Lesson 5: Graphs of Functions and Equations

Student Outcomes

- Students define the graph of a numerical function to be the set of all points (x, y) with x an input of the function and y its matching output.
- Students realize that if a numerical function can be described by an equation, then the graph of the function precisely matches the graph of the equation.

Classwork

Exploratory Challenge/Exercises 1–3 (15 minutes)

Students work independently or in pairs to complete Exercises 1–3.

Exploratory Challenge/Exercises 1–3

1. **The distance that Giselle can run is a function of the amount of time she spends running. Giselle runs 3 miles in 21 minutes. Assume she runs at a constant rate.**

   a. **Write an equation in two variables that represents her distance run, y, as a function of the time, x, she spends running.**

   $$\frac{3}{21} = \frac{y}{x}$$
   $$y = \frac{1}{7}x$$

   b. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 14 minutes.**

   $$y = \frac{1}{7}(14)$$
   $$y = 2$$

   Giselle can run 2 miles in 14 minutes.

   c. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 28 minutes.**

   $$y = \frac{1}{7}(28)$$
   $$y = 4$$

   Giselle can run 4 miles in 28 minutes.

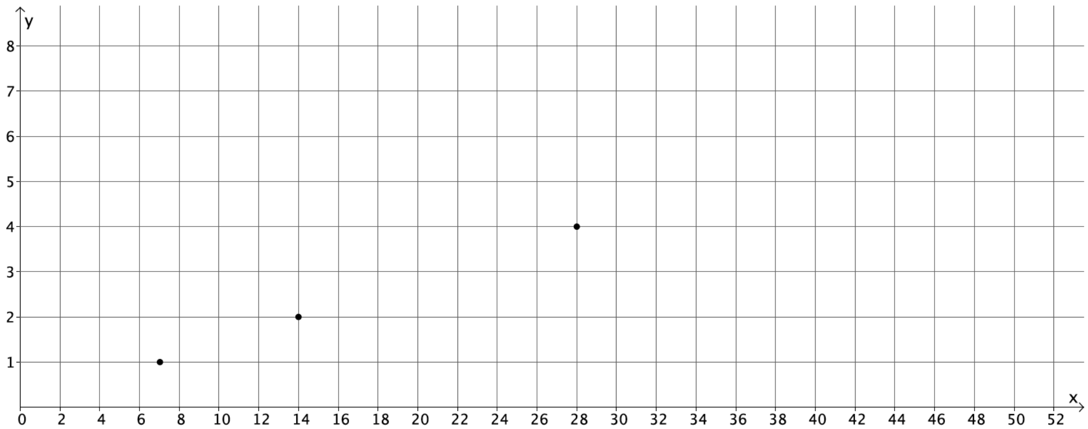
   d. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 7 minutes.**

   $$y = \frac{1}{7}(7)$$
   $$y = 1$$

   Giselle can run 1 mile in 7 minutes.

e. For a given input x of the function, a time, the matching output of the function, y, is the distance Giselle ran in that time. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.

$$(14, 2), (28, 4), (7, 1)$$



f. What do you notice about the points you plotted?

The points appear to be in a line.

g. Is the function discrete?

The function is not discrete because we can find the distance Giselle runs for any given amount of time she spends running.

h. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 36 minutes. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.

$$y = \frac{1}{7}(36)$$
$$y = \frac{36}{7}$$
$$y = 5\frac{1}{7}$$

$\left(36, 5\frac{1}{7}\right)$ *The point is where I expected it to be because it is in line with the other points.*

i. Assume you used the rule that describes the function to determine how many miles Giselle can run for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?

I think all of the points would fall on a line.

j. What do you think the graph of all the possible input/output pairs would look like? Explain.

I know the graph will be a line as we can find all of the points that represent fractional intervals of time too. We also know that Giselle runs at a constant rate, so we would expect that as the time she spends running increases, the distance she can run will increase at the same rate.

k. Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.

Answers will vary. Sample student work:

The point $(42, 6)$ is a point on the graph.

$$y = \frac{1}{7}x$$
$$6 = \frac{1}{7}(42)$$
$$6 = 6$$

The function assigns the output of 6 to the input of 42.

