Lesson 30: Conversion Between Celsius and Fahrenheit

Student Outcomes

- Students learn a real-world application of linear equations with respect to the conversion of temperatures from Celsius to Fahrenheit and Fahrenheit to Celsius.

Classwork

Mathematical Modeling Exercise (20 minutes)

- There are two methods for measuring temperature: (1) Fahrenheit, which assigns the number 32 to the temperature of water freezing and the number 212 to the temperature of water boiling; and (2) Celsius, which assigns the numbers 0 and 100, respectively, to the same temperatures. These numbers will be denoted by $32°F$, $212°F$, $0°C$, $100°C$, respectively.

- Our goal is to address the following two questions:

> **Mathematical Modeling Exercise**
>
> (1) If t is a number, what is the degree in Fahrenheit that corresponds to $t°C$?
>
> (2) If t is a number, what is the degree in Fahrenheit that corresponds to $(-t)°C$?

- Instead of trying to answer these questions directly, let's try something simpler. With this in mind, can we find out what degree in Fahrenheit corresponds to $1°C$? Explain.

- We can use the following diagram (double number line) to organize our thinking.
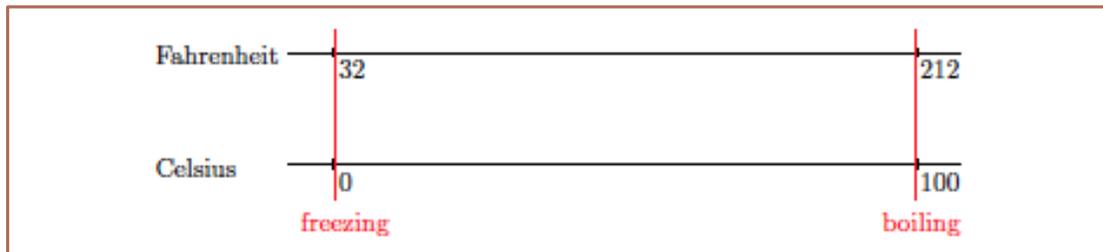
- At this point, the only information we have is that $0°C = 32°F$, and $100°C = 212°F$. We want to figure out what degree of Fahrenheit corresponds to $1°C$. Where on the diagram would $1°C$ be located? Be specific.

MP.3   Provide students time to talk to their partners about a plan, and then have them share. Ask them to make conjectures about what degree in Fahrenheit corresponds to $1°C$, and have them explain their rationale for the numbers they chose. Consider recording the information, and have the class vote on which answer they think is closest to correct.

    □ *We need to divide the Celsius number line from 0 to 100 into 100 equal parts. The first division to the right of zero will be the location of $1°C$.*

Now that we know where to locate 1°C on the lower number line, we need to figure out what number it corresponds to on the upper number line representing Fahrenheit. Like we did with Celsius, we divide the number line from 32 to 212 into 100 equal parts. The number line from 32 to 212 is actually a length of 180 units $(212 - 32 = 180)$. Now, how would we determine the precise number in Fahrenheit that corresponds to 1°C?

Provide students time to talk to their partners and compute the answer.

   □   *We need to take the length* 180 *and divide it into* 100 *equal parts.*

$$\frac{180}{100} = \frac{9}{5} = 1\frac{4}{5} = 1.8$$

■   If we look at a magnified version of the number line with this division, we have the following diagram:
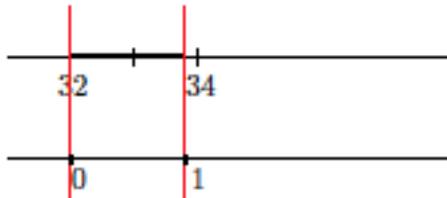
■   Based on your computation, what number falls at the intersection of the Fahrenheit number line and the red line that corresponds to 1°C? Explain.

   □   *Since we know that each division on the Fahrenheit number line has a length of* 1.8*, then when we start from* 32 *and add* 1.8*, we get* 33.8*. Therefore,* 1°C *is equal to* 33.8°F*.*

Revisit the conjecture made at the beginning of the activity, and note which student came closest to guessing 33.8°F. Ask the student to explain how he arrived at such a close answer.

