



## Lesson 17: Characterize Points on a Perpendicular Bisector

### Student Outcomes

- Students understand that any point on a line of reflection is equidistant from any pair of pre-image and image points in a reflection.

### Lesson Notes

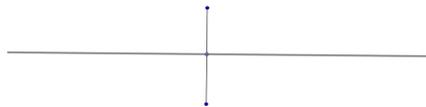
In Lesson 17, students review the three types of rigid motions previously explored and examine their relationship to the construction of perpendicular lines and, more specifically, perpendicular bisectors. They further explore the *distance preservation* characteristic of rigid motions and see how this preservation applies in each type of transformation.

### Classwork

#### Opening Exercise (4 minutes)

##### Opening Exercise

In Lesson 3, you bisected angles, including straight angles. You related the bisection of straight angles in Lesson 3 to the construction of perpendicular bisectors in Lesson 4. Review the process of constructing a perpendicular bisector with the segment below. Then complete the definition of perpendicular lines below your construction.



Use the compass and straightedge construction from Lesson 4.

Two lines are perpendicular if they intersect, and if any of the angles formed by the intersection of the lines is a right (or  $90^\circ$ ) angle. Two segments are perpendicular if the lines containing them are perpendicular.

Ask students to discuss the definition of a perpendicular bisector and how having already learned the construction helps them to know and understand the definition.

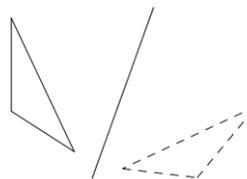
#### Discussion/Examples 1–3 (15 minutes)

##### Discussion

The line you constructed in the Opening Exercise is called the perpendicular bisector of the segment. As you learned in Lesson 14, the perpendicular bisector is also known as the line of reflection of the segment. With a line of reflection, any point on one side of the line (pre-image) is the same distance from the line as its image on the opposite side of the line.

**Example 1**

Is it possible to find or construct a line of reflection that is *not* a perpendicular bisector of a segment connecting a point on the pre-image to its image? Try to locate a line of reflection between the two figures to the right without constructing any perpendicular bisectors.



**Discussion**

Why were your attempts impossible? Look back at the definition of *reflection* from Lesson 14.

For a line  $l$  in the plane, a *reflection across  $l$*  is the transformation  $r_l$  of the plane defined as follows:

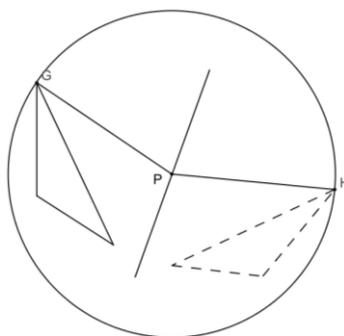
1. For any point  $P$  on the line  $l$ ,  $r_l(P) = P$ , and
2. For any point  $P$  not on  $l$ ,  $r_l(P)$  is the point  $Q$  so that  $l$  is the perpendicular bisector of the segment  $PQ$ .

The key lies in the use of the term *perpendicular bisector*. For a point  $P$  not on  $l$ , explain how to construct the point  $Q$  so that  $l$  is the perpendicular bisector of the segment  $PQ$ .

Now, let's think about the problem from another perspective. We have determined that any point on the pre-image figure is the same distance from the line of reflection as its image. Therefore, the two points are equidistant from the point at which the line of reflection (perpendicular bisector) intersects the segment connecting the pre-image point to its image. What about other points on the perpendicular bisector? Are they also equidistant from the pre-image and image points? Let's investigate.

**Example 2**

Using the same figure from the previous investigation, but with the line of reflection, is it possible to conclude that any point on the perpendicular bisector is equidistant from any pair of pre-image and image points? For example, is  $GP = HP$  in the figure? The point  $P$  is clearly *not* on the segment connecting the pre-image point  $G$  to its image  $H$ . How can you be certain that  $GP = HP$ ? If  $r$  is the reflection, then  $r(G) = H$  and  $r(P) = P$ . Since  $r$  preserves distances,  $GP = HP$ .

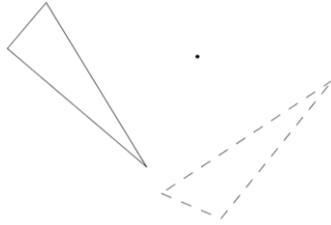


**Discussion**

We have explored perpendicular bisectors as they relate to reflections and have determined that they are essential to reflections. Are perpendicular lines, specifically, perpendicular bisectors, essential to the other two types of rigid motions: rotations and translations? Translations involve constructing parallel lines (which can certainly be done by constructing perpendiculars but are not essential to constructing parallels). However, perpendicular bisectors play an important role in rotations. In Lesson 13, we found that the intersection of the perpendicular bisectors of two segments connecting pairs of pre-image to image points determined the center of rotation.

**Example 3**

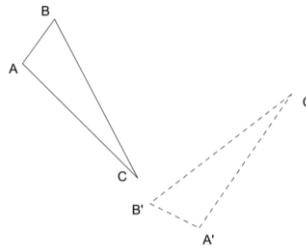
Find the center of rotation for the transformation below. How are perpendicular bisectors a major part of finding the center of rotation? Why are they essential?