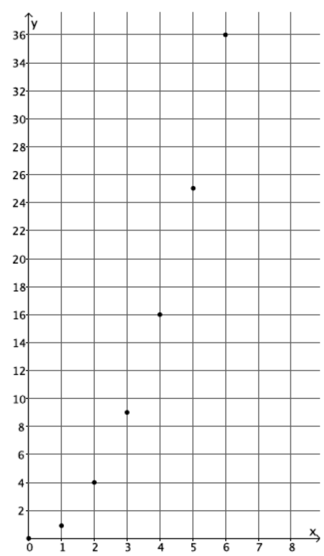
l. Sketch the graph of the equation $y = \frac{1}{7}x$ using the same coordinate plane in part (e). What do you notice about the graph of all the input/output pairs that describes Giselle's constant rate of running and the graph of the equation $y = \frac{1}{7}x$?

The graphs of the equation and the function coincide completely.

2. Sketch the graph of the equation $y = x^2$ for positive values of x. Organize your work using the table below, and then answer the questions that follow.

x	y
0	0
1	1
2	4
3	9
4	16
5	25
6	36

a. Plot the ordered pairs on the coordinate plane.



b. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

c. Is this equation a linear equation? Explain.

No, the equation $y = x^2$ is not a linear equation because the exponent of x is greater than 1.

d. Consider the function that assigns to each square of side length s units its area A square units. Write an equation that describes this function.

$$A = s^2$$

e. **What do you think the graph of all the input/output pairs (s, A) of this function will look like? Explain.**

I think the graph of input/output pairs will look like the graph of the equation $y = x^2$. The inputs and outputs would match the solutions to the equation exactly. For the equation, the y value is the square of the x value. For the function, the output is the square of the input.

f. **Use the function you wrote in part (d) to determine the area of a square with side length 2.5 units. Write the input and output as an ordered pair. Does this point appear to belong to the graph of $y = x^2$?**

$$A = (2.5)^2$$
$$A = 6.25$$

The area of the square is 6.25 units squared. $(2.5, 6.25)$ *The point looks like it would belong to the graph of $y = x^2$; it looks like it would be on the curve that the shape of the graph is taking.*

3. **The number of devices a particular manufacturing company can produce is a function of the number of hours spent making the devices. On average, 4 devices are produced each hour. Assume that devices are produced at a constant rate.**

a. **Write an equation in two variables that describes the number of devices, y, as a function of the time the company spends making the devices, x.**

$$\frac{4}{1} = \frac{y}{x}$$
$$y = 4x$$

b. **Use the equation you wrote in part (a) to determine how many devices are produced in 8 hours.**

$$y = 4(8)$$
$$y = 32$$

The company produces 32 devices in 8 hours.

c. **Use the equation you wrote in part (a) to determine how many devices are produced in 6 hours.**

$$y = 4(6)$$
$$y = 24$$

The company produces 24 devices in 6 hours.

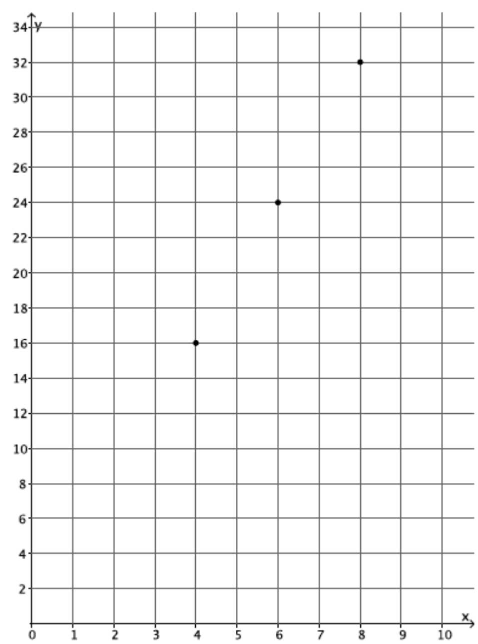
d. **Use the equation you wrote in part (a) to determine how many devices are produced in 4 hours.**

$$y = 4(4)$$
$$y = 16$$

The company produces 16 devices in 4 hours.

e. **The input of the function, x, is time, and the output of the function, y, is the number of devices produced. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.**

$(8, 32)$, $(6, 24)$, $(4, 16)$



f. What shape does the graph of the points appear to take?

The points appear to be in a line.

g. Is the function discrete?

The function is not discrete because we can find the number of devices produced for any given time, including fractions of an hour.

h. Use the equation you wrote in part (a) to determine how many devices are produced in 1.5 hours. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.

$$y = 4(1.5)$$
$$y = 6$$

$(1.5, 6)$ *The point is where I expected it to be because it is in line with the other points.*

i. Assume you used the equation that describes the function to determine how many devices are produced for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?

I think all of the points would fall on a line.

j. What do you think the graph of all possible input/output pairs will look like? Explain.

