

## Lesson 12: Unique Triangles—Two Sides and a Non-Included Angle

### Classwork

#### Exploratory Challenge

1. Use your tools to draw  $\triangle ABC$  in the space below, provided  $AB = 5$  cm,  $BC = 3$  cm, and  $\angle A = 30^\circ$ . Continue with the rest of the problem as you work on your drawing.

- a. What is the relationship between the given parts of  $\triangle ABC$ ?
- b. Which parts of the triangle can be drawn without difficulty? What makes this drawing challenging?
- c. A ruler and compass are instrumental in determining where  $C$  is located.
- ✓ Even though the length of segment  $AC$  is unknown, extend the ray  $AC$  in anticipation of the intersection with segment  $BC$ .
  - ✓ Draw segment  $BC$  with length 3 cm away from the drawing of the triangle.
  - ✓ Adjust your compass to the length of  $\overline{BC}$ .
  - ✓ Draw a circle with center  $B$  and a radius equal to  $BC$ , or 3 cm.
- d. How many intersections does the circle make with segment  $AC$ ? What does each intersection signify?
- e. Complete the drawing of  $\triangle ABC$ .
- f. Did the results of your drawing differ from your prediction?

2. Now attempt to draw  $\triangle DEF$  in the space below, provided  $DE = 5$  cm,  $EF = 3$  cm, and  $\angle F = 90^\circ$ . Continue with the rest of the problem as you work on your drawing.

- a. How are these conditions different from those in Exercise 1, and do you think the criteria will determine a unique triangle?
- b. What is the relationship between the given parts of  $\triangle DEF$ ?
- c. Describe how you will determine the position of  $\overline{DE}$ .
- d. How many intersections does the circle make with  $\overline{FD}$ ?
- e. Complete the drawing of  $\triangle DEF$ . How is the outcome of  $\triangle DEF$  different from that of  $\triangle ABC$ ?
- f. Did your results differ from your prediction?

3. Now attempt to draw  $\triangle JKL$ , provided  $KL = 8$  cm,  $KJ = 4$  cm, and  $\angle J = 120^\circ$ . Use what you drew in Exercises 1 and 2 to complete the full drawing.

4. Review the conditions provided for each of the three triangles in the Exploratory Challenge, and discuss the uniqueness of the resulting drawing in each case.

**Lesson Summary**

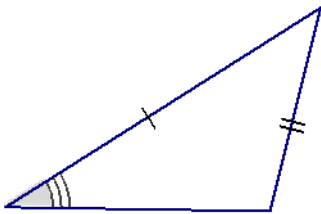
Consider a triangle correspondence  $\triangle ABC \leftrightarrow \triangle XYZ$  that corresponds to two pairs of equal sides and one pair of equal, non-included angles. If the triangles are not identical, then  $\triangle ABC$  can be made to be identical to  $\triangle XYZ$  by swinging the appropriate side along the path of a circle with a radius length of that side.

A triangle drawn under the condition of two sides and a non-included angle, where the angle is  $90^\circ$  or greater, creates a unique triangle.

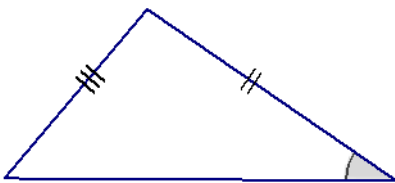
**Problem Set**

- In each of the triangles below, two sides and a non-included acute angle are marked. Use a compass to draw a nonidentical triangle that has the same measurements as the marked angle and marked sides (look at Exercise 1, part (e) of the Exploratory Challenge as a reference). Draw the new triangle on top of the old triangle. What is true about the marked angles in each triangle that results in two non-identical triangles under this condition?

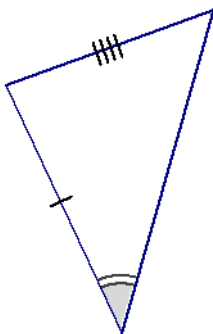
a.



b.

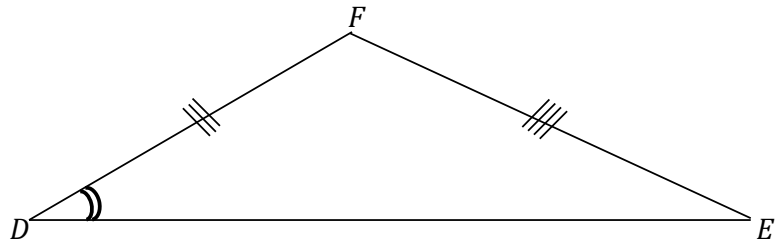
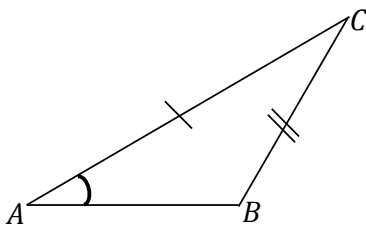


c.



2. Sometimes two sides and a non-included angle of a triangle determine a unique triangle, even if the angle is acute. In the following two triangles, copy the marked information (i.e., two sides and a non-included acute angle), and discover which determines a unique triangle. Measure and label the marked parts.

In each triangle, how does the length of the marked side adjacent to the marked angle compare with the length of the side opposite the marked angle? Based on your drawings, specifically state when the two sides and acute non-included angle condition determines a unique triangle.



3. A sub-condition of the two sides and non-included angle is provided in each row of the following table. Decide whether the information determines a unique triangle. Answer with a *yes*, *no*, or *maybe* (for a case that may or may not determine a unique triangle).

	Condition	Determines a Unique Triangle?
1	Two sides and a non-included $90^\circ$ angle.	
2	Two sides and an acute, non-included angle.	
3	Two sides and a non-included $140^\circ$ angle.	
4	Two sides and a non-included $20^\circ$ angle, where the side adjacent to the angle is shorter than the side opposite the angle.	
5	Two sides and a non-included angle.	
6	Two sides and a non-included $70^\circ$ angle, where the side adjacent to the angle is longer than the side opposite the angle.	

4. Choose one condition from the table in Problem 3 that does not determine a unique triangle, and explain why.
5. Choose one condition from the table in Problem 3 that does determine a unique triangle, and explain why.