■   Eventually, we want to revisit the original two questions. But first, let's look at a few more concrete questions. What is 37°C in Fahrenheit? Explain.

Provide students time to talk to their partners about how to answer the question. Ask students to share their ideas and explain their thinking.

   □   *Since the unit length on the Celsius scale is equal to the unit length on the Fahrenheit scale, then* 37°C *means we need to multiply* (37×1.8) *to determine the corresponding location on the Fahrenheit scale. But, because* 0 *on the Celsius scale is* 32 *on the Fahrenheit scale, we will need to add* 32 *to our answer. In other words,* 37°C $= (32 + 37 \times 1.8)°F = (32 + 66.6)°F = 98.6°F$*.*

Exercises (8 minutes)

Have students work in pairs or small groups to determine the corresponding Fahrenheit temperature for each given Celsius temperature. The goal is for students to be consistent in their use of repeated reasoning to lead them to the general equation for the conversion between Celsius and Fahrenheit.

MP.7

Exercises

Determine the corresponding Fahrenheit temperature for the given Celsius temperatures in Exercises 1–5.

1.   **How many degrees Fahrenheit is $25°C$?**

   $25°C = (32 + 25 \times 1.8)°F = (32 + 45)°F = 77°F$

MP.7

2. **How many degrees Fahrenheit is 42°C?**

    $42°C = (32 + 42 \times 1.8)°F = (32 + 75.6)°F = 107.6°F$

3. **How many degrees Fahrenheit is 94°C?**

    $94°C = (32 + 94 \times 1.8)°F = (32 + 169.2)°F = 201.2°F$

4. **How many degrees Fahrenheit is 63°C?**

    $63°C = (32 + 63 \times 1.8)°F = (32 + 113.4)°F = 145.4°F$

5. **How many degrees Fahrenheit is t°C?**

    $t°C = (32 + 1.8t)°F$

Discussion (10 minutes)

Have students share their answers from Exercise 5. Select several students to explain how they derived the equation to convert between Celsius and Fahrenheit. Close that part of the Discussion by letting them know that they answered Question (1) that was posed at the beginning of the lesson:

   (1) If t is a number, what is the degree in Fahrenheit that corresponds to t°C?

The following discussion answers Question (2):

   (2) If t is a number, what is the degree in Fahrenheit that corresponds to $(-t)$°C?

- Now that Question (1) has been answered, let's begin thinking about Question (2). Where on the number line would we find a negative Celsius temperature?
    - *A negative Celsius temperature will be to the left of zero on the number line.*
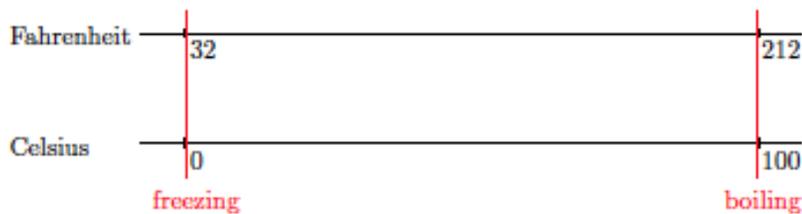
- Again, we will start simply. How can we determine the Fahrenheit temperature that corresponds to -1°C?

Provide students time to think, confirm with a partner, and then share with the class.

   - *We know that each unit on the Celsius scale is equal to 1.8°F. Then (-1)°C will equal $(32 - 1.8)°F = 30.2°F$.*
- How many degrees Fahrenheit corresponds to (-15)°C?

Provide students time to think, confirm with a partner, and then share with the class.

   - $(-15)°C = (32 - 15 \times 1.8)°F = (32 - 27)°F = 5°F$

- How many degrees Fahrenheit corresponds to $(-36)°$C?

Provide students time to think, confirm with a partner, and then share with the class.