I think the graph of this function will be a line. Since the rate is continuous, we can find all of the points that represent fractional intervals of time. We also know that devices are produced at a constant rate, so we would expect that as the time spent producing devices increases, the number of devices produced would increase at the same rate.

k. Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.

Answers will vary. Sample student work:

The point $(5, 20)$ *is a point on the graph.*

$$y = 4x$$
$$20 = 4(5)$$
$$20 = 20$$

The function assigns the output of 20 *to the input of* 5.

l. Sketch the graph of the equation $y = 4x$ using the same coordinate plane in part (e). What do you notice about the graph of input/output pairs that describes the company's constant rate of producing devices and the graph of the equation $y = 4x$?

The graphs of the equation and the function coincide completely.

Discussion (10 minutes)

- What was the equation that described the function in Exercise 1, Giselle's distance run over given time intervals?
    - *The equation was* $y = \frac{1}{7}x$.
- Given an input, how did you determine the output the function would assign?
    - *We used the equation. In place of* x*, we put the input. The number that was computed was the output.*

MP.6

- So each input and its matching output correspond to a pair of numbers (x, y) that makes the equation $y = \frac{1}{7}x$ a true number sentence?
    - *Yes*

Give students a moment to make sense of this, verifying that each pair of input/output values in Exercise 1 is indeed a pair of numbers (x, y) that make $y = \frac{1}{7}x$ a true statement.

- And suppose we have a pair of numbers (x, y) that make $y = \frac{1}{7}x$ a true statement with x positive. If x is an input of the function, the number of minutes Giselle runs, would y be its matching output, the distance she covers?
    - *Yes. We computed the outputs precisely by following the equation* $y = \frac{1}{7}x$*. So* y *will be the matching output to* x.
- So can we conclude that any pair of numbers (x, y) that make the equation $y = \frac{1}{7}x$ a true number statement correspond to an input and its matching output for the function?
    - *Yes*
- And, backward, any pair of numbers (x, y) that represent an input/output pair for the function is a pair of numbers that make the equation $y = \frac{1}{7}x$ a true number statement?
    - *Yes*
- Can we make similar conclusions about Exercise 3, the function that gives the devices built over a given number of hours?

Give students time to verify that the conclusions about Exercise 3 are the same as the conclusions about Exercise 1. Then continue with the discussion.

- The function in Exercise 3 is described by the equation $y = 4x$.
- We have that the ordered pairs (x, y) that make the equation $y = 4x$ a true number sentence precisely match the ordered pairs (x, y) with x an input of the function and y its matching output.
- Recall, in previous work, we defined the *graph of an equation* to be the set of all ordered pairs (x, y) that make the equation a true number sentence. Today we define the *graph of a function* to be the set of all the ordered pairs (x, y) with x an input of the function and y its matching output.
- And our discussion today shows that if a function can be described by an equation, then the graph of the function is precisely the same as the graph of the equation.
- It is sometimes possible to draw the graph of a function even if there is no obvious equation describing the function. (Consider having students plot some points of the function that assigns to each positive whole number its first digit, for example.)

- For Exercise 2, you began by graphing the equation $y = x^2$ for positive values of x. What was the shape of the graph?
  - *It looked curved.*
- The graph had a curve in it because it was not the graph of a linear equation. All linear equations graph as lines. That is what we learned in Module 4. Since this equation was not linear, we should expect it to graph as something other than a line.
- What did you notice about the ordered pairs of the equation $y = x^2$ and the inputs and corresponding outputs for the function $A = s^2$?
  - *The ordered pairs were exactly the same for the equation and the function.*
- What does that mean about the graphs of functions, even those that are not linear?
  - *It means that the graph of a function will be identical to the graph of an equation.*

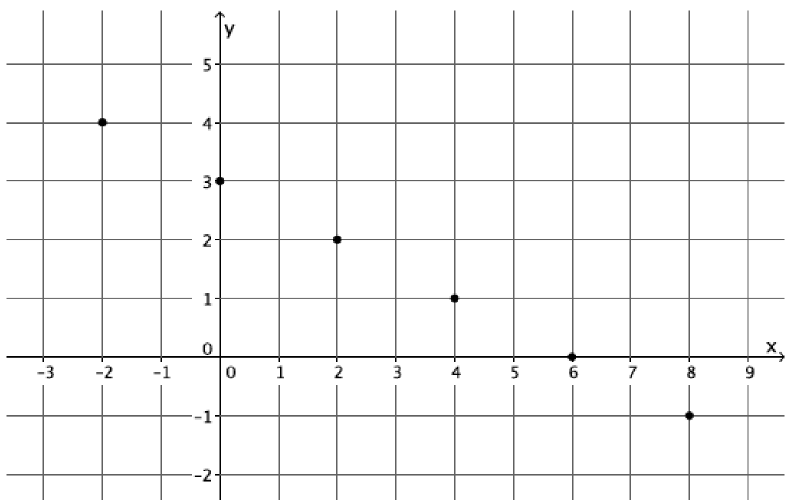
Exploratory Challenge/Exercise 4 (7 minutes)

Students work in pairs to complete Exercise 4.

