Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes

- Students use fraction bars and area models to divide fractions by fractions with different denominators.
- Students make connections between visual models and multiplication of fractions.

Classwork

Opening Exercise (2 minutes)

Begin class with a review of equivalent fractions. Ask each student for a new example of an equivalent fraction. Students need to share how they know that the new fraction is equivalent to the old fraction.

Opening Exercise

Write at least three equivalent fractions for each fraction below.

a. \( \frac{2}{3} \)
   
   Sample solutions include \( \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18} \)

b. \( \frac{10}{12} \)
   
   Sample solutions include \( \frac{5}{6}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36} \)

Example 1 (Optional)

This example is a review of the problems completed in the previous lesson. Therefore, it is decided by the teacher if this example is necessary or not.

For the first example, students are asked to solve a word problem using the skills they used in Lesson 3 to divide fractions with the same denominator.

- Molly has \( \frac{3}{8} \) cups of strawberries. This can also be represented as \( \frac{11}{8} \). She needs \( \frac{3}{8} \) cup of strawberries to make one batch of muffins. How many batches can Molly make?
  
  This question is really asking me how many \( \frac{3}{8} \) are in \( \frac{11}{8} \) or, in other words, to divide 11 eighths by 3 eighths. I can use a model to show that there are enough strawberries to make \( 3 \frac{2}{3} \) batches of muffins.
Example 1

Molly has $\frac{3}{8}$ cups of strawberries. She needs $\frac{3}{6}$ cup of strawberries to make one batch of muffins. How many batches can Molly make?

Use a model to support your answer.

$$\frac{11}{8} \div \frac{3}{8} = \frac{11}{3} = 3 \frac{2}{3}$$

Molly can make $3 \frac{2}{3}$ batches of muffins.

Example 2 (3 minutes)

- Molly’s friend, Xavier, also has $\frac{11}{8}$ cups of strawberries. He needs $\frac{3}{4}$ cup strawberries to make a batch of tarts. How many batches can he make?
  - He has purchased $\frac{11}{8}$ cups, which makes $1$ and $\frac{5}{6}$ batches. (This would be answered last after a brief discussion using the questions that follow.)
  - What is this question asking us to do?
    - I am being asked to divide $\frac{11}{8}$ cups into $\frac{3}{4}$-cup units.
  - How does the problem differ from the first example?
    - The denominators are different.
  - What are some possible ways that we could divide these two fractions?
    - I could rename $\frac{3}{4}$ as $\frac{6}{8}$. These fractions are equivalent. I created an equivalent fraction by multiplying $\frac{3}{4} \times \frac{2}{2}$. 

MP.1
Example 2

Molly’s friend, Xavier, also has \( \frac{11}{8} \) cups of strawberries. He needs \( \frac{3}{4} \) cup of strawberries to make a batch of tarts. How many batches can he make? Draw a model to support your solution.

\[
\frac{11}{8} \div \frac{6}{8} = \frac{11}{6} = 1 \frac{5}{6}
\]

\( \frac{11}{8} \) divided by \( \frac{6}{8} \) is 11 eighths divided by 6 eighths, which equals \( \frac{11}{6} \) or 1 and \( \frac{5}{6} \) batches.

Example 3 (3 minutes)

- \( \frac{6}{8} \div \frac{2}{8} \)

- Yesterday we focused on measurement division. Let’s solve this problem using partitive division. Therefore, what is the question asking?
  - \( \frac{6}{8} \) is \( \frac{2}{8} \) of what number?

- How could we model this problem?
  - Using the model, how could we solve the problem?
    - 2 units = 6 eighths
    - 1 unit = 6 eighths ÷ 2
    - 1 unit = 3 eighths
    - 8 units = 3 eighths × 8
    - 8 units = 3
Therefore, \( \frac{6}{8} \div \frac{2}{8} = 3 \).

What do you notice about this solution that is similar to the solutions we found yesterday?

- The units, eighths, cancel out.

This shows that we can follow the same process when solving both measurement and partitive division.

Example 3
Find the quotient: \( \frac{6}{8} \div \frac{2}{8} \). Use a model to show your answer.

\[
\begin{align*}
2 \text{ units} &= 6 \text{ eighths} \\
1 \text{ unit} &= 6 \text{ eighths} \div 2 = 3 \text{ eighths} \\
8 \text{ units} &= 8 \times 3 \text{ eighths} = 24 \text{ eighths} = 3
\end{align*}
\]

Example 4 (3 minutes)

\( \frac{3}{4} \div \frac{2}{3} \)

What is this question asking?

- It could be either \( \frac{2}{3} \) of what is \( \frac{3}{4} \) or how many \( \frac{2}{3} \) are in \( \frac{3}{4} \)?

Lead students through a brief discussion about this example:

- Is your answer larger or smaller than one? Why?
  - Since \( \frac{2}{3} \) is less than \( \frac{3}{4} \), we will have an answer that is larger than 1.