- $(-36)°C = (32 - 36 \times 1.8)°F = (32 - 64.8)°F = -32.8°F$

- How many degrees Fahrenheit corresponds to $(-t)°$C?

Provide students time to think, confirm with a partner, and then share with the class.

- $(-t)°C = (32 - 1.8t)°F$

- Each of the previous four temperatures was negative. Then $(32 - 1.8t)°F$ can be rewritten as $\big(32 + 1.8(-t)\big)°F$ where the second equation looks a lot like the one we wrote for $t°C$; that is, $t°C = (32 + 1.8t)°F$. Are they the same equation? In other words, given any number t, positive or negative, would the result be the correct answer? We already know that the equation works for positive Celsius temperatures, so now let's focus on negative Celsius temperatures. Use $t°C = (32 + 1.8t)°F$ where $t = -15$. We expect the same answer as before: $(-15)°C = (32 - 15 \times 1.8)°F = (32 - 27)°F = 5°F$. Show that it is true.

- $(-15)°C = \big(32 + 1.8(-15)\big)°F = \big(32 + (-27)\big) = 5°F$

- Therefore, the equation $t°C = (32 + 1.8t)°F$ will work for any t.

- On a coordinate plane, if we let x be the given temperature, which in each case above has been given in Celsius and y be the temperature in Celsius, then we have the equation $y = x$. But when we let x be the given temperature in Celsius and y be the temperature in Fahrenheit, we have the equation $y = 32 + 1.8x$.
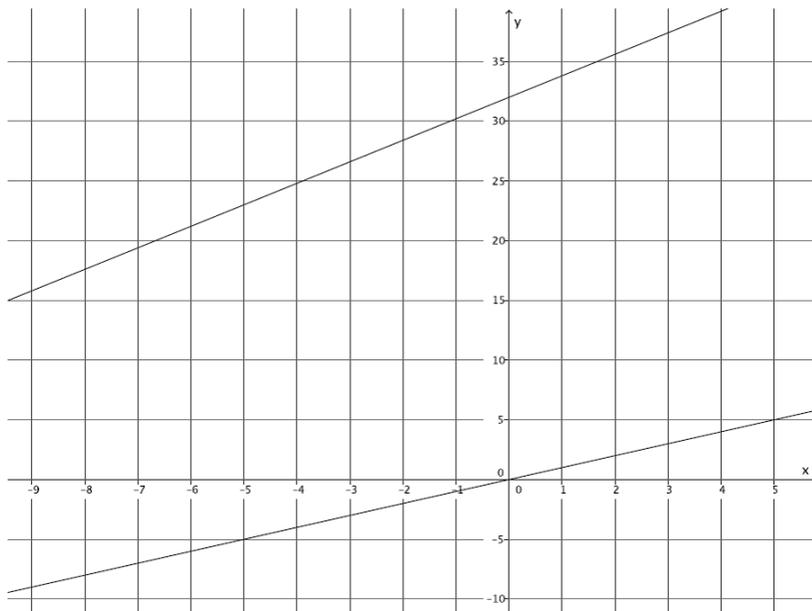
- Will these lines intersect? Explain?

  - *Yes. They have different slopes, so at some point they will intersect.*

- What will that point of intersection represent?
  - *It represents the solution to the system $\begin{cases} y = x \\ y = 1.8x + 32 \end{cases}$. That point will represent when the given temperature is the same number in Celsius and in Fahrenheit.*
- Solve the system of equations algebraically to determine at what number $t°C = t°F$.
  - *Sample student work:*

$$\begin{cases} y = x \\ y = 1.8x + 32 \end{cases}$$

$$x = 1.8x + 32$$
$$-0.8x = 32$$
$$x = -40$$

At -40 degrees, the temperatures will be equal in both units. In other words, at -40 degrees Celsius, the temperature in Fahrenheit will also be -40 degrees.