Exploratory Challenge/Exercise 4

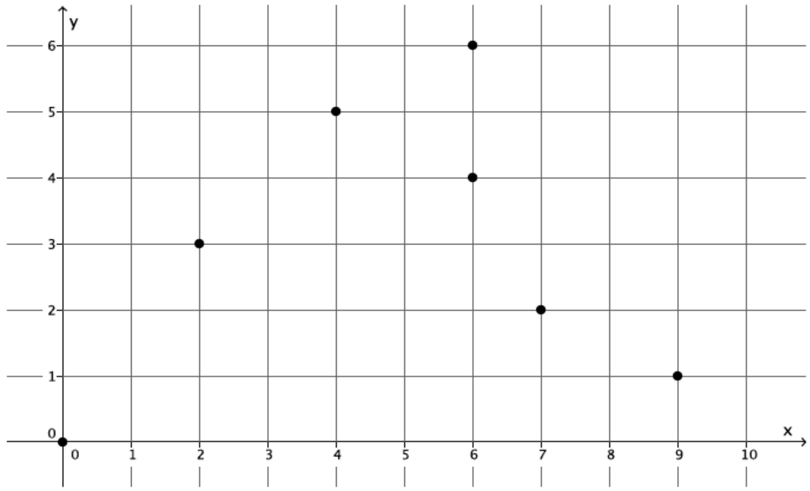
4. Examine the three graphs below. Which, if any, could represent the graph of a function? Explain why or why not for each graph.

Graph 1:



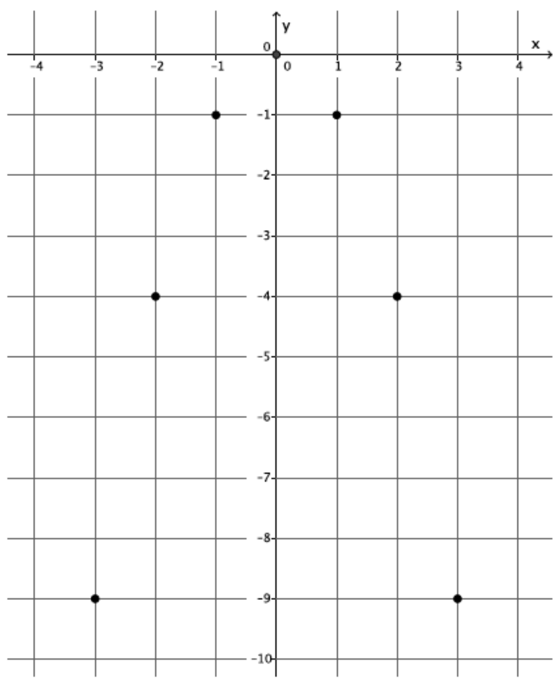
This is the graph of a function. Each input is a real number x, and we see from the graph that there is an output y to associate with each such input. For example, the ordered pair $(-2, 4)$ on the line associates the output 4 to the input -2.

Graph 2:



This is not the graph of a function. The ordered pairs $(6, 4)$ and $(6, 6)$ show that for the input of 6 there are two different outputs, both 4 and 6. We do not have a function.

Graph 3:



This is the graph of a function. The ordered pairs $(-3, -9)$, $(-2, -4)$, $(-1, -1)$, $(0, 0)$, $(1, -1)$, $(2, -4)$, and $(3, -9)$ represent inputs and their unique outputs.

Discussion (3 minutes)

- The graph of a function is the set of all points (x, y) with x an input for the function and y its matching output. How did you use this definition to determine which graphs, if any, were functions?
  - *By the definition of a function, we need each input to have only one output. On a graph, this means there cannot be two different ordered pairs with the same x value.*
- Assume the following set of ordered pairs is from some graph. Could this be the graph of a function? Explain.

