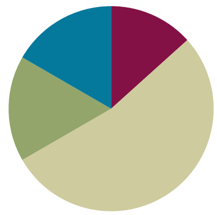
Lesson 11

Objective: Subtract fractions making like units numerically.

Suggested Lesson Structure

Fluency Practice	(8 minutes)
Application Problem	(10 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (8 minutes)

- Subtracting Fractions from Whole Numbers **4.NF.3a** (5 minutes)
- Adding and Subtracting Fractions with Like Units **4.NF.3c** (3 minutes)

Subtracting Fractions from Whole Numbers (5 minutes)

Note: This mental math fluency activity strengthens part–part–whole understanding as it relates to fractions and mixed numbers.

T: I'll say a subtraction sentence. You say the subtraction sentence with the answer. 1 – 1 half.

S: 1 – 1 half = 1 half.

T: 2 – 1 half.

S: 2 – 1 half = 1 and 1 half.

T: 3 – 1 half.

S: 3 – 1 half = 2 and 1 half.

T: 7 – 1 half.

S: 7 – 1 half = 6 and 1 half.

Continue with the following possible sequence:

$1-\frac{1}{3}$, $1-\frac{2}{3}$, $2-\frac{2}{3}$, $2-\frac{1}{3}$, $5-\frac{1}{4}$, and $5-\frac{3}{4}$.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

If students struggle to answer any fluency activity verbally, the teacher can always scaffold the activity by writing what is said. Teachers may consider an alternative that includes drawing on personal white boards, as outlined here:

T: Draw 2 units. (Students draw.)

T: Subtract 1 half. Are we subtracting $\frac{1}{2}$ of 1 unit or both units?

S: Half of 1 unit!

T: Write the number sentence.

S: (Write $2-\frac{1}{2}=1\frac{1}{2}$.)

Adding and Subtracting Fractions with Like Units (3 minutes)

Note: This fluency activity reviews adding and subtracting like units mentally.

T: I'll say an addition or subtraction sentence. You say the answer. 3 sevenths + 1 seventh.

S: 4 sevenths.

T: 3 sevenths – 1 seventh.

S: 2 sevenths.

T: 3 sevenths + 3 sevenths.

S: 6 sevenths.

T: 3 sevenths – 3 sevenths.

S: 0.

T: 4 sevenths + 3 sevenths.

S: 1.

T: I'll write an addition sentence. You say true or false. (Write $\frac{2}{5} + \frac{2}{5} = \frac{4}{10}$.)

S: False.

T: Say the sum that makes the addition sentence true.

S: 2 fifths + 2 fifths = 4 fifths.

T: (Write $\frac{5}{8} + \frac{3}{8} = 1$.)

S: True.

T: (Write $\frac{5}{6} + \frac{1}{6} = \frac{6}{12}$.)

S: False.

T: Say the sum that makes the addition sentence true.

S: 5 sixths + 1 sixth = 1.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Assign extension problems to students working above grade level. For example, assign fraction addition and subtraction problems that include the largest fractional unit:

True or false?

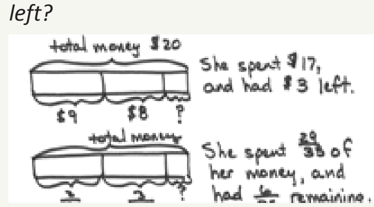
$$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$$\frac{4}{8} + \frac{2}{8} = \frac{3}{4}$$

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

The language of whole numbers is much more familiar to English language learners and students working below grade level. Possibly start by presenting the Application Problem with whole numbers.

Meredith went to the movies. She spent $9 of her money on a movie and $8 of her money on popcorn.
How much money did she spend?
If she started with $20, how much is left?



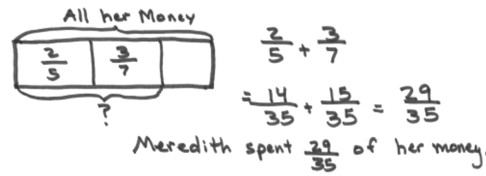
Application Problem (10 minutes)

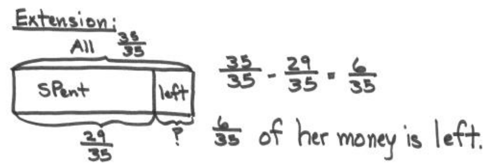
Meredith went to the movies. She spent $\frac{2}{5}$ of her money on a ticket and $\frac{3}{7}$ of her money on popcorn. How much of her money did she spend? (Extension: How much of her money is left?)

