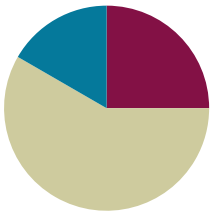
Lesson 16

Objective: Explore part-to-whole relationships.

Suggested Lesson Structure



- Fluency Practice (15 minutes)
- Concept Development (35 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)

Fluency Practice (15 minutes)

- Break Apart the Whole **4.NF.3c** (5 minutes)
- Make a Like Unit **5.NF.1** (5 minutes)
- Add Fractions with Answers Greater than 1 **5.NF.1** (5 minutes)

Break Apart the Whole (5 minutes)

Materials: (S) Personal white board

Note: Students decompose fractions greater than 1 (i.e., improper fractions) into a whole and a fraction in preparation for today's Concept Development.

T: I'll give you a fraction greater than one, and you'll break out the whole by writing the addition fraction sentence. For example, I say $\frac{3}{2}$, and you write $1 + \frac{1}{2}$. (The teacher can also ask students to write out the whole number fraction plus fraction, i.e., $\frac{2}{2} + \frac{1}{2}$.)

T: $\frac{4}{3}$.

S: (Write $1 + \frac{1}{3}$.)

T: $\frac{7}{5}$.

S: (Write $1 + \frac{2}{5}$.)

T: $\frac{19}{17}$.

S: (Write $1 + \frac{2}{17}$.)

T: $\frac{13}{3}$.

S: (Write $4 + \frac{1}{3}$.)

T: $\frac{31}{6}$.

S: (Write $5 + \frac{1}{6}$.)

T: (Continue with a sequence appropriate for students.) Share with your partner. What's your strategy of breaking out the whole?

S: (Share.)

T: Excellent!

Make a Like Unit (5 minutes)

Materials: (S) Personal white board

Note: Students make like units, which is a prerequisite skill for advanced work with fractions.

T: What does *like unit* mean?

S: When you add or subtract fractions, if the denominators are the same, then they are like units.

T: Tell your partner how we find like units.

S: (Share.)

T: I'll say two numbers. You make a like unit and write it on your personal white board.

T: 3 and 2.

S: (Write and show 6.)

Continue with the following possible sequence: 4 and 3; 2 and 4; 2 and 6; 3 and 9; 3 and 12; 4 and 8; 6 and 8.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students may provide the smallest like unit; others may not. Accept a range of answers. Notice which students consistently do not show the smallest or easiest like unit. It may be that they need extra support.

Add Fractions with Answers Greater than 1 (5 minutes)

Materials: (S) Personal white board

Note: Students recognize and analyze fractions greater than 1 in preparation for today's problem-solving set.

T: I'll say an equation. You write and solve it. If the answer is greater than 1, put a dot next to it. Leave room to write all of the equations on your personal white board without erasing.

T: $\frac{3}{3} + \frac{1}{3} =$ ___.

S: (Show $\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$ •)

T: $\frac{2}{2} + \frac{3}{2} =$ ___.

S: (Show $\frac{2}{2} + \frac{3}{2} = \frac{5}{2}$ •)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Depending on the class, make the activity slightly more complex by eliminating the personal white board. Have students complete the activity orally.

T: $\frac{2}{4}+\frac{1}{4}=$ ___.

S: (Show $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$.)

T: (Continue, alternating between equations that have answers greater than 1 or 2 with those that don't.) What is different about answers that are greater than 1 and those that are less?

S: Answers greater than 1 all have a numerator that is greater than the denominator.

T: Some of these answers are greater than 2. Circle those.

S: (Find and circle the appropriate answers.)

T: Talk to your partner about the difference between answers that are greater than 1 and those that are greater than 2.

S: Numerators are greater than denominators in both. → Yes, but greater than 2 means the denominator has to fit inside the numerator at least twice, too.