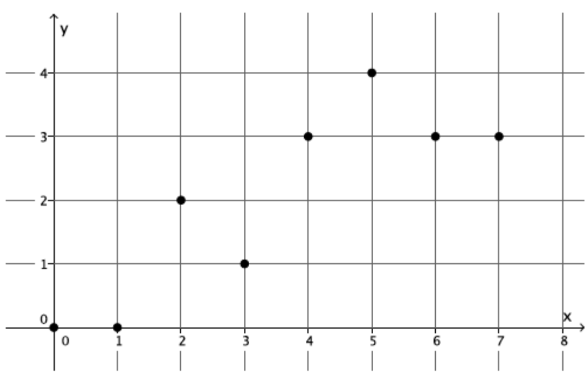
$$(3, 5), (4, 7), (3, 9), (5, -2)$$

  - *No, because the input of* 3 *has two different outputs. It does not fit the definition of a function.*
- Assume the following set of ordered pairs is from some graph. Could this be the graph of a function? Explain.

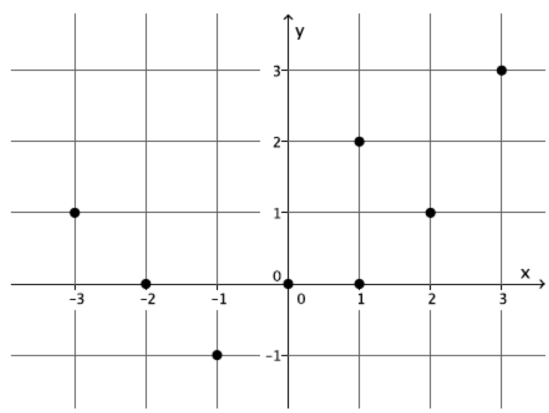
$$(-1, 6), (-3, 8), (5, 10), (7, 6)$$

  - *Yes, it is possible as each input has a unique output. It satisfies the definition of a function so far.*
- Which of the following four graphs are functions? Explain.

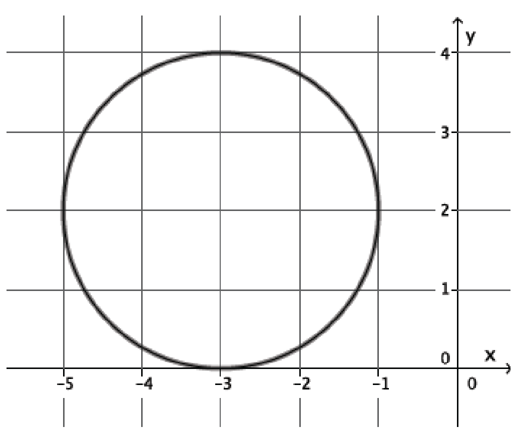
Graph 1:



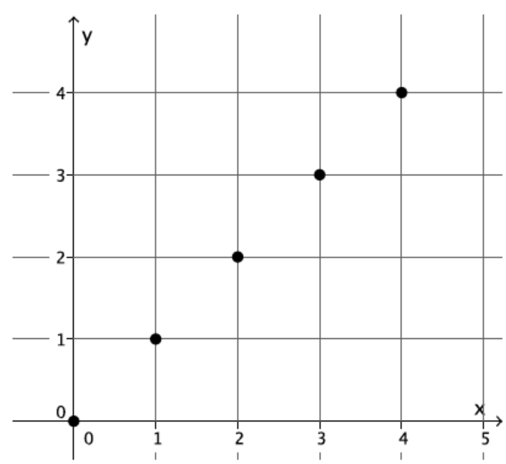
Graph 2:



Graph 3:



Graph 4:



- *Graphs 1 and 4 are functions. Graphs 2 and 3 are not. Graphs 1 and 4 show that for each input of x, there is a unique output of y. For Graph 2, the input of $x = 1$ has two different outputs, $y = 0$ and $y = 2$, which means it cannot be a function. For Graph 3, it appears that each value of x between -5 and -1, excluding -5 and -1, has two outputs, one on the lower half of the circle and one on the upper half, which means it does not fit the definition of function.*

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- The graph of a function is defined to be the set of all points (x, y) with x an input for the function and y its matching output.
- If a function can be described by an equation, then the graph of the function matches the graph of the equation (at least at points which correspond to valid inputs of the function).
- We can look at plots of points and determine if they could be the graphs of functions.

