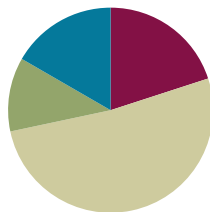


Lesson 3

Objective: Write and interpret numerical expressions, and compare expressions using a visual model.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(7 minutes)
■ Concept Development	(31 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Multiply by Multiples of 10 **5.NBT.2** (3 minutes)
- Estimate Products **5.NBT.6** (5 minutes)
- Decompose a Factor: The Distributive Property **3.OA.5** (4 minutes)

Multiply by Multiples of 10 (3 minutes)

Note: This review fluency activity helps preserve skills students learned and mastered in Module 1 and lays the groundwork for future concepts.

Follow the same process and procedure as Lesson 2 for the following possible sequence: 21×40 , 213×30 , and $4,213 \times 20$.

Estimate Products (5 minutes)

Materials: (S) Personal white board

T: (Write $421 \times 18 \approx \underline{\quad} \times \underline{\quad} = \underline{\quad}$.) Round 421 to the nearest hundred.

S: 400.

T: (Write $421 \times 18 \approx 400 \times \underline{\quad} = \underline{\quad}$.) Round 18 to the nearest ten.

S: 20.

T: (Write $421 \times 18 \approx 400 \times 20 = \underline{\quad}$.) What's 400×20 ?

S: 8,000.

T: (Write $421 \times 18 \approx 400 \times 20 = 8,000$.)

T: (Write $323 \times 21 \approx \underline{\quad} \times \underline{\quad} = \underline{\quad}$.) On your personal white board, write the multiplication sentence rounding each factor to arrive at a reasonable estimate of the product.

S: (Write $323 \times 21 \approx 300 \times 20 = 6,000$.)

Repeat the process and procedure for $1,950 \times 42$ and $2,480 \times 27$. Ask students to explain the reasoning behind their estimates.

Decompose a Factor: The Distributive Property (4 minutes)

Note: Review of multiplication decomposition with low numbers prepares students for decomposing multiplication sentences with bigger numbers in the upcoming lessons. Students could be encouraged to generate their own decomposition to be used in the distribution (e.g., for the first, possible decompositions of 9 include 2 and 7 or 3 and 6). However, this will increase the time needed for this fluency activity.

Materials: (S) Personal white board

T: (Write $9 \times 3 = \underline{\quad}$.) Write the multiplication sentence.

S: (Write.)

T: (Write $(5 \times 3) + (\underline{\quad} \times 3) = \underline{\quad}$ below $9 \times 3 = \underline{\quad}$.)
9 is the same as 5 and what number?

S: 4.

T: (Write $(5 \times 3) + (4 \times 3) = \underline{\quad}$. Below it, write
 $15 + \underline{\quad} = \underline{\quad}$.) Fill in the blanks.

S: (Write $9 \times 3 = 27$. Below it, write $(5 \times 3) + (4 \times 3) = 27$.
Below that line, write $15 + 12 = 27$.)

$$9 \times 3 = \underline{\quad}$$

$$(5 \times 3) + (\underline{4} \times 3) = \underline{\quad}$$

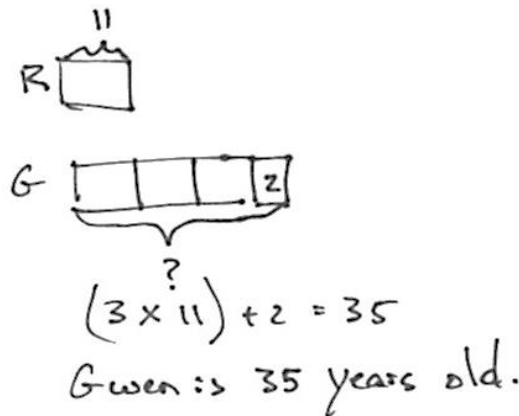
$$15 + \underline{12} = \underline{\quad}$$

Repeat using the following possible sequence of 7×4 , 8×2 , and 9×6 .

Application Problem (7 minutes)

Robin is 11 years old. Her mother, Gwen, is 2 years more than 3 times Robin's age. How old is Gwen?

Note: This problem is simple enough that students can solve it prior to Lesson 3; however, in the Student Debrief, students are asked to return to the Application Problem and create a numerical expression to represent Gwen's age (i.e., $(3 \times 11) + 2$). Accept any valid approach to solving the problem. The tape diagram is but one approach. Allow students to share.