Concept Development (35 minutes)

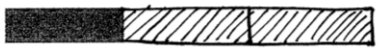
Materials: (S) Problem Set

T: Today, you are going to work in pairs to solve some ribbon and wire problems. I am going to be an observer for the most part, just listening and watching until the Debrief. You have 30 minutes to reason about and solve 3 problems. I will let you know when you have 10 minutes and then 5 minutes remaining. You can use any materials in the classroom, but I ask that you work only with your partner. The work will be scored with a rubric. Each question can earn 4 points.

- Question 1: Each correct answer including the drawing is 1 point.
- Questions 2 and 3: Clear drawing: 1 point. Labeled drawing: 1 point. Correct equation and answer: 1 point. Correct statement of your answer: 1 point. The total possible number of points is 12.

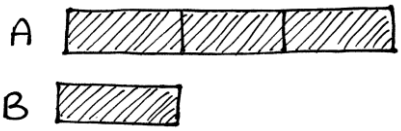
1. Draw the following ribbons. When finished, compare your work to your partner's.

   a. 1 ribbon. The piece shown below is only $\frac{1}{3}$ of the whole. Complete the drawing to show the whole ribbon.

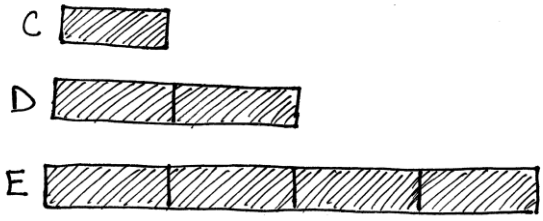


   b. 1 ribbon. The piece shown below is $\frac{4}{5}$ of the whole. Complete the drawing to show the whole ribbon.

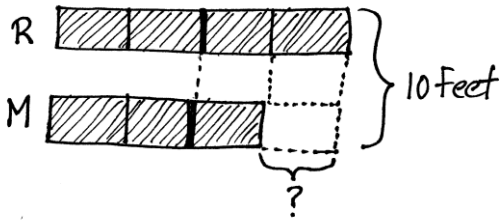
c. 2 ribbons, A and B. One third of A is equal to all of B. Draw a picture of the ribbons.



d. 3 ribbons, C, D, and E. C is half the length of D. E is twice as long as D. Draw a picture of the ribbons.



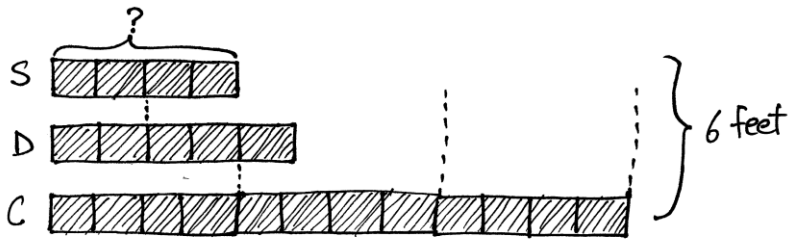
2. Half of Robert's piece of wire is equal to $\frac{2}{3}$ of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?



$10 \div 7 = \frac{10}{7}$

Robert's wire is $\frac{10}{7}$ feet longer than Maria's.

3. Half of Sarah's wire is equal to $\frac{2}{5}$ of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire in feet?



$6 \text{ ft} \div 21 = \frac{6}{21} \text{ ft}$

$\frac{6}{21} + \frac{6}{21} + \frac{6}{21} + \frac{6}{21} = \frac{24}{21} = \frac{8}{7}$

Sarah's wire is $\frac{8}{7}$ feet long.

MP.1

This lesson is an opportunity for students to *make sense of problems and persevere in solving them* (MP.1). Observe with as little interference as possible. For students who have language barriers, support by pairing appropriately for primary language, or provide a translation of the problem. As students work, circulate and make decisions about which work to share with the class and in what order.