> **Lesson Summary**
>
> **The graph of a function is defined to be the set of all points (x, y) with x an input for the function and y its matching output.**
>
> **If a function can be described by an equation, then the graph of the function is the same as the graph of the equation that represents it (at least at points which correspond to valid inputs of the function).**
>
> **It is not possible for two different points in the plot of the graph of a function to have the same x-coordinate.**

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 5: Graphs of Functions and Equations

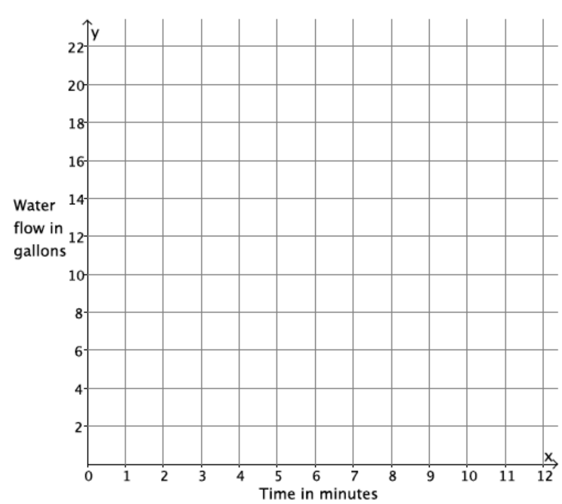
Exit Ticket

Water flows from a hose at a constant rate of 11 gallons every 4 minutes. The total amount of water that flows from the hose is a function of the number of minutes you are observing the hose.

a. Write an equation in two variables that describes the amount of water, y, in gallons, that flows from the hose as a function of the number of minutes, x, you observe it.

b. Use the equation you wrote in part (a) to determine the amount of water that flows from the hose during an 8-minute period, a 4-minute period, and a 2-minute period.

c. An input of the function, x, is time in minutes, and the output of the function, y, is the amount of water that flows out of the hose in gallons. Write the inputs and outputs from part (b) as ordered pairs, and plot them as points on the coordinate plane.



Exit Ticket Sample Solutions

Water flows from a hose at a constant rate of 11 gallons every 4 minutes. The total amount of water that flows from the hose is a function of the number of minutes you are observing the hose.

a. Write an equation in two variables that describes the amount of water, y, in gallons, that flows from the hose as a function of the number of minutes, x, you observe it.

$$\frac{11}{4} = \frac{y}{x}$$
$$y = \frac{11}{4}x$$

b. Use the equation you wrote in part (a) to determine the amount of water that flows from the hose during an 8-minute period, a 4-minute period, and a 2-minute period.

$$y = \frac{11}{4}(8)$$
$$y = 22$$

In 8 minutes, 22 gallons of water flow out of the hose.

$$y = \frac{11}{4}(4)$$
$$y = 11$$

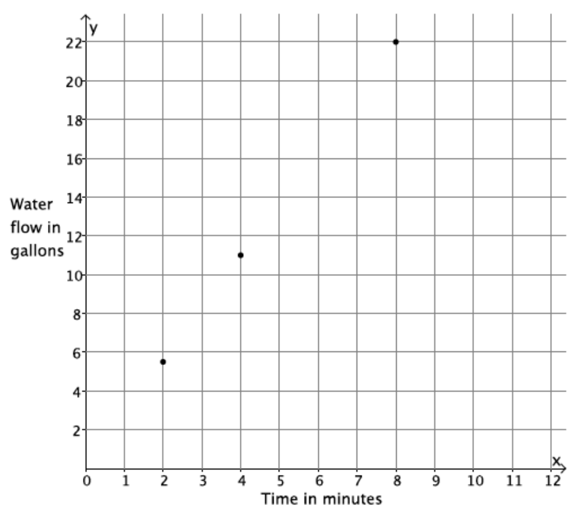
In 4 minutes, 11 gallons of water flow out of the hose.

$$y = \frac{11}{4}(2)$$
$$y = 5.5$$

In 2 minutes, 5.5 gallons of water flow out of the hose.

c. An input of the function, x, is time in minutes, and the output of the function, y, is the amount of water that flows out of the hose in gallons. Write the inputs and outputs from part (b) as ordered pairs, and plot them as points on the coordinate plane.

$(8, 22)$, $(4, 11)$, $(2, 5.5)$



Problem Set Sample Solutions

1. The distance that Scott walks is a function of the time he spends walking. Scott can walk $\frac{1}{2}$ mile every 8 minutes. Assume he walks at a constant rate.

   a. Predict the shape of the graph of the function. Explain.

   The graph of the function will likely be a line because a linear equation can describe Scott's motion, and I know that the graph of the function will be the same as the graph of the equation.

   b. Write an equation to represent the distance that Scott can walk in miles, y, in x minutes.

   $$\frac{0.5}{8} = \frac{y}{x}$$
   $$y = \frac{0.5}{8}x$$
   $$y = \frac{1}{16}x$$

   c. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 24 minutes.

   $$y = \frac{1}{16}(24)$$
   $$y = 1.5$$

   Scott can walk 1.5 miles in 24 minutes.

   d. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 12 minutes.

   $$y = \frac{1}{16}(12)$$
   $$y = \frac{3}{4}$$

   Scott can walk 0.75 miles in 12 minutes.