- What is the difference between this problem and the problems we completed in Lesson 3?
  - The fractions in this problem do not have common denominators, but the problems in Lesson 3 did.

- Draw a model.
Lesson 4

Interpreting and Computing Division of a Fraction by a Fraction—More Models

How can we rewrite this question to make it easier to model?

- We can create equivalent fractions with like denominators and then model and divide.
- We can also think of this as \( \frac{9}{12} \div \frac{8}{12} \) or 9 twelfths divided by 8 twelfths. 9 units ÷ 8 units = \( \frac{9}{8} \) or 1 \( \frac{1}{8} \) units

Example 4

Find the quotient: \( \frac{3}{4} \div \frac{2}{3} \). Use a model to show your answer.

We could rewrite this problem to ask \( \frac{9}{12} \div \frac{8}{12} = \frac{9}{12} \div 8 \) twelfths = \( \frac{9}{8} \) twelfths = \( \frac{9}{8} = 1 \frac{1}{8} \).

Exercises 1–5 (19 minutes)

Students work in pairs or alone to solve more questions about division of fractions with unlike denominators.

Students are no longer required to draw models; however, models are provided in the answers in case some students still need the visual to complete the problems.

Exercises 1–5

Find each quotient.

1. \( \frac{6}{2} \div \frac{3}{4} \)

We could rewrite this expression and solve as \( \frac{12}{4} \div \frac{3}{4} = \frac{12}{3} = 4 \).
2. \( \frac{2}{3} \div \frac{2}{5} \)

We could rewrite this expression and solve as \( \frac{10}{15} \div \frac{6}{15} = \frac{10}{6} = 1 \frac{4}{6} \).

3. \( \frac{7}{8} \div \frac{1}{2} \)

We could rewrite this as \( \frac{7}{8} \div \frac{4}{8} = \frac{7}{4} = 1 \frac{3}{4} \).

4. \( \frac{3}{5} \div \frac{1}{4} \)

This can be rewritten as \( \frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2 \frac{2}{5} \).
Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction — More Models

5. \( \frac{5}{4} \div \frac{1}{3} \)

We can rewrite this as \( \frac{15}{12} \div \frac{4}{12} = \frac{15}{4} = 3 \frac{3}{4} \).

Closing (10 minutes)

- When dividing fractions, is it possible to get a whole number quotient?
  - It is possible to get a whole number quotient when dividing fractions.
  - When the dividend is larger than the divisor, the quotient will be greater than 1.

- When dividing fractions, is it possible to get a quotient that is larger than the dividend?
  - It is possible to get a quotient that is larger than the dividend when dividing fractions. For example, \( 1 \div \frac{1}{4} = 4 \) fourths \( \div 1 \) fourth = 4.

- When you are asked to divide two fractions with different denominators, what is one possible way to solve?
  - To divide fractions with different denominators, we can find equivalent fractions with like denominators in order to solve.

Exit Ticket (5 minutes)
Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Exit Ticket

Calculate each quotient. If needed, draw a model.

1. \( \frac{9}{4} \div \frac{3}{8} \)

2. \( \frac{3}{5} \div \frac{2}{3} \)
Exit Ticket Sample Solutions

Calculate each quotient. If needed, draw a model.

1. \( \frac{9}{4} \div \frac{3}{8} \)

   This can be rewritten as \( \frac{18}{8} \div \frac{3}{8} = 18 \text{ eighths divided by } 3 \text{ eighths} = \frac{18}{3} = 6 \).

   \[ 1 + 1 + 1 + 1 + 1 + 1 = 6 \]

2. \( \frac{3}{5} \div \frac{2}{3} \)

   This can be rewritten as \( \frac{9}{15} \div \frac{10}{15} = 9 \text{ fifteenths divided by } 10 \text{ fifteenths}, \text{ or } 9 \text{ units } \div 10 \text{ units} \).

   So, this is equal to \( \frac{9}{10} \).
Problem Set Sample Solutions

The following problems can be used as extra practice or a homework assignment.

Calculate the quotient. If needed, draw a model.

1. \( \frac{8}{9} \div \frac{4}{9} \)
   - 8 ninths ÷ 4 ninths = 2

2. \( \frac{9}{10} \div \frac{4}{10} \)
   - 9 tenths ÷ 4 tenths = 2 \( \frac{1}{4} \)

3. \( \frac{3}{5} \div \frac{1}{3} \)
   - \( \frac{9}{15} \div \frac{5}{15} \) = 9 fifteenths ÷ 5 fifteenths = \( \frac{9}{5} = 1 \frac{4}{5} \)
Lesson 4

Interpreting and Computing Division of a Fraction by a Fraction—More Models

4. \[\frac{3}{4} \div \frac{1}{5}\]

\[
\frac{15}{20} \div \frac{4}{20} = 15 \text{ twentieths} \div 4 \text{ twentieths} = \frac{15}{4}
\]

\[\frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{3}{4}\]

\[\frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{15}{4}\]