Closing (3 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to use a linear equation in a real-world situation like converting between Celsius and Fahrenheit.
- We can use the computations we make for specific numbers to help us determine a general linear equation for a situation.
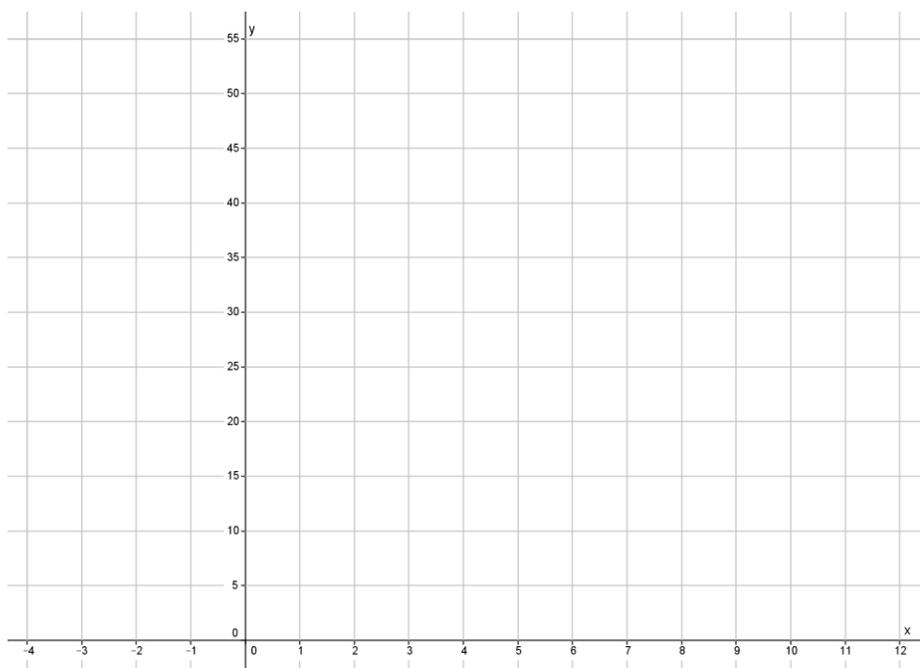
Exit Ticket (4 minutes)

Name _____   Date _____

Lesson 30: Conversion Between Celsius and Fahrenheit

Exit Ticket

Use the equation developed in class to answer the following questions:

1. How many degrees Fahrenheit is $11°C$?

2. How many degrees Fahrenheit is $-3°C$?

3. Graph the equation developed in class, and use it to confirm your results from Problems 1 and 2.

Exit Ticket Sample Solutions

Use the equation developed in class to answer the following questions:

1. **How many degrees Fahrenheit is $11°C$?**

$$11°C = (32 + 11 \times 1.8)°F$$
$$11°C = (32 + 19.8)°F$$
$$11°C = 51.8°F$$

2. **How many degrees Fahrenheit is $-3°C$?**

$$-3°C = (32 + (-3) \times 1.8)°F$$
$$-3°C = (32 - 5.4)°F$$
$$-3°C = 26.6°F$$

3. **Graph the equation developed in class, and use it to confirm your results from Problems 1 and 2.**
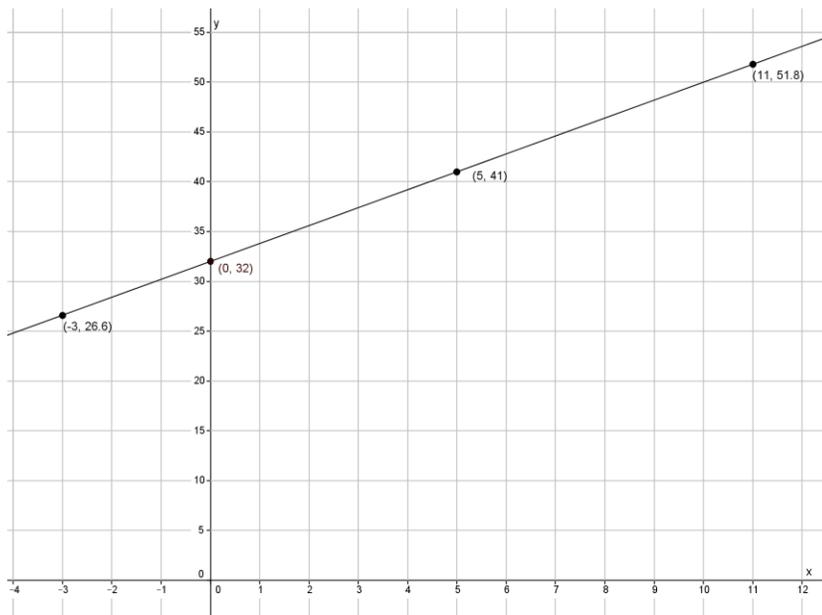
When I graph the equation developed in class, $t°C = (32 + 1.8t)°F$, the results from Problems 1 and 2 are on the line, confirming they are solutions to the equation.