Concept Development (31 minutes)

Materials: (S) Personal white board

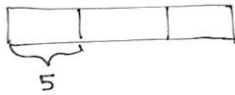
Problems 1–3: From word form to numerical expressions and diagrams.

3 times the sum of 26 and 4.

6 times the difference between 60 and 51.

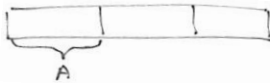
The sum of 2 twelves and 4 threes.

T: What expression describes the total value of these 3 equal units? Show a tape diagram.



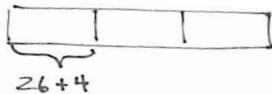
S: 3×5 .

T: How about 3 times an unknown amount called A. Show a tape diagram and expression.



S: $3 \times A$.

T: 3 times the sum of 26 and 4? Show a tape diagram and expression.



S: $3 \times (26 + 4)$ or $(26 + 4) \times 3$.

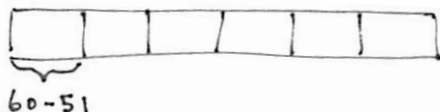
T: Why are **parentheses** necessary around $26 + 4$? Talk to your partner.

S: We want 3 times as much as the total of $26 + 4$. → If we don't put the parentheses, it doesn't show what we are counting. → We are counting the total of 26 and 4 three times.

T: Evaluate the expression.

S: 90.

T: (Write *6 times the difference between 60 and 51* on the board.) Work with a partner to show a tape diagram and expression to match these words.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

A review of relevant vocabulary may be in order for some students. Words such as *sum*, *product*, *difference*, and *quotient* might be reviewed, or a scaffold such as a word wall in the classroom might be appropriate.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

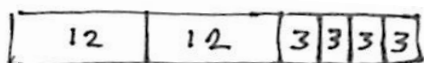
For some students, it may be more appropriate to begin with expressions in a more direct order such as, *the sum of 4 and 3 multiplied by 2* or *the difference between 14 and 6 times 5*.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Some students may have difficulty understanding a number word like *twelves* as a noun—a unit to be counted. Substitute another more concrete noun like *apples* in the phrases, then transition to the noun *dozens* before using *twelves*. Use a concrete model of twelves like egg cartons to act out the problem.

- S: $6 \times (60 - 51)$ or $(60 - 51) \times 6$.
- T: You've offered two different expressions for these words: $6 \times (60 - 51)$ and $(60 - 51) \times 6$. Are these expressions equal? Why or why not?
- S: Yes, they are equal. The two factors are just reversed.
- T: What is the name of this property?
- S: It's the commutative property.
- T: Explain it in your own words to your partner.
- S: (Share with partners.)
- T: (Write *the sum of 2 twelves and 4 threes* on the board.) Represent this with a tape diagram and expression.



$$(2 \times 12) + (4 \times 3)$$

Repeat as necessary with examples such as *the sum of 2 nineteens and 8 nineteens* or *5 times the sum of 16 and 14*.

Problems 4–6: From numerical expressions to word form.

$$8 \times (43 - 13)$$

$$(16 + 9) \times 4$$

$$(20 \times 3) + (5 \times 3)$$

- T: (Show $8 \times (43 - 13)$ on the board.) Read this expression in words.
- S: Eight times 43 minus 13.
- T: Let me write down what I hear you saying. (Write $8 \times 43 - 13$.) It sounds like you are saying that we should multiply 8 and 43 and then subtract 13. Is that what you meant? Is this second expression equivalent to the one I wrote at first? Why or why not?
- S: No. It's not the same. → You didn't write any parentheses. Without them, you will get a different answer because you won't subtract first. → We are supposed to subtract 13 from 43 and then multiply by 8.
- T: Why can't we simply read every expression left to right and translate it?
- S: We need to use words that tell which operation we should do first.
- T: Let's name the two factors we are multiplying. Turn and talk.
- S: 8 and the answer to $43 - 13$. → We need to multiply the answer to the expression inside the parentheses by 8.
- T: Since one of the factors is the answer to *this part* (make a circular motion around $43 - 13$), what could we say to make sure we are talking about the answer to this subtraction problem? What do we call the answer to a subtraction problem?
- S: The difference between 43 and 13.
- T: What is happening to the difference of 43 and 13?
- S: It's being multiplied by 8.

