Lesson 1: Exponential Notation

Student Outcomes

- Students know what it means for a number to be raised to a power and how to represent the repeated multiplication symbolically.
- Students know the reason for some bases requiring parentheses.

Lesson Notes

This lesson is foundational for the topic of properties of integer exponents. For the first time in this lesson, students are seeing the use of exponents with negative valued bases. It is important that students explore and understand the importance of parentheses in such cases, just as with rational base values. It may also be the first time that students are seeing the notation (dots and braces) used in this lesson. If students have already mastered the skills in this lesson, it is optional to move forward and begin with Lesson 2 or provide opportunities for students to explore how to rewrite expressions in a different base, 4^2 as 2^4, for example.

Classwork

Discussion (15 minutes)

MP.2 & MP.7

When we add 5 copies of 3, we devise an abbreviation (i.e., a new notation) for this purpose.

$$3 + 3 + 3 + 3 + 3 = 5 \times 3$$

Now if we multiply 5 factors of 3, how should we abbreviate this?

$$3 \times 3 \times 3 \times 3 \times 3 = ?$$

Allow students to make suggestions (see sidebar for scaffolds).

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

Similarly, we also write $3^3 = 3 \times 3 \times 3$; $3^4 = 3 \times 3 \times 3 \times 3$; etc.

We see that when we add 5 summands of 3, we write 5×3, but when we multiply 5 factors of 3, we write 3^5. Thus, the *multiplication by* 5 in the context of addition corresponds exactly to the superscript 5 in the context of multiplication.

Make students aware of the correspondence between addition and multiplication because what they know about *repeated addition* will help them learn exponents as *repeated multiplication* as we go forward.

Scaffolding:

Remind students of their previous experiences:

- The square of a number (e.g., 3×3 is denoted by 3^2).
- From the expanded form of a whole number, we also learned that 10^3 stands for $10 \times 10 \times 10$.

5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$, and $\left(\frac{9}{7}\right)^4$ means $\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7}$.

You have seen this kind of notation before; it is called exponential notation. In general, for any number x and any positive integer n,

$$x^n = \underbrace{(x \cdot x \cdots x)}_{n \text{ times}}.$$

The number x^n is called x raised to the n^{th} power, where n is the exponent of x in x^n and x is the base of x^n.

Examples 1–5

Work through Examples 1–5 as a group, and supplement with additional examples if needed.

Example 1

$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

Example 2

$\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} = \left(\frac{9}{7}\right)^4$

Example 3

$\left(-\frac{4}{11}\right)^3 = \left(-\frac{4}{11}\right) \times \left(-\frac{4}{11}\right) \times \left(-\frac{4}{11}\right)$

Example 4

$(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$

Example 5

$3.8^4 = 3.8 \times 3.8 \times 3.8 \times 3.8$

- Notice the use of parentheses in Examples 2, 3, and 4. Do you know why we use them?
  - *In cases where the base is either fractional or negative, parentheses tell us what part of the expression is included in the base and, therefore, going to be multiplied repeatedly.*
- Suppose n is a fixed **positive integer**. Then 3^n by definition is $3^n = \underbrace{(3 \times \cdots \times 3)}_{n \text{ times}}$.
- Again, if n is a fixed positive integer, then by definition:

$$7^n = \underbrace{(7 \times \cdots \times 7)}_{n \text{ times}},$$

$$\left(\frac{4}{5}\right)^n = \underbrace{\left(\frac{4}{5} \times \cdots \times \frac{4}{5}\right)}_{n \text{ times}},$$

$$(-2.3)^n = \underbrace{\left((-2.3) \times \cdots \times (-2.3)\right)}_{n \text{ times}}.$$

If students ask about values of n that are not positive integers, ask them to give an example and to consider what such an exponent would indicate. Let them know that *integer* exponents will be discussed later in this module, so they should continue examining their question as we move forward. Positive and negative fractional exponents are a topic that will be introduced in Algebra II.

MP.6

- In general, for any number x, $x^1 = x$, and for any positive integer $n > 1$, x^n is by definition:
$$x^n = \underbrace{(x \cdot x \cdots x)}_{n \text{ times}}.$$
- The number x^n is called **x raised to the n^{th} power**, where n is the **exponent** of x in x^n, and x is the **base** of x^n.
- x^2 is called the **square** of x, and x^3 is its **cube**.
- You have seen this kind of notation before when you gave the expanded form of a whole number for powers of 10; it is called **exponential notation**.

Students might ask why we use the terms *square* and *cube* to represent exponential expressions with exponents of 2 and 3, respectively. Refer them to earlier grades and finding the area of a square and the volume of a cube. These geometric quantities are obtained by multiplying equal factors. The area of a square with side lengths of 4 units is $4 \text{ units} \times 4 \text{ units} = 4^2 \text{ units}^2$ or 16 units^2. Similarly, the volume of a cube with edge lengths of 4 units is $4 \text{ units} \times 4 \text{ units} \times 4 \text{ units} = 4^3 \text{ units}^3$ or 64 units^3.