T: Talk with your partner for 30 seconds about strategies to solve this problem. What equation will you use? (Circulate and listen to student responses.)

T: Jackie, will you share?

S: I thought about when I go to the movies and buy a ticket and popcorn. I have to add those two things. So, I am going to add to solve this problem.





T: Good. David, can you expand on Jackie's comment with your strategy?

S: The units don't match. I need to make like units first, and then I can add the price of the ticket and popcorn together.

T: Nice observation. You have 90 seconds to work with your partner to solve this problem.

S: (Work.)

T: Using the strategies that we learned about adding fractions with unlike units, how can I make like units from fifths and sevenths?

S: Multiply 2 fifths by 7 sevenths, and multiply 3 sevenths by 5 fifths.

T: Everyone, say your addition sentence with your new like units.

S: 14 thirty-fifths plus 15 thirty-fifths equals 29 thirty-fifths.

T: Please share a sentence about the money Meredith spent.

S: Meredith spent 29 thirty-fifths of her money at the theater.

T: Is 29 thirty-fifths more than or less than a whole? How do you know?

S: Less than a whole because the numerator is less than the denominator.

T: (If time allows.) Did anyone answer the extension question?

S: Yes! Her total money would be $\frac{35}{35}$.
She spent $\frac{29}{35}$, so $\frac{6}{35}$ is left.

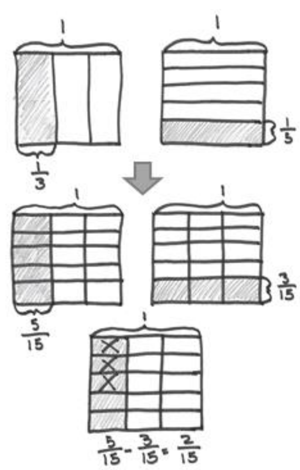
Note: Students solve this Application Problem by making like units numerically to add. This problem also serves as an introduction to this lesson's topic of making like units numerically to subtract.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

The vignette demonstrated before Problem 1 in the Concept Development uses a conceptual model for finding like units in order to subtract. If students do not need this review, move directly to Problem 1.

Concept Development (32 minutes)

Materials: (S) Personal white board



T: (Write $\frac{1}{3} - \frac{1}{5}$ on the board.) Look at this problem. Tell your partner how you might solve it. (Give 30 seconds for discussion.)

S: I would draw two fraction models. First, I would divide one whole into thirds and bracket $\frac{1}{3}$. Then, I would horizontally divide the other whole into fifths and bracket $\frac{1}{5}$. Then, I would divide both wholes the way the other was divided. That way, I would create like units. Finally, I could subtract.

T: What is a like unit for thirds and fifths?

S: Fifteenths.

T: Since we know how to find like units for addition using an equation, let's use that knowledge to subtract using an equation instead of a picture.

Problem 1: $\frac{1}{3} - \frac{1}{5}$

T: How many fifteenths are equal to 1 third?

S: 5 fifteenths.

T: (Write the following on the board.)

$\left(\frac{1}{3} \times \frac{5}{5}\right)$ 5 times as many selected units. 5 times as many units in the whole.

T: How many fifteenths are equal to 1 fifth?

S: 3 fifteenths.

T: (Write the following on the board.)

$\left(\frac{1}{5} \times \frac{3}{3}\right)$ 3 times as many selected units. 3 times as many units in the whole.

T: (Write the following equation on the board.)

$$\left(\frac{1}{3} \times \frac{5}{5}\right) - \left(\frac{1}{5} \times \frac{3}{3}\right) = \frac{5}{15} - \frac{3}{15} =$$

T: As with addition, the equation supports what we drew in our model. Say the subtraction sentence with like units and the answer.

S: 5 fifteenths – 3 fifteenths = 2 fifteenths.

T: (As shown below, write the difference to the subtraction problem.)

$$\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$$

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Be aware of cognates—words that sound similar and have the same meaning—between English and English language learners' first languages. Related cognates for Spanish speakers, for example, are listed below:

- Fraction = fracción
- Find the sum (add) = sumar
- Numerator = numerador
- Denominator = denominador

Encourage students to listen for cognates and share them with the class. This may help English language learners actively listen, and also boost auditory comprehension as they make links between prior knowledge and new learning.