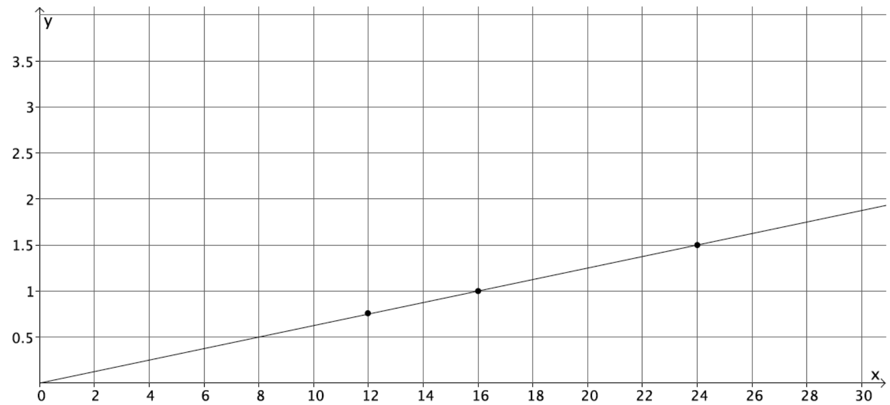
   e. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 16 minutes.

   $$y = \frac{1}{16}(16)$$
   $$y = 1$$

   Scott can walk 1 mile in 16 minutes.

f. **Write your inputs and corresponding outputs as ordered pairs, and then plot them on a coordinate plane.**

$(24, 1.5)$, $(12, 0.75)$, $(16, 1)$



g. **What shape does the graph of the points appear to take? Does it match your prediction?**

The points appear to be in a line. Yes, as I predicted, the graph of the function is a line.

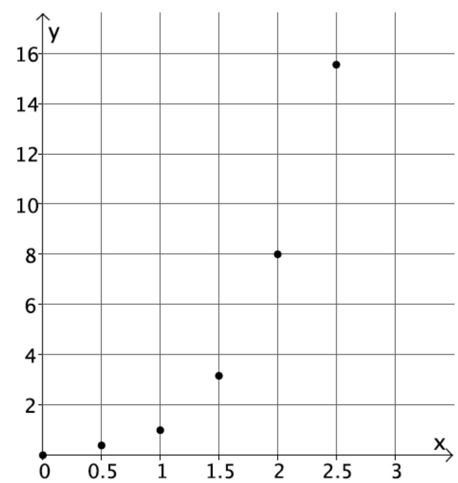
h. **Connect the points to make a line. What is the equation of the line?**

It is the equation that described the function: $y = \frac{1}{16}x$.

2. Graph the equation $y = x^3$ for positive values of x. Organize your work using the table below, and then answer the questions that follow.

x	y
0	0
0.5	0.125
1	1
1.5	3.375
2	8
2.5	15.625

a. Plot the ordered pairs on the coordinate plane.



b. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

c. Is this the graph of a linear function? Explain.

No, this is not the graph of a linear function. The equation $y = x^3$ is not a linear equation.

d. Consider the function that assigns to each positive real number s the volume V of a cube with side length s units. An equation that describes this function is $V = s^3$. What do you think the graph of this function will look like? Explain.

I think the graph of this function will look like the graph of the equation $y = x^3$. The inputs and outputs would match the solutions to the equation exactly. For the equation, the y-value is the cube of the x-value. For the function, the output is the cube of the input.

e. Use the function in part (d) to determine the volume of a cube with side length of 3 units. Write the input and output as an ordered pair. Does this point appear to belong to the graph of $y = x^3$?

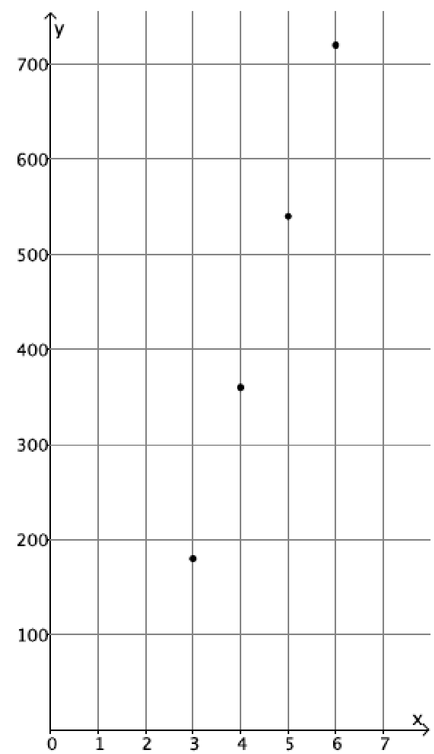
$$V = (3)^3$$
$$V = 27$$

$(3, 27)$ *The point looks like it would belong to the graph of $y = x^3$; it looks like it would be on the curve that the shape of the graph is taking.*

