of some examples?

S: Yes. → If 4 cookies are divided between 2 people, each person receives half of the cookies.

T: So, if that’s true, would this also be true:

\[ \frac{\frac{1}{4}}{2} = \frac{1}{4} \times \frac{1}{2} \]

(Write and allow quiet time.) Can you think of some examples?

S: Yes. → If there is only 1 fourth of a candy bar and 2 people share it, they would each receive half of the fourth. That would be 1 eighth of the whole candy bar.

Once this idea is introduced, search for opportunities in visual models to highlight it. For example, in today’s lesson, Problem 3’s tape diagram was drawn to show \( \frac{1}{2} \) divided into 4 equal parts. The model highlights that the answer is \( \frac{1}{4} \) of that \( \frac{1}{2} \), an answer that can be obtained by multiplying \( \frac{1}{4} \times \frac{1}{2} \).

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students’ understanding of the concepts that were presented in today’s lesson and planning more effectively for future lessons. The questions may be read aloud to the students.
1. Draw a model or tape diagram to solve. Use the thought bubble to show your thinking. Write your quotient in the blank. Use the example to help you.

Example: \( \frac{1}{2} \div 3 \)

\[ \frac{1}{2} \div 3 = \frac{1}{6} \]

\[ \frac{1}{2} \div 3 = \frac{1}{6} \]

a. \( \frac{1}{3} \div 2 = \) ________

b. \( \frac{1}{3} \div 4 = \) ________
2. Divide. Then, multiply to check.

<table>
<thead>
<tr>
<th>a. 1/2 ÷ 7</th>
<th>b. 1/3 ÷ 6</th>
<th>c. 1/4 ÷ 5</th>
<th>d. 1/5 ÷ 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>e. 1/5 ÷ 2</td>
<td>f. 1/6 ÷ 3</td>
<td>g. 1/8 ÷ 2</td>
<td>h. 1/10 ÷ 10</td>
</tr>
</tbody>
</table>
3. Tasha eats half her snack and gives the other half to her two best friends for them to share equally. What portion of the whole snack does each friend get? Draw a picture to support your response.

4. Mrs. Appler used \( \frac{1}{2} \) gallon of olive oil to make 8 identical batches of salad dressing.
   a. How many gallons of olive oil did she use in each batch of salad dressing?
   b. How many cups of olive oil did she use in each batch of salad dressing?
5. Mariano delivers newspapers. He always puts $\frac{3}{4}$ of his weekly earnings in his savings account and then divides the rest equally into 3 piggy banks for spending at the snack shop, the arcade, and the subway.

a. What fraction of his earnings does Mariano put into each piggy bank?

b. If Mariano adds $2.40 to each piggy bank every week, how much does Mariano earn per week delivering papers?
Lesson 26 Exit Ticket

Name _______________________________ Date __________________

1. Solve. Support at least one of your answers with a model or tape diagram.

   a. \( \frac{1}{2} \div 4 = \) _____

   b. \( \frac{1}{8} \div 5 = \) _____

2. Larry spends half of his workday teaching piano lessons. If he sees 6 students, each for the same amount of time, what fraction of his workday is spent with each student?
1. Solve and support your answer with a model or tape diagram. Write your quotient in the blank.
   
   a. \( \frac{1}{2} \div 4 = \) _____  
   b. \( \frac{1}{3} \div 6 = \) _____
   
   c. \( \frac{1}{4} \div 3 = \) _____  
   d. \( \frac{1}{5} \div 2 = \) _____

2. Divide. Then, multiply to check.

   a. \( \frac{1}{2} \div 10 \)  
   b. \( \frac{1}{4} \div 10 \)  
   c. \( \frac{1}{3} \div 5 \)  
   d. \( \frac{1}{5} \div 3 \)

   e. \( \frac{1}{8} \div 4 \)  
   f. \( \frac{1}{7} \div 3 \)  
   g. \( \frac{1}{10} \div 5 \)  
   h. \( \frac{1}{5} \div 20 \)
3. Teams of four are competing in a quarter-mile relay race. Each runner must run the same exact distance. What is the distance each teammate runs?

4. Solomon has read \( \frac{1}{3} \) of his book. He finishes the book by reading the same amount each night for 5 nights.
   
   a. What fraction of the book does he read each of the 5 nights?

   b. If he reads 14 pages on each of the 5 nights, how long is the book?