Problem 2: $\frac{3}{5} - \frac{1}{6}$

T: What do we need to multiply by to make 3 fifths into smaller units?

S: 6 sixths.

T: What do we multiply by to make 1 sixth into smaller units?

S: 5 fifths.

T: (Write the following expression on the board.)

$$\left(\frac{3}{5} \times \frac{6}{6}\right) - \left(\frac{1}{6} \times \frac{5}{5}\right)$$

T: What happened to each fraction?

S: The fractions are still equivalent, but just renamed into smaller units. → We are renaming the fractions into like units so we can subtract them. → We are partitioning our original fractions into smaller units. The value of the fraction doesn't change.

T: Say your subtraction sentence with the like units.

S: 18 thirtieths – 5 thirtieths = 13 thirtieths.

T: (As shown below, write the equation on the board.)

$$\frac{18}{30} - \frac{5}{30} = \frac{13}{30}$$

Problem 3: $1\frac{3}{4} - \frac{3}{5}$

T: What are some different ways we can solve this problem?

S: You can solve it as 2 fifths plus $\frac{3}{4}$. Just take the $\frac{3}{5}$ from 1 to get 2 fifths, and add the 3 fourths. (Shown as Method 1.) → You can subtract the fractional units, and then add the whole number. → I noticed before we started that 3 fifths is less than 3 fourths, so I changed only the fractional units to twentieths. (Shown as Method 2.) → The whole number can be represented as 4 fourths and added to 3 fourths to equal 7 fourths. Then, subtract. (Shown as Method 3.)

Method 1

$1\frac{3}{4} - \frac{3}{5}$

$\frac{5}{5}$ $\frac{3}{4}$

$= \frac{2}{5} + \frac{3}{4}$

$= \frac{8}{20} + \frac{15}{20}$

$= \frac{23}{20}$

$= 1\frac{3}{20}$

Method 2

$1\frac{3}{4} - \frac{3}{5}$

$= 1 + \frac{15}{20} - \frac{12}{20}$

$= 1\frac{3}{20}$

Method 3

$1\frac{3}{4} - \frac{3}{5}$

$\frac{7}{4} - \frac{3}{5}$

$= \frac{35}{20} - \frac{12}{20}$

$= \frac{23}{20}$

$= 1\frac{3}{20}$

Problem 4: $3\frac{3}{5} - 2\frac{1}{2}$

Method 1

$3\frac{3}{5} - 2\frac{1}{2}$

3 $\frac{3}{5}$

$= \frac{1}{2} + \frac{3}{5}$

$= \frac{5}{10} + \frac{6}{10}$

$= 1\frac{1}{10}$

Method 2

$3\frac{3}{5} - 2\frac{1}{2}$

$= 1\frac{3}{5} - \frac{1}{2}$

$= 1 + \frac{6}{10} - \frac{5}{10}$

$= 1\frac{1}{10}$

Method 3

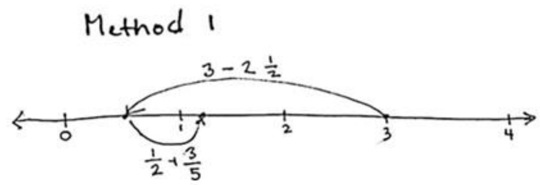
$3\frac{3}{5} - 2\frac{1}{2}$

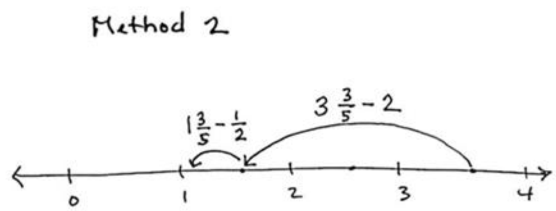
$= \frac{18}{5} - \frac{5}{2}$

$= \frac{36}{10} - \frac{25}{10}$

$= \frac{11}{10}$

$= 1\frac{1}{10}$





T: (Write the problem on the board.) Solve this problem.

S: (Solve.)

T: (Look for students who solved as above, and have them display their work. Make sure all three methods are represented.)

T: Let's confirm the reasonableness of our answers using the number line to show 2 of our methods.

MP.3

T: For Method 1, draw a number line from 0 to 4.