3. Sketch the graph of the equation $y = 180(x - 2)$ for whole numbers. Organize your work using the table below, and then answer the questions that follow.

x	y
3	180
4	360
5	540
6	720

a. Plot the ordered pairs on the coordinate plane.



b. What shape does the graph of the points appear to take?

It appears to take the shape of a line.

c. Is this graph a graph of a function? How do you know?

It appears to be a function because each input has exactly one output.

d. Is this a linear equation? Explain.

Yes, $y = 180(x - 2)$ is a linear equation. It can be rewritten as $y = 180x - 360$.

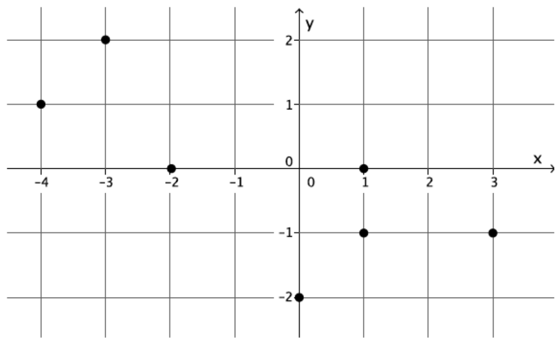
e. The sum S of interior angles, in degrees, of a polygon with n sides is given by $S = 180(n - 2)$. If we take this equation as defining S as a function of n, how do you think the graph of this S will appear? Explain.

I think the graph of this function will look like the graph of the equation $y = 180(x - 2)$. The inputs and outputs would match the solutions to the equation exactly.

f. Is this function discrete? Explain.

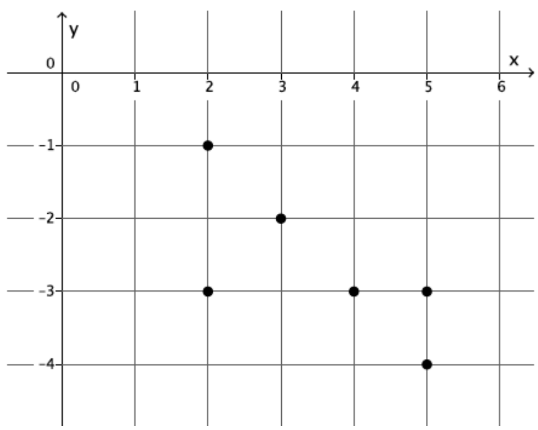
The function $S = 180(n - 2)$ is discrete. The inputs are the number of sides, which are integers. The input, n, must be greater than 2 since three sides is the smallest number of sides for a polygon.

4. **Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.**



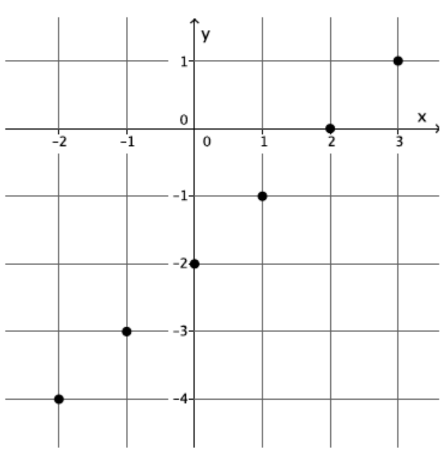
This is not the graph of a function. The ordered pairs $(1, 0)$ and $(1, -1)$ show that for the input of 1 there are two different outputs, both 0 and -1. For that reason, this cannot be the graph of a function because it does not fit the definition of a function.

5. **Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.**



This is not the graph of a function. The ordered pairs $(2, -1)$ and $(2, -3)$ show that for the input of 2 there are two different outputs, both -1 and -3. Further, the ordered pairs $(5, -3)$ and $(5, -4)$ show that for the input of 5 there are two different outputs, both -3 and -4. For these reasons, this cannot be the graph of a function because it does not fit the definition of a function.

6. **Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.**



This is the graph of a function. The ordered pairs $(-2, -4)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(2, 0)$, and $(3, 1)$ represent inputs and their unique outputs. By definition, this is a function.