- T: We can say and write, “8 times the difference of 43 and 13.” Compare these words to the ones we said at first. Do they make sure we are multiplying the right numbers together? What other ways are there to say it?
- S: Yes, the words clearly tell us what to multiply. → The product of 8 and the difference between 43 and 13. → 8 times as much as the difference between 43 and 13. → The difference of 43 and 13 multiplied 8 times.

Repeat the process with the following:

$$(16 + 9) \times 4$$

Students should write *the sum of 16 and 9 times 4*. If students say *16 plus 9 times 4*, follow the sequence above to correct their thinking.

$$(20 \times 3) + (5 \times 3)$$

Students may write *the sum of 20 threes and 5 threes* or *the sum of 3 twenties and 3 fives*, or *the product of 20 and 3 plus the product of 5 and 3*, and so on. Similarly, discuss why *twenty times 3 plus 5 times 3* is unclear and imprecise.

Problems 7–9: Comparison of expressions in word form and numerical form.

$$9 \times 13$$

8 thirteens

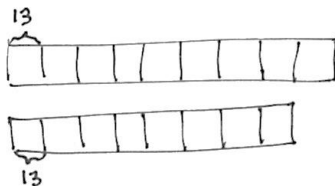
The sum of 10 and 9, doubled

$$(2 \times 10) + (2 \times 9)$$

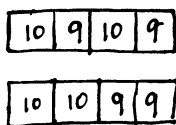
30 fifteens minus 1 fifteen

$$29 \times 15$$

- T: Let’s use $<$, $>$, or $=$ to compare expressions. (Write 9×13 and 8 thirteens on the board.) Draw a tape diagram for each expression and compare them.



- S: (Draw and write $9 \times 13 > 8$ thirteens.)
- T: We don’t even need to evaluate the solutions in order to compare them.
- T: Now compare the next two expressions without evaluating, using diagrams.
- S: They are equal because the sum of 10 and 9, doubled is $(10 + 9) \times 2$. The expression on the right is the sum of 2 tens and 2 nines. There are 2 tens and 2 nines in each tape.



Repeat the process with the final example.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Write and interpret numerical expressions, and compare expressions using a visual model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Return to the Application Problem. Create a numerical expression to represent Gwen’s age.
- In Problem 1(b) some of you wrote $4 \times (14 + 26)$ and others wrote $(14 + 26) \times 4$. Are both expressions acceptable? Explain.
- When evaluating the expression in Problem 2(a), a student got 85. Can you identify the error in thinking?
- Look at Problem 3(b). Talk in groups about how you know the expressions are not equal. How can you change the second expression to make it equivalent to 18×27 ?
- In Problem 5, be sure to point out that MeiLing’s expression, while equivalent, does not accurately reflect what Mr. Huynh wrote on the board. As an extension, ask students to put the expressions that MeiLing and Angeline wrote into words.

Expression	Words	The Value of the Expression
a. $12 \times (5 + 25)$	12 times the sum of 5 and 25	360
b. $(62 - 12) \times 11$	11 times the difference between 62 and 12	550
c. $(45 + 55) \times 23$	23 times the sum of 45 and 55	2,300
d. $(30 \times 2) + (8 \times 2)$	the sum of 30 twos and 8 twos	76

3. Compare the two expressions using $>$, $<$, or $=$. In the space beneath each pair of expressions, explain how you can compare without calculating. Draw a model if it helps you.

a. $24 \times (20 + 5)$ $>$ $(20 + 5) \times 12$
 It's greater than because the left side shows 24 groups of (20+5), but the right side only has 12 groups of (20+5).

b. 18×27 $<$ 20 twenty-sevens minus 1 twenty-seven
 It's less than because the left side shows 18 twenty-sevens, and the right side shows 19 twenty-sevens.

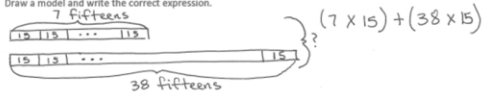
c. 19×9 $=$ 3 nineteens, tripled
 Both sides are equal. They're both 9 groups of 19.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students’ understanding of the concepts that were presented in today’s lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 5•2

4. Mr. Huynh wrote the sum of 7 fifteens and 38 fifteens on the board. Draw a model and write the correct expression.



5. Two students wrote the following numerical expressions.
 Angeline: $(7 + 15) \times (38 + 15)$
 Meiling: $15 \times (7 + 38)$

Are the students’ answers equivalent to your answer in Problem 4? Explain your answer.
 Meiling’s answer is equivalent. She found the sum (added) of the 2 bars together.
 Angeline’s answer is not equivalent. She multiplied the values of the two bars.