T: (Support students to see that they would start at 3. Subtract $2\frac{1}{2}$ and add back the $\frac{3}{5}$. As students work, circulate and observe.)

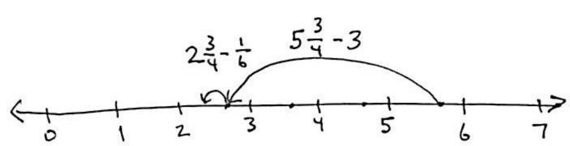
T: To show Method 2, draw your number line from 0 to 4, and then estimate the location of 3 and 3 fifths.

T: Take away 2 first, and then take away the half.

T: Discuss with your partner if the answer of $1\frac{1}{10}$ is reasonable based on both of the number lines.

Problem 5: $5\frac{3}{4} - 3\frac{1}{6}$

T: Estimate the answer first by drawing a number line. The difference between $5\frac{3}{4}$ and $3\frac{1}{6}$ is between which 2 whole numbers?



S: Three fourths is much larger than one sixth, so the answer will be between 2 and 3.

T: Will it be closer to 2 or $2\frac{1}{2}$? Discuss your thinking with a partner.

S: (Discuss.)

T: Solve this problem, and find the difference independently. (Circulate and observe as students work.)

S: (Solve.)

T: Some of you used twenty-fourths, and some of you used twelfths to solve this problem. Were your answers the same?

S: They had the same value. $\frac{14}{24}$ can be made into larger units: twelfths. The units are twice as big, so we need half as many to name an equal fraction.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Subtract fractions making like units numerically.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

T: Please take 2 minutes to check your answers with your partner.

T: I will say the subtraction problem. You say your answer out loud. Problem 1(a), 1 half – 1 third.

S: 1 sixth.

T: (Continue with the remaining problems.)

T: Take the next 2 minutes to discuss with your partner any insights you had while solving these problems.

Allow students to discuss, circulating and listening for conversations that can be shared with the whole class.

MP.7

T: Sandy, will you share your thinking about Problem 2?

S: George is wrong. He just learned a rule and thinks it is the only way. It's a good way, but you can also make eighths and sixths into twenty-fourths or ninety-sixths.

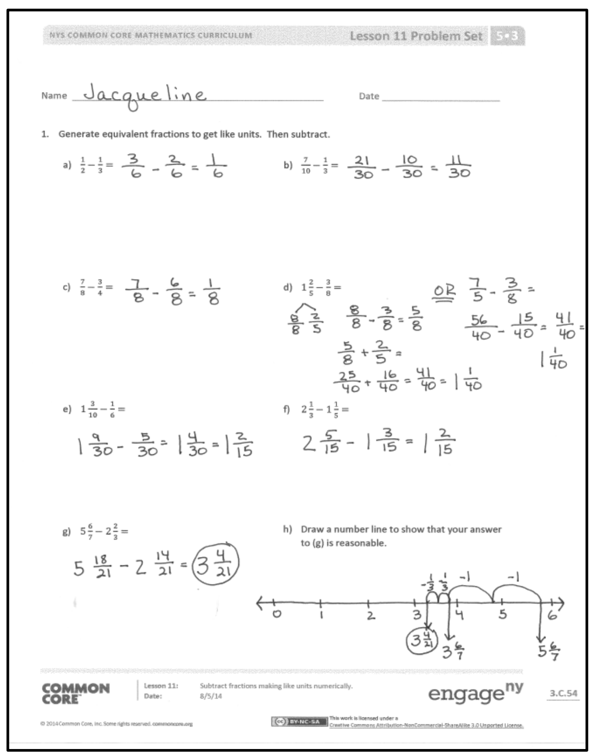
T: Discuss in pairs if there are advantages to using twenty-fourths or forty-eighths.

S: Sometimes, it's easier to multiply by the opposite denominator. → Sometimes, larger denominators just get in the way. → Sometimes, they are right. Like if you have to find the minutes, you want to keep your fraction out of 60.

S: An example of this is Problem 1(c). I didn't need to multiply both fractions. I could have just multiplied 3 fourths by 2 halves. Then, I would have had eighths as the like unit for both fractions. Then, the answer is already simplified.

T: Did anyone notice George's issue applying to any of the other problems on the Problem Set?