Exercises 1–10 (5 minutes)

Have students complete these independently and check their answers before moving on.

Exercise 1

$$\underbrace{4 \times \cdots \times 4}_{7 \text{ times}} = 4^7$$

Exercise 2

$$\underbrace{3.6 \times \cdots \times 3.6}_{\_\_\_\_ \text{ times}} = 3.6^{47}$$

47 times

Exercise 3

$$\underbrace{(-11.63) \times \cdots \times (-11.63)}_{34 \text{ times}} = (-11.63)^{34}$$

Exercise 4

$$\underbrace{12 \times \cdots \times 12}_{\_\_\_\_ \text{ times}} = 12^{15}$$

15 times

Exercise 5

$$\underbrace{(-5) \times \cdots \times (-5)}_{10 \text{ times}} = (-5)^{10}$$

Exercise 6

$$\underbrace{\frac{7}{2} \times \cdots \times \frac{7}{2}}_{21 \text{ times}} = \left(\frac{7}{2}\right)^{21}$$

Exercise 7

$$\underbrace{(-13) \times \cdots \times (-13)}_{6 \text{ times}} = (-13)^6$$

Exercise 8

$$\underbrace{\left(-\frac{1}{14}\right) \times \cdots \times \left(-\frac{1}{14}\right)}_{10 \text{ times}} = \left(-\frac{1}{14}\right)^{10}$$

Exercise 9

$$\underbrace{x \cdot x \cdots x}_{185 \text{ times}} = x^{185}$$

Exercise 10

$$\underbrace{x \cdot x \cdots x}_{\_\_\_\_ \text{ times}} = x^n$$

n times

Exercises 11–14 (15 minutes)

Allow students to complete Exercises 11–14 individually or in small groups. As an alternative, provide students with several examples of exponential expressions whose bases are negative values, and whose exponents alternate between odd and even whole numbers. Ask students to discern a pattern from their calculations, form a conjecture, and work to justify their conjecture. They should find that a negative value raised to an even exponent results in a positive value since the product of two negative values yields a positive product. They should also find that having an even number of negative factors means each factor pairs with another, resulting in a set of positive products. Likewise, they should conclude that a negative number raised to an odd exponent always results in a negative value. This is because any odd whole number is 1 greater than an even number (or zero). This means that while the even set of negative factors results in a positive value, there will remain one more negative factor to negate the resulting product.

- When a negative number is raised to an odd power, what is the sign of the result?
- When a negative number is raised to an even power, what is the sign of the result?

Point out that when a negative number is raised to an odd power, the sign of the answer is negative. Conversely, if a negative number is raised to an even power, the sign of the answer is positive.

Exercise 11

Will these products be positive or negative? How do you know?

$$\underbrace{(-1)\times(-1)\times\cdots\times(-1)}_{12\text{ times}}=(-1)^{12}$$

This product will be positive. Students may state that they computed the product and it was positive. If they say that, let them show their work. Students may say that the answer is positive because the exponent is positive; however, this would not be acceptable in view of the next example.

$$\underbrace{(-1)\times(-1)\times\cdots\times(-1)}_{13\text{ times}}=(-1)^{13}$$

This product will be negative. Students may state that they computed the product and it was negative. If so, ask them to show their work. Based on the discussion of the last problem, you may need to point out that a positive exponent does not always result in a positive product.

The two problems in Exercise 12 force the students to think beyond the computation level. If students struggle, revisit the previous two problems, and have them discuss in small groups what an even number of negative factors yields and what an odd number of negative factors yields.

Exercise 12

Is it necessary to do all of the calculations to determine the sign of the product? Why or why not?

$$\underbrace{(-5)\times(-5)\times\cdots\times(-5)}_{95\text{ times}}=(-5)^{95}$$

Students should state that an odd number of negative factors yields a negative product.

$$\underbrace{(-1.8)\times(-1.8)\times\cdots\times(-1.8)}_{122\text{ times}}=(-1.8)^{122}$$

Students should state that an even number of negative factors yields a positive product.

Exercise 13

Fill in the blanks indicating whether the number is positive or negative.

If n is a positive even number, then $(-55)^n$ is ***positive.***

If n is a positive odd number, then $(-72.4)^n$ is ***negative.***

Exercise 14

Josie says that $\underbrace{(-15) \times \cdots \times (-15)}_{6 \text{ times}} = -15^6$. Is she correct? How do you know?

Students should state that Josie is not correct for the following two reasons: (1) They just stated that an even number of factors yields a positive product, and this conflicts with the answer Josie provided, and (2) the notation is used incorrectly because, as is, the answer is the negative of 15^6, instead of the product of 6 copies of -15. The base is (-15). Recalling the discussion at the beginning of the lesson, when the base is negative it should be written clearly by using parentheses. Have students write the answer correctly.

Closing (5 minutes)

- Why should we bother with exponential notation? Why not just write out the multiplication?