6. A box contains 24 oranges. Mr. Lee ordered 8 boxes for his store and 12 boxes for his restaurant.

a. Write an expression to show how to find the total number of oranges ordered.
 $(8 \times 24) + (12 \times 24)$ OR $24(8 + 12)$ OR 24×20

b. Next week, Mr. Lee will both double the number of boxes he orders. Write a new expression to represent the number of oranges in next week’s order.
 $((8 \times 24) + (12 \times 24)) \times 2$ OR $(24 \times 20) \times 2$

c. Evaluate your expression from Part (b) to find the total number of oranges ordered in both weeks.
 Week 1: $(24 \times 20) = 480$
 Week 2: $(24 \times 20) \times 2 = 960$

In both weeks, Mr. Lee ordered 1,440 oranges.

$\begin{array}{r} 480 \\ + 960 \\ \hline 1,440 \end{array}$	
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COMMON CORE Lesson 3: Write and interpret numerical expressions and compare expressions using a visual model. 2.8.12
 Date: 2/12/14 engage^{ny}

Name _____

Date _____




1. Draw a model. Then, write the numerical expressions.

a. The sum of 8 and 7, doubled	b. 4 times the sum of 14 and 26
c. 3 times the difference between 37.5 and 24.5	d. The sum of 3 sixteens and 2 nines
e. The difference between 4 twenty-fives and 3 twenty-fives	f. Triple the sum of 33 and 27

2. Write the numerical expressions in words. Then, solve.

Expression	Words	The Value of the Expression
a. $12 \times (5 + 25)$		
b. $(62 - 12) \times 11$		
c. $(45 + 55) \times 23$		
d. $(30 \times 2) + (8 \times 2)$		

3. Compare the two expressions using $>$, $<$, or $=$. In the space beneath each pair of expressions, explain how you can compare without calculating. Draw a model if it helps you.

a. $24 \times (20 + 5)$		$(20 + 5) \times 12$
b. 18×27		20 twenty-sevens minus 1 twenty-seven
c. 19×9		3 nineteens, tripled

4. Mr. Huynh wrote *the sum of 7 fifteens and 38 fifteens* on the board.
Draw a model, and write the correct expression.

5. Two students wrote the following numerical expressions.

Angeline: $(7 + 15) \times (38 + 15)$

MeiLing: $15 \times (7 + 38)$

Are the students' expressions equivalent to your answer in Problem 4? Explain your answer.

6. A box contains 24 oranges. Mr. Lee ordered 8 boxes for his store and 12 boxes for his restaurant.
- Write an expression to show how to find the total number of oranges ordered.
 - Next week, Mr. Lee will double the number of boxes he orders. Write a new expression to represent the number of oranges in next week's order.
 - Evaluate your expression from Part (b) to find the total number of oranges ordered in both weeks.


Name _____

Date _____

1. Draw a model. Then, write the numerical expressions.

a. The difference between 8 forty-sevens and 7 forty-sevens	b. 6 times the sum of 12 and 8
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2. Compare the two expressions using $>$, $<$, or $=$.

$62 \times (70 + 8)$		$(70 + 8) \times 26$
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Name _____

Date _____



1. Draw a model. Then, write the numerical expressions.

a. The sum of 21 and 4, doubled	b. 5 times the sum of 7 and 23
c. 2 times the difference between 49.5 and 37.5	d. The sum of 3 fifteens and 4 twos
e. The difference between 9 thirty-sevens and 8 thirty-sevens	f. Triple the sum of 45 and 55

2. Write the numerical expressions in words. Then, solve.

Expression	Words	The Value of the Expression
a. $10 \times (2.5 + 13.5)$		
b. $(98 - 78) \times 11$		
c. $(71 + 29) \times 26$		
d. $(50 \times 2) + (15 \times 2)$		

3. Compare the two expressions using $>$, $<$, or $=$. In the space beneath each pair of expressions, explain how you can compare without calculating. Draw a model if it helps you.

a. $93 \times (40 + 2)$		$(40 + 2) \times 39$
b. 61×25		60 twenty-fives minus 1 twenty-five

4. Larry claims that $(14 + 12) \times (8 + 12)$ and $(14 \times 12) + (8 \times 12)$ are equivalent because they have the same digits and the same operations.

a. Is Larry correct? Explain your thinking.

b. Which expression is greater? How much greater?