S: Yes, Problem 1(e). You could use sixtieths or thirtieths. → Yes, in Problem 1(e), the denominator of sixtieths is a small unit, but easy. Thirtieths are a bigger unit and a multiple of both 6 and 10. → Either common unit can be used.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Problem Set 5•3

Name Jacqueline Date

1. Generate equivalent fractions to get like units. Then subtract.

a) $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$

b) $\frac{7}{10}-\frac{1}{3}=\frac{21}{30}-\frac{10}{30}=\frac{11}{30}$

c) $\frac{7}{8}-\frac{3}{4}=\frac{7}{8}-\frac{6}{8}=\frac{1}{8}$

d) $1\frac{2}{5}-\frac{3}{8}=$ $\frac{8}{8}-\frac{3}{8}=\frac{5}{8}$ $\frac{5}{8}+\frac{2}{5}=\frac{25}{40}+\frac{16}{40}=\frac{41}{40}=1\frac{1}{40}$ OR $\frac{7}{5}-\frac{3}{8}=\frac{56}{40}-\frac{15}{40}=\frac{41}{40}=1\frac{1}{40}$

e) $1\frac{3}{10}-\frac{1}{6}=1\frac{9}{30}-\frac{5}{30}=1\frac{4}{30}=1\frac{2}{15}$

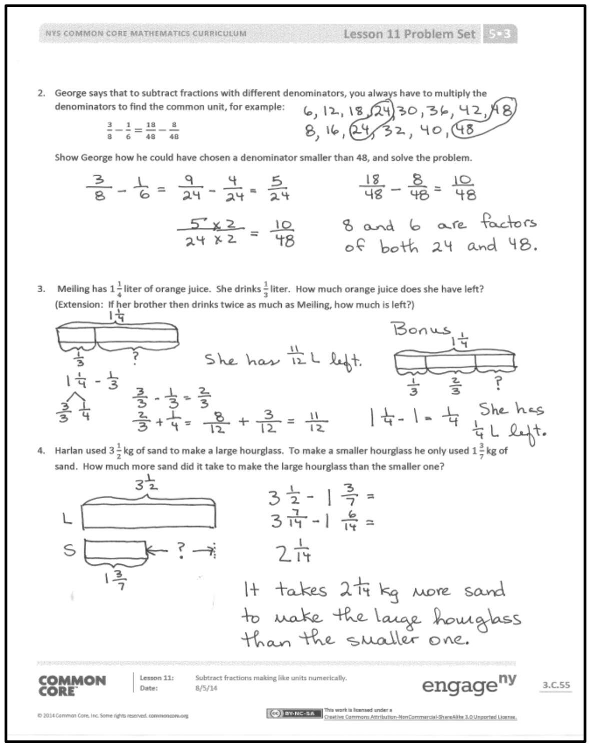
f) $2\frac{1}{3}-1\frac{1}{5}=2\frac{5}{15}-1\frac{3}{15}=1\frac{2}{15}$

g) $5\frac{6}{7}-2\frac{2}{3}=5\frac{18}{21}-2\frac{14}{21}=3\frac{4}{21}$

h) Draw a number line to show that your answer to (g) is reasonable.

T: I notice that many of you are becoming so comfortable with this equation when subtracting unlike units that you don't have to write the multiplication. You are doing it mentally. However, you still have to check your answers to see if they are reasonable. Discuss with your partner how you use mental math, and also how you make sure your methods and answers are reasonable.

S: It's true. I just look at the other denominator and multiply. It's easy. → I added instead of subtracted and wouldn't have even noticed if I hadn't checked my answer to see that it was greater than the whole amount I started with! → We are learning to find like units, and we may not always need to multiply both fractions. If I don't slow down, I won't even notice there are other choices for solving the problem. → I like choosing the strategy I want to use. Sometimes, it's easier to use the number bond method, and sometimes, it's just easier to subtract from the whole.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Problem Set 5•3

2. George says that to subtract fractions with different denominators, you always have to multiply the denominators to find the common unit, for example:

$\frac{3}{8} - \frac{1}{6} = \frac{18}{48} - \frac{8}{48}$

6, 12, 18, 24, 30, 36, 42, 48
8, 16, 24, 32, 40, 48

Show George how he could have chosen a denominator smaller than 48, and solve the problem.