Student Debrief (10 minutes)

Lesson Objective: Explore part-to-whole relationships.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

T: Let's work together now to analyze your solutions. Compare your solutions from the first page with the solutions of your neighbor to the right.

T: What surprises did you have on this first page?

S: We don't usually think about a shape being a fraction of something. We usually fold it or partition it to make a fraction, so it was new to think of something as a fraction.

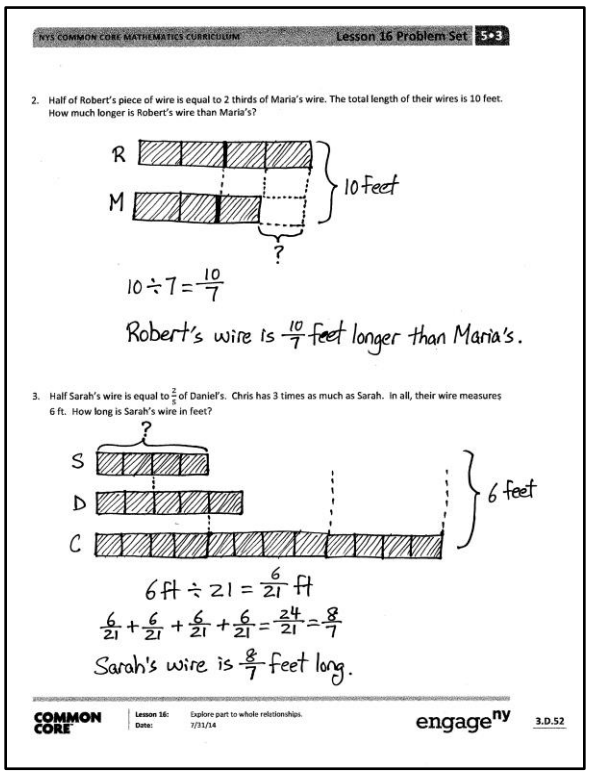
T: (After reviewing work from the first page.) Let's analyze John and Erica's work from the two story problems (Students' Work 1). Take a moment to talk to your partner about precisely what you see when you look at their work and how that relates to the questions.

T: (After students have analyzed the work.) Does anyone have a question for John and Erica?

S: Can you explain how you got 21 units in Problem 3?

S: (Authors explain.)

Students' Work 1: John and Erica


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 16 Problem Set 5•3

2. Half of Robert's piece of wire is equal to 2 thirds of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

R
M
10 feet
?

$10 \div 7 = \frac{10}{7}$

Robert's wire is $\frac{10}{7}$ feet longer than Maria's.

3. Half Sarah's wire is equal to $\frac{2}{5}$ of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire in feet?

?
S
D
C
6 feet

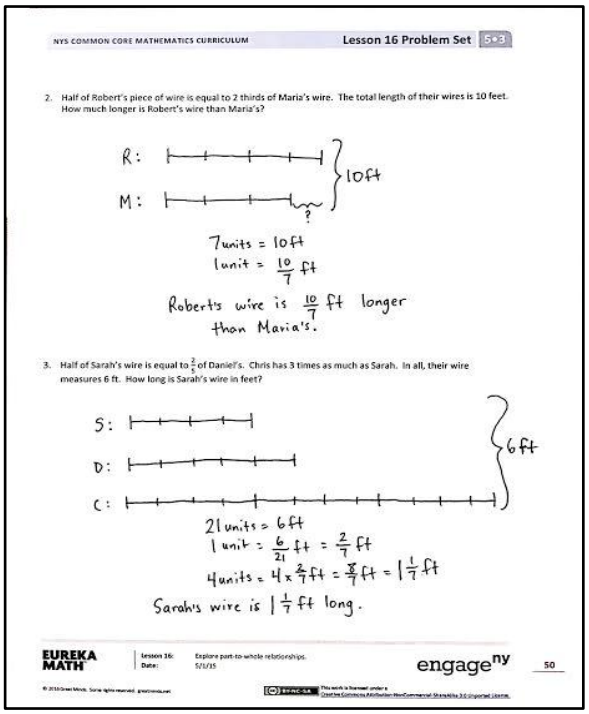
$6 \text{ ft} \div 21 = \frac{6}{21} \text{ ft}$

$\frac{6}{21} + \frac{6}{21} + \frac{6}{21} = \frac{24}{21} = \frac{8}{7}$

Sarah's wire is $\frac{8}{7}$ feet long.