Problem Set Sample Solutions

1. Does the equation $t°C = (32 + 1.8t)°F$ work for any rational number t? Check that it does with $t = 8\frac{2}{3}$ and $t = -8\frac{2}{3}$.

$$\left(8\frac{2}{3}\right)°C = \left(32 + 8\frac{2}{3} \times 1.8\right)°F = (32 + 15.6)°F = 47.6°F$$

$$\left(-8\frac{2}{3}\right)°C = \left(32 + \left(-8\frac{2}{3}\right) \times 1.8\right)°F = (32 - 15.6)°F = 16.4°F$$

2. Knowing that $t°C = \left(32 + \frac{9}{5}t\right)°F$ for any rational t, show that for any rational number d, $d°F = \left(\frac{5}{9}(d - 32)\right)°C$.

   Since $d°F$ can be found by $\left(32 + \frac{9}{5}t\right)$, then $d = \left(32 + \frac{9}{5}t\right)$, and $d°F = t°C$. Substituting $d = \left(32 + \frac{9}{5}t\right)$ into $d°F$, we get

$$d°F = \left(32 + \frac{9}{5}t\right)°F$$
$$d = 32 + \frac{9}{5}t$$
$$d - 32 = \frac{9}{5}t$$
$$\frac{5}{9}(d - 32) = t.$$

   Now that we know $t = \frac{5}{9}(d - 32)$, then $d°F = \left(\frac{5}{9}(d - 32)\right)°C$.

3. Drake was trying to write an equation to help him predict the cost of his monthly phone bill. He is charged $35 just for having a phone, and his only additional expense comes from the number of texts that he sends. He is charged $0.05 for each text. Help Drake out by completing parts (a)–(f).

   a. How much was his phone bill in July when he sent 750 texts?

   $$35 + 750(0.05) = 35 + 37.5 = 72.5$$

   His bill in July was $72.50.

   b. How much was his phone bill in August when he sent 823 texts?

   $$35 + 823(0.05) = 35 + 41.15 = 76.15$$

   His bill in August was $76.15.

   c. How much was his phone bill in September when he sent 579 texts?

   $$35 + 579(0.05) = 35 + 28.95 = 63.95$$

   His bill in September was $63.95.

   d. Let y represent the total cost of Drake's phone bill. Write an equation that represents the total cost of his phone bill in October if he sends t texts.

   $$y = 35 + t(0.05)$$

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e.   Another phone plan charges $20 for having a phone and $0.10 per text. Let y represent the total cost of the phone bill for sending t texts. Write an equation to represent his total bill.

$$y = 20 + t(0.10)$$

f.   Write your equations in parts (d) and (e) as a system of linear equations, and solve. Interpret the meaning of the solution in terms of the phone bill.

$$\begin{cases} y = 35 + t(0.05) \\ y = 20 + t(0.10) \end{cases}$$

$$35 + (0.05)t = 20 + (0.10)t$$
$$15 + (0.05)t = (0.10)t$$
$$15 = 0.05t$$
$$300 = t$$

$$y = 20 + 300(0.10)$$
$$y = 50$$

The solution is $(300, 50)$*, meaning that when Drake sends* 300 *texts, the cost of his bill will be* $50 *using his current phone plan or the new one.*