Engage the class in discussion, but make sure to address at least the following two reasons:

1. Like all good notation, exponential notation saves writing.
2. Exponential notation is used for recording scientific measurements of very large and very small quantities. It is indispensable for the clear indication of the magnitude of a number (see Lessons 10–13).

- Here is an example of the labor-saving aspect of the exponential notation: Suppose a colony of bacteria doubles in size every 8 hours for a few days under tight laboratory conditions. If the initial size is B, what is the size of the colony after 2 days?
  - *In 2 days, there are six 8-hour periods; therefore, the size will be 2^6B.*

If time allows, give more examples as a lead in to Lesson 2. Example situations: (1) exponential decay with respect to heat transfer, vibrations, ripples in a pond, or (2) exponential growth with respect to interest on a bank deposit after some years have passed.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Exponential Notation

Exit Ticket

1.

a. Express the following in exponential notation:

$$\underbrace{(-13)\times\cdots\times(-13)}_{35\text{ times}}.$$

b. Will the product be positive or negative? Explain.

2. Fill in the blank:

$$\underbrace{\frac{2}{3}\times\cdots\times\frac{2}{3}}_{\_\_\_\_\_\text{times}}=\left(\frac{2}{3}\right)^4$$

3. Arnie wrote:

$$\underbrace{(-3.1)\times\cdots\times(-3.1)}_{4\text{ times}}=-3.1^4$$

Is Arnie correct in his notation? Why or why not?

Exit Ticket Sample Solutions

1.

a. **Express the following in exponential notation:**

$$\underbrace{(-13) \times \cdots \times (-13)}_{35 \text{ times}}$$

$(-13)^{35}$

b. **Will the product be positive or negative? Explain.**

The product will be negative. The expanded form shows 34 negative factors plus one more negative factor. Any even number of negative factors yields a positive product. The remaining 35^{th} negative factor negates the resulting product.

2. **Fill in the blank:**

$$\underbrace{\frac{2}{3} \times \cdots \times \frac{2}{3}}_{\_\_\_\_ \text{ times}} = \left(\frac{2}{3}\right)^4$$

4 times

3. **Arnie wrote:**

$$\underbrace{(-3.1) \times \cdots \times (-3.1)}_{4 \text{ times}} = -3.1^4$$

Is Arnie correct in his notation? Why or why not?

Arnie is not correct. The base, -3.1, should be in parentheses to prevent ambiguity. At present the notation is not correct.

Problem Set Sample Solutions

1. **Use what you know about exponential notation to complete the expressions below.**

$$\underbrace{(-5) \times \cdots \times (-5)}_{17 \text{ times}} = (-5)^{17}$$

$$\underbrace{3.7 \times \cdots \times 3.7}_{\_\_\_ \text{ times}} = 3.7^{19}$$

19 times

$$\underbrace{7 \times \cdots \times 7}_{\_\_\_ \text{ times}} = 7^{45}$$

45 times

$$\underbrace{6 \times \cdots \times 6}_{4 \text{ times}} = 6^4$$

$$\underbrace{4.3 \times \cdots \times 4.3}_{13 \text{ times}} = 4.3^{13}$$

$$\underbrace{(-1.1) \times \cdots \times (-1.1)}_{9 \text{ times}} = (-1.1)^9$$

$$\underbrace{\left(\frac{2}{3}\right) \times \cdots \times \left(\frac{2}{3}\right)}_{19 \text{ times}} = \left(\frac{2}{3}\right)^{19}$$

$$\underbrace{\left(-\frac{11}{5}\right) \times \cdots \times \left(-\frac{11}{5}\right)}_{\_\_\_ \text{ times}} = \left(-\frac{11}{5}\right)^x$$

x times

$\underbrace{(-12) \times \cdots \times (-12)}_{\_\_\_ \text{ times}} = (-12)^{15}$ $\underbrace{a \times \cdots \times a}_{m \text{ times}} = a^m$

15 times

2. Write an expression with (-1) as its base that will produce a positive product, and explain why your answer is valid.

   Accept any answer with (-1) to an exponent that is even.

3. Write an expression with (-1) as its base that will produce a negative product, and explain why your answer is valid.

   Accept any answer with (-1) to an exponent that is odd.

4. Rewrite each number in exponential notation using 2 as the base.

   $8 = 2^3$ $16 = 2^4$ $32 = 2^5$

   $64 = 2^6$ $128 = 2^7$ $256 = 2^8$

5. Tim wrote 16 as $(-2)^4$. Is he correct? Explain.

   Tim is correct that $16 = (-2)^4$. $(-2)(-2)(-2)(-2) = (4)(4) = 16$.

6. Could -2 be used as a base to rewrite 32? 64? Why or why not?

   A base of -2 cannot be used to rewrite 32 because $(-2)^5 = -32$. A base of -2 can be used to rewrite 64 because $(-2)^6 = 64$. If the exponent, n, is even, $(-2)^n$ will be positive. If the exponent, n, is odd, $(-2)^n$ cannot be a positive number.