COMMON CORE Lesson 16: Explore part to whole relationships. Date: 7/31/14 engage^ny 3.D.52

Students' work #2: Jacqueline and Perry


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 16 Problem Set 5•3

2. Half of Robert's piece of wire is equal to 2 thirds of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

R:
M:
10ft
?

7 units = 10 ft
1 unit = $\frac{10}{7}$ ft

Roberts wire is $\frac{10}{7}$ ft longer than Maria's.

3. Half of Sarah's wire is equal to $\frac{2}{5}$ of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire in feet?

S:
D:
C:
6ft

21 units = 6 ft
1 unit = $\frac{6}{21}$ ft = $\frac{2}{7}$ ft
4 units = $4 \times \frac{2}{7}$ ft = $\frac{8}{7}$ ft = $1\frac{1}{7}$ ft

Sarah's wire is $1\frac{1}{7}$ ft long.

EUREKA MATH Lesson 16: Explore part-to-whole relationships. Date: 5/1/15 engage^ny 50

T: Retell the explanation to your partner in your own words.
T: (After time for conversation.) Let's compare their work with Jacqueline and Perry's (Students' Work 2). First, analyze the new team's work by itself for a minute.
T: (After analysis and questioning the authors.)
T: Now, let's compare these two pieces of student work. What is the same, and what is different?
S: (Discuss in pairs and at times as a whole group.)
T: Be sure to compare their work numerically, too. What number sentences did they use? How do their number sentences relate to each other's work? For example, where do we see $4 \times \frac{2}{7}$ in John and Erica's work?

Once the analysis is complete, encourage students to score their own work according to the predetermined rubric. This task brings students back to a fraction as a quotient and a quotient as a fraction. It also heightens their fraction number sense right as they are about to embark on the multiplication and division module.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Names _____ and _____ Date _____

1. Draw the following ribbons. When finished, compare your work to your partner's.

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   b. 1 ribbon. The piece shown below is $\frac{4}{5}$ of the whole. Complete the drawing to show the whole ribbon.

   c. 2 ribbons, A and B. One third of A is equal to all of B. Draw a picture of the ribbons.

   d. 3 ribbons, C, D, and E. C is half the length of D. E is twice as long as D. Draw a picture of the ribbons.

2. Half of Robert's piece of wire is equal to $\frac{2}{3}$ of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

3. Half of Sarah's wire is equal to $\frac{2}{5}$ of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire in feet?

Name _____ Date _____

Draw the following ribbons.

a. 1 ribbon. The piece shown below is only $\frac{2}{3}$ of the whole. Complete the drawing to show the whole ribbon.

b. 1 ribbon. The piece shown below is $\frac{1}{4}$ of the whole. Complete the drawing to show the whole ribbon.

c. 3 ribbons, A, B, and C. 1 third of A is the same length as B. C is half as long as B. Draw a picture of the ribbons.

Name _____ Date _____

Draw the following roads.

a. 1 road. The piece shown below is only $\frac{3}{7}$ of the whole. Complete the drawing to show the whole road.

b. 1 road. The piece shown below is $\frac{1}{6}$ of the whole. Complete the drawing to show the whole road.

c. 3 roads, A, B, and C. B is three times longer than A. C is twice as long as B. Draw the roads. What fraction of the total length of the roads is the length of A? If Road B is 7 miles longer than Road A, what is the length of Road C?

d. Write your own road problem with 2 or 3 lengths.