*The center of rotation is the intersection of two perpendicular bisectors, each to a segment that joins a pair of corresponding points (between the figure and its image).*

As you explore this figure, also note another feature of rotations. As with all rigid motions, rotations preserve distance. A transformation is said to be distance-preserving (or length-preserving) if the distance between the images of two points is always equal to the distance between the original two points. Which of the statements below is true of the distances in the figure? Justify your response.

1.  $AB = A'B'$
2.  $AA' = BB'$



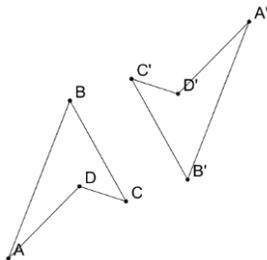
*$AB = A'B'$  is a distance in the figure. The distance between A and B remains the same after the transformation. The second statement  $AA' = BB'$  is incorrect because it involves distances between the original figure and the new figures. Rotations are distance-preserving; thus, we are only assured that distance between the images of two points is always equal to the distances between the original two points.*

**Exercises (20 minutes)**

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In each pre-image/image combination below, (a) identify the type of transformation; (b) state whether perpendicular bisectors play a role in constructing the transformation and, if so, what role; and (c) cite an illustration of the distance-preserving characteristic of the transformation (e.g., identify two congruent segments from the pre-image to the image). For the last requirement, you have to label vertices on the pre-image and image.

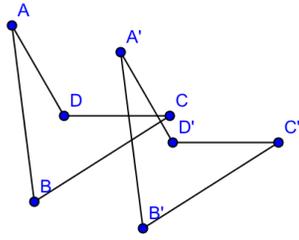
1.



Transformation	Perpendicular Bisectors?	Examples of Distance Preservation
<i>Rotation</i>	<i>Yes, to construct the center of rotation</i>	$AB = A'B'$ $DC = D'C'$

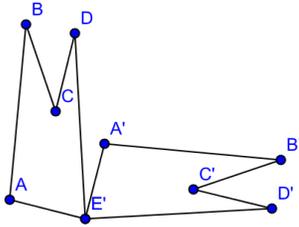
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2.



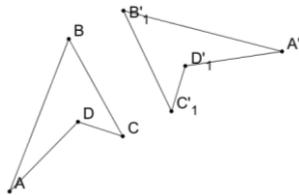
Transformation	Perpendicular Bisectors?	Examples of Distance Preservation
<i>Translation</i>	<i>No</i>	$AD = A'D'$ $DC = D'C'$

3.



Transformation	Perpendicular Bisectors?	Examples of Distance Preservation
<i>Rotation</i>	<i>In general, yes; however, not required in this case</i>	$AB = A'B'$ $BC = B'C'$

4.

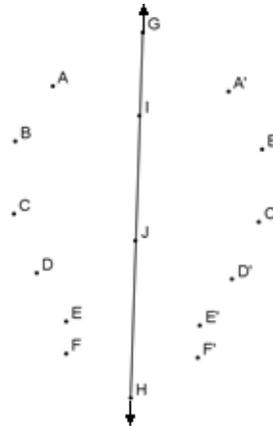


Transformation	Perpendicular Bisectors?	Examples of Distance Preservation
<i>Reflection</i>	<i>Yes, to construct the line of reflection</i>	$BC = B'C'$ $AD = A'D'$

5. In the figure to the right,  $\overleftrightarrow{GH}$  is a segment of reflection. State and justify two conclusions about distances in this figure. At least one of your statements should refer to perpendicular bisectors.

*Each point and its reflected image are the same distance from the line of reflection.*

*The line of reflection is the perpendicular bisector of the line connecting each point with its reflected image.*



**Closing (1 minute)**

- Perpendicular bisectors are essential to both reflections and rotations. The perpendicular bisector is the line of reflection or line of symmetry in a reflection. In a rotation, the intersection of the perpendicular bisectors of two segments connecting pairs of pre-image to image points determine the center of rotation.

**Exit Ticket (5 minutes)**

Name \_\_\_\_\_

Date \_\_\_\_\_

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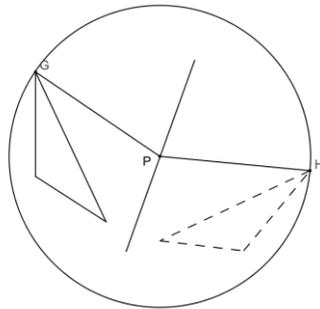
### Exit Ticket

Using your understanding of rigid motions, explain why *any* point on the perpendicular bisector is equidistant from *any* pair of pre-image and image points. Use your construction tools to create a figure that supports your explanation.

## Exit Ticket Sample Solutions

Using your understanding of rigid motions, explain why *any* point on the perpendicular bisector is equidistant from *any* pair of pre-image and image points. Use your construction tools to create a figure that supports your explanation.

If  $r$  is the reflection, then  $r(G) = H$  and  $r(P) = P$ . Since  $r$  preserves distances,  $GP = HP$ .



## Problem Set Sample Solutions

Create/construct two problems involving transformations—one reflection and one rotation—that require the use of perpendicular bisectors. Your reflection problem may require locating the line of reflection or using the line of reflection to construct the image. Your rotation problem should require location of the point of rotation. (Why should your rotation problem *not* require construction of the rotated image?) Create the problems on one page, and construct the solutions on another. Another student will be solving your problems in the next class period.

*Answers will vary.*