Lesson 2: Generating Equivalent Expressions

Classwork

Opening Exercise

Additive inverses have a sum of zero. Fill in the center column of the table with the opposite of the given number or expression, then show the proof that they are opposites. The first row is completed for you.

Expression	Opposite	Proof of Opposites
1	-1	$1 + (-1) = 0$
3		
-7		
$-\frac{1}{2}$		
x		
$3x$		
$x + 3$		
$3x - 7$		

Example 1: Subtracting Expressions

a. Subtract: $(40+9)-(30+2)$.

b. Subtract: $(3x+5y-4)-(4x+11)$.

Example 2: Combining Expressions Vertically

a. Find the sum by aligning the expressions vertically.
$(5a+3b-6c)+(2a-4b+13c)$

b. Find the difference by aligning the expressions vertically.
$(2x+3y-4)-(5x+2)$

Example 3: Using Expressions to Solve Problems

A stick is x meters long. A string is 4 times as long as the stick.

a. Express the length of the string in terms of x.

b. If the total length of the string and the stick is 15 meters long, how long is the string?

Example 4: Expressions from Word Problems

It costs Margo a processing fee of $\$3$ to rent a storage unit, plus $\$17$ per month to keep her belongings in the unit. Her friend Carissa wants to store a box of her belongings in Margo's storage unit and tells her that she will pay her $\$1$ toward the processing fee and $\$3$ for every month that she keeps the box in storage. Write an expression in standard form that represents how much Margo will have to pay for the storage unit if Carissa contributes. Then, determine how much Margo will pay if she uses the storage unit for 6 months.

Example 5: Extending Use of the Inverse to Division

Multiplicative inverses have a product of 1. Find the multiplicative inverses of the terms in the first column. Show that the given number and its multiplicative inverse have a product of 1. Then, use the inverse to write each corresponding expression in standard form. The first row is completed for you.

Given	Multiplicative Inverse	Proof—Show that their product is 1.	Use each inverse to write its corresponding expression below in standard form.
3	$\frac{1}{3}$	$3\cdot\frac{1}{3}$ $\frac{3}{1}\cdot\frac{1}{3}$ $\frac{3}{3}$ 1	$12\div 3$ $12\cdot\frac{1}{3}$ 4
5			$65\div 5$
-2			$18\div(-2)$
$-\frac{3}{5}$			$6\div\left(-\frac{3}{5}\right)$
x			$5x\div x$
$2x$			$12x\div 2x$

Relevant Vocabulary

AN EXPRESSION IN EXPANDED FORM: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form.* A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, $3x$, $5x + 3 - 40$, and $x + 2x + 3x$.

TERM: Each summand of an expression in expanded form is called a *term.* For example, the expression $2x + 3x + 5$ consists of 3 terms: $2x$, $3x$, and 5.

COEFFICIENT OF THE TERM: The number found by multiplying just the numbers in a term together is called the *coefficient*. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is $8x$. The number 8 is called the coefficient of the term $8x$.

AN EXPRESSION IN STANDARD FORM: An expression in expanded form with all of its like terms collected is said to be in *standard form. For example,* $2x + 3x + 5$ *is an expression written in expanded form; however, to be written in standard form, the like terms* $2x$ *and* $3x$ *must be combined. The equivalent expression* $5x + 5$ *is written in standard form.*

Lesson Summary

- Rewrite subtraction as adding the opposite before using any order, any grouping.
- Rewrite division as multiplying by the reciprocal before using any order, any grouping.
- The opposite of a sum is the sum of its opposites.
- Division is equivalent to multiplying by the reciprocal.

Problem Set

1. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression using $x = 5$.

a. $3x + (2 - 4x)$	b. $3x + (-2 + 4x)$	c. $-3x + (2 + 4x)$
d. $3x + (-2 - 4x)$	e. $3x - (2 + 4x)$	f. $3x - (-2 + 4x)$
g. $3x - (-2 - 4x)$	h. $3x - (2 - 4x)$	i. $-3x - (-2 - 4x)$

   j. In problems (a)–(d) above, what effect does addition have on the terms in parentheses when you removed the parentheses?

   k. In problems (e)–(i), what effect does subtraction have on the terms in parentheses when you removed the parentheses?

2. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression for the given value of the variable.

a. $4y - (3 + y);\ y = 2$	b. $(2b + 1) - b;\ b = -4$	c. $(6c - 4) - (c - 3);\ c = -7$
d. $(d + 3d) - (-d + 2);\ d = 3$	e. $(-5x - 4) - (-2 - 5x);\ x = 3$	f. $11f - (-2f + 2);\ f = \frac{1}{2}$
g. $-5g + (6g - 4);\ g = -2$	h. $(8h - 1) - (h + 3);\ h = -3$	i. $(7 + w) - (w + 7); w = -4$
j. $(2g + 9h - 5) - (6g - 4h + 2); g = -2$ and $h = 5$		

3. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $-3(8x);\ x = \frac{1}{4}$	b. $5 \cdot k \cdot (-7);\ k = \frac{3}{5}$	c. $2(-6x) \cdot 2;\ x = \frac{3}{4}$
d. $-3(8x) + 6(4x);\ x = 2$	e. $8(5m) + 2(3m);\ m = -2$	f. $-6(2v) + 3a(3);\ v = \frac{1}{3}; a = \frac{2}{3}$

4. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $8x \div 2;\ x = -\frac{1}{4}$	b. $18w \div 6;\ w = 6$	c. $25r \div 5r;\ r = -2$
d. $33y \div 11y;\ y = -2$	e. $56k \div 2k;\ k = 3$	f. $24xy \div 6y;\ x = -2; y = 3$

5. For each problem (a)–(g), write an expression in standard form.
   a. Find the sum of $-3x$ and $8x$.
   b. Find the sum of $-7g$ and $4g + 2$.
   c. Find the difference when $6h$ is subtracted from $2h - 4$.
   d. Find the difference when $-3n - 7$ is subtracted from $n + 4$.
   e. Find the result when $13v + 2$ is subtracted from $11 + 5v$.
   f. Find the result when $-18m - 4$ is added to $4m - 14$.
   g. What is the result when $-2x + 9$ is taken away from $-7x + 2$?

6. Marty and Stewart are stuffing envelopes with index cards. They are putting x index cards in each envelope. When they are finished, Marty has 15 stuffed envelopes and 4 extra index cards, and Stewart has 12 stuffed envelopes and 6 extra index cards. Write an expression in standard form that represents the number of index cards the boys started with. Explain what your expression means.

7. The area of the pictured rectangle below is $24b$ ft^2. Its width is $2b$ ft. Find the height of the rectangle and name any properties used with the appropriate step.