$\frac{3}{8} - \frac{1}{6} = \frac{9}{24} - \frac{4}{24} = \frac{5}{24}$

$\frac{18}{48} - \frac{8}{48} = \frac{10}{48}$

$\frac{5 \times 2}{24 \times 2} = \frac{10}{48}$

8 and 6 are factors of both 24 and 48.

3. Meiling has $1\frac{1}{4}$ liter of orange juice. She drinks $\frac{1}{3}$ liter. How much orange juice does she have left? (Extension: If her brother then drinks twice as much as Meiling, how much is left?)

$1\frac{1}{4} - \frac{1}{3}$

$\frac{3}{3}$ $\frac{1}{4}$

$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

She has $\frac{11}{12}$ L left.

Bonus

$1\frac{1}{4} - 1 = \frac{1}{4}$

She has $\frac{1}{4}$ L left.

4. Harlan used $3\frac{1}{2}$ kg of sand to make a large hourglass. To make a smaller hourglass he only used $1\frac{3}{7}$ kg of sand. How much more sand did it take to make the large hourglass than the smaller one?

$3\frac{1}{2} - 1\frac{3}{7} =$

$3\frac{7}{14} - 1\frac{6}{14} =$

$2\frac{1}{14}$

It takes $2\frac{1}{14}$ kg more sand to make the large hourglass than the smaller one.

COMMON CORE Lesson 11: Subtract fractions making like units numerically. Date: 8/5/14 engage^ny 3.C.55

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name ____ Date ______________________

1. Generate equivalent fractions to get like units. Then, subtract.

a. $\frac{1}{2} - \frac{1}{3} =$

b. $\frac{7}{10} - \frac{1}{3} =$

c. $\frac{7}{8} - \frac{3}{4} =$

d. $1\frac{2}{5} - \frac{3}{8} =$

e. $1\frac{3}{10} - \frac{1}{6} =$

f. $2\frac{1}{3} - 1\frac{1}{5} =$

g. $5\frac{6}{7} - 2\frac{2}{3} =$

h. Draw a number line to show that your answer to (g) is reasonable.

2. George says that, to subtract fractions with different denominators, you always have to multiply the denominators to find the common unit; for example:

$$\frac{3}{8}-\frac{1}{6}=\frac{18}{48}-\frac{8}{48}.$$

Show George how he could have chosen a denominator smaller than 48, and solve the problem.

3. Meiling has $1\frac{1}{4}$ liter of orange juice. She drinks $\frac{1}{3}$ liter. How much orange juice does she have left? (Extension: If her brother then drinks twice as much as Meiling, how much is left?)

4. Harlan used $3\frac{1}{2}$ kg of sand to make a large hourglass. To make a smaller hourglass, he only used $1\frac{3}{7}$ kg of sand. How much more sand did it take to make the large hourglass than the smaller one?

Name ____ Date ____________________

Generate equivalent fractions to get like units. Then, subtract.

a. $\frac{3}{4} - \frac{3}{10} =$

b. $3\frac{1}{2} - 1\frac{1}{3} =$

Name ______________________________ Date ____________

1. Generate equivalent fractions to get like units. Then, subtract.

a. $\frac{1}{2} - \frac{1}{5} =$

b. $\frac{7}{8} - \frac{1}{3} =$

c. $\frac{7}{10} - \frac{3}{5} =$

d. $1\frac{5}{6} - \frac{2}{3} =$

e. $2\frac{1}{4} - 1\frac{1}{5} =$

f. $5\frac{6}{7} - 3\frac{2}{3} =$

g. $15\frac{7}{8} - 5\frac{3}{4} =$

h. $15\frac{5}{8} - 3\frac{1}{3} =$

2. Sandy ate $\frac{1}{6}$ of a candy bar. John ate $\frac{3}{4}$ of it. How much more of the candy bar did John eat than Sandy?

3. $4\frac{1}{2}$ yards of cloth are needed to make a woman's dress. $2\frac{2}{7}$ yards of cloth are needed to make a girl's dress. How much more cloth is needed to make a woman's dress than a girl's dress?

4. Bill reads $\frac{1}{5}$ of a book on Monday. He reads $\frac{2}{3}$ of the book on Tuesday. If he finishes reading the book on Wednesday, what fraction of the book did he read on Wednesday?

5. Tank A has a capacity of 9.5 gallons. $6\frac{1}{3}$ gallons of the tank's water are poured out. How many gallons of water are left in the tank?