Lesson 27: Real-World Volume Problems

Classwork

Example 1

A swimming pool holds 10,000 ft$^3$ of water when filled. Jon and Anne want to fill the pool with a garden hose. The garden hose can fill a five-gallon bucket in 30 seconds. If each cubic foot is about 7.5 gallons, find the flow rate of the garden hose in gallons per minute and in cubic feet per minute. About how long will it take to fill the pool with a garden hose? If the hose is turned on Monday morning at 8:00 a.m., approximately when will the pool be filled?

Example 2

A square pipe (a rectangular prism-shaped pipe) with inside dimensions of 2 in. × 2 in. has water flowing through it at a flow speed of $\frac{3}{5}$ ft/s. The water flows into a pool in the shape of a right triangular prism, with a base in the shape of a right isosceles triangle and with legs that are each 5 feet in length. How long will it take for the water to reach a depth of 4 feet?
Exercise 1

A park fountain is about to be turned on in the spring after having been off all winter long. The fountain flows out of the top level and into the bottom level until both are full, at which point the water is just recycled from top to bottom through an internal pipe. The outer wall of the top level, a right square prism, is five feet in length; the thickness of the stone between outer and inner wall is 1 ft.; and the depth is 1 ft. The bottom level, also a right square prism, has an outer wall that is 11 ft. long with a 2 ft. thickness between the outer and inner wall and a depth of 2 ft. Water flows through a 3 in. × 3 in. square pipe into the top level of the fountain at a flow speed of $4 \frac{ft}{s}$. Approximately how long will it take for both levels of the fountain to fill completely?
Exercise 2

A decorative bathroom faucet has a 3 in. × 3 in. square pipe that flows into a basin in the shape of an isosceles trapezoid prism like the one shown in the diagram. If it takes one minute and twenty seconds to fill the basin completely, what is the approximate speed of water flowing from the faucet in feet per second?
Lesson Summary

The formulas $V = Bh$ and $V = rt$, where $r$ is flow rate, can be used to solve real-world volume problems involving flow speed and flow rate. For example, water flowing through a square pipe can be visualized as a right rectangular prism. If water is flowing through a 2 in. $\times$ 2 in. square pipe at a flow speed of $4 \frac{ft}{s}$, then for every second the water flows through the pipe, the water travels a distance of 4 ft. The volume of water traveling each second can be thought of as a prism with a 2 in. $\times$ 2 in. base and a height of 4 ft. The volume of this prism is:

$$V = Bh$$
$$= \frac{1}{6} \text{ ft} \times \frac{1}{6} \text{ ft} \times 4 \text{ ft}$$
$$= \frac{1}{9} \text{ ft}^3$$

Therefore, $\frac{1}{9} \text{ ft}^3$ of water flows every second, and the flow rate is $\frac{1}{9} \frac{\text{ft}^3}{\text{s}}$.

Problem Set

1. Harvey puts a container in the shape of a right rectangular prism under a spot in the roof that is leaking. Rainwater is dripping into the container at an average rate of 12 drops a minute. The container Harvey places under the leak has a length and width of 5 cm and a height of 10 cm. Assuming each raindrop is roughly 1 cm$^3$, approximately how long does Harvey have before the container overflows?

2. A large square pipe has inside dimensions 3 in. $\times$ 3 in., and a small square pipe has inside dimensions 1 in. $\times$ 1 in. Water travels through each of the pipes at the same constant flow speed. If the large pipe can fill a pool in 2 hours, how long will it take the small pipe to fill the same pool?

3. A pool contains 12,000 ft$^3$ of water and needs to be drained. At 8:00 a.m., a pump is turned on that drains water at a flow rate of 10 ft$^3$ per minute. Two hours later, at 10:00 a.m., a second pump is activated that drains water at a flow rate of 8 ft$^3$ per minute. At what time will the pool be empty?

4. In the previous problem, if water starts flowing into the pool at noon at a flow rate of 3 ft$^3$ per minute, how much longer will it take to drain the pool?

5. A pool contains 6,000 ft$^3$ of water. Pump A can drain the pool in 15 hours, Pump B can drain it in 12 hours, and Pump C can drain it in 10 hours. How long will it take all three pumps working together to drain the pool?

6. A 2,000-gallon fish aquarium can be filled by water flowing at a constant rate in 10 hours. When a decorative rock is placed in the aquarium, it can be filled in 9.5 hours. Find the volume of the rock in cubic feet (1 ft$^3 = 7.5$ gal.)