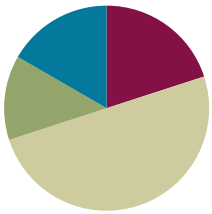
Lesson 10

Objective: Use the area model and division to show the equivalence of two fractions.

Suggested Lesson Structure



Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

Fluency Practice (12 minutes)

- Count by Equivalent Fractions **3.NF.3** (4 minutes)
- Find Equivalent Fractions **4.NF.1** (4 minutes)
- Draw Equivalent Fractions **4.NF.1** (4 minutes)

Count by Equivalent Fractions (4 minutes)

Note: This fluency activity reinforces Module 5 fraction concepts.

T: Count by threes to 24. Start at zero.
S: 0, 3, 6, 9, 12, 15, 18, 21, 24.
T: Count by 3 fourths to 24 fourths. Start at 0 fourths. (Write as students count.)
S: $\frac{0}{4}, \frac{3}{4}, \frac{6}{4}, \frac{9}{4}, \frac{12}{4}, \frac{15}{4}, \frac{18}{4}, \frac{21}{4}, \frac{24}{4}$.
T: 1 is the same as how many fourths?
S: 4 fourths.
T: 2 is the same as how many fourths?
S: 8 fourths.
T: 3 is the same as how many fourths?
S: 12 fourths.
T: (Beneath $\frac{12}{4}$, write 3.) 4 is the same as how many fourths?

$\frac{0}{4}$	$\frac{3}{4}$	$\frac{6}{4}$	$\frac{9}{4}$	$\frac{12}{4}$	$\frac{15}{4}$	$\frac{18}{4}$	$\frac{21}{4}$	$\frac{24}{4}$
0	$\frac{3}{4}$	$\frac{6}{4}$	$\frac{9}{4}$	3	$\frac{15}{4}$	$\frac{18}{4}$	$\frac{21}{4}$	6

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

While leading the Count by Equivalent Fractions fluency activity, enunciate the ending digraph */th/* of fraction names to help English language learners distinguish fractions from whole numbers (e.g., *fourths*, not *fours*).

Couple numbers on the board with prepared visuals, if beneficial.

S: 16 fourths.

T: 5 is the same as how many fourths?

S: 20 fourths.

T: 6 is the same as how many fourths?

S: 24 fourths.

T: (Beneath $\frac{24}{4}$, write 6.) Count by 3 fourths again. This time, say the whole numbers when you arrive at them. Start at zero.

S: 0, $\frac{3}{4}$, $\frac{6}{4}$, $\frac{9}{4}$, 3, $\frac{15}{4}$, $\frac{18}{4}$, $\frac{21}{4}$, 6.

Repeat the process, counting by 3 fifths to 30 fifths.

Find Equivalent Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

T: (Write $\frac{3}{4} = \frac{\times}{\times} = \frac{\_}{8}$. Point to $\frac{3}{4}$.) Say the fraction.

S: 3 fourths.

T: On your personal white board, complete the number sentence.

S: (Write $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$.)

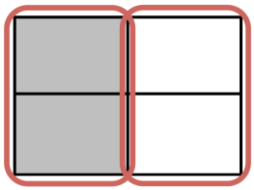
Continue with the following possible suggestions: $\frac{3}{4} = \frac{9}{12}$, $\frac{2}{3} = \frac{4}{6}$, $\frac{2}{5} = \frac{4}{10}$, $\frac{4}{5} = \frac{8}{10}$, and $\frac{3}{5} = \frac{9}{15}$.

Draw Equivalent Fractions (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 9.

T: (Project a model with 2 out of 4 equal units shaded.) Draw the model, and write the fraction that is shaded.



S: (Draw a model with 2 out of 4 equal units shaded. Write $\frac{2}{4}$.)

T: (Write $\frac{2}{4} = \frac{\div}{\div} = \frac{\_}{\_}$.) Compose the shaded units into 1 larger unit by circling. Then, complete the number sentence.

S: (Circle the shaded units into 1 larger unit. Write $\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$.)

Continue with the following possible sequence: $\frac{3}{9} = \frac{1}{3}$, $\frac{4}{8} = \frac{1}{2}$, $\frac{2}{8} = \frac{1}{4}$, $\frac{5}{10} = \frac{1}{2}$, and $\frac{4}{12} = \frac{1}{3}$.

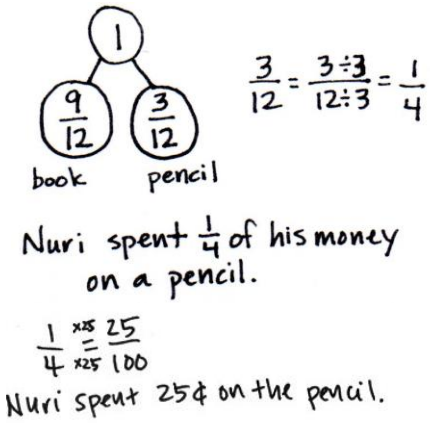
Application Problem (8 minutes)

Nuri spent $\frac{9}{12}$ of his money on a book and the rest of his money on a pencil.

a. Express how much of his money he spent on the pencil in fourths.

b. Nuri started with $1. How much did he spend on the pencil?

Note: This Application Problem connects Topic A and Lesson 9 by finding the other fractional part of the whole and expressing equivalent fractions. Using what students know about money, ask why it is preferable to answer in fourths rather than twelfths. Students connect fourths to quarters of a dollar. Revisit this problem in the Student Debrief to express how much money was spent on the book in fourths.



Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1: Simplify a fraction by drawing to find a common factor, and relate it to division.

T: Draw an area model that represents $\frac{10}{12}$.

T: If we want to compose an equivalent fraction, what do we do?

S: We make equal groups. → We divide the numerator and the denominator by the same number. → We should divide by 10. We divided by the same number that was in the numerator yesterday.

T: Can I divide both the numerator and denominator by 10?

S: No.

T: Discuss with your partner how to determine the largest possible unit.

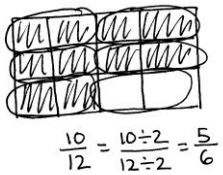
S: We can try to make groups of 2, then 3, then 4, until we have the largest number of units in a group with no remainder. → We can only make equal groups of 2. The other numbers don't divide evenly into both the numerator and denominator.

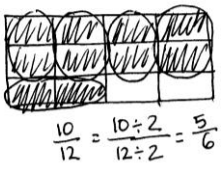
T: Show me. (Allow time for students to compose an area model.) What happened to the number of shaded units?

S: There were 10 units shaded, and now there are 5 groups of 2 units shaded!

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

There are multiple ways of showing a given fraction using an area model. Area models may, therefore, look different from student to student. Allow students to share how they have drawn different area models, and be accepting of those that are mathematically correct.





T: Consider the unit fractions $\frac{1}{12}$ and $\frac{1}{6}$. What do you notice about their denominators?

S: 6 is a factor of 12.

T: What about the numerators 10 and 5?

S: 5 is a factor of 10.

T: List the factors of 10 and 12.

S: The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 10 are 1, 2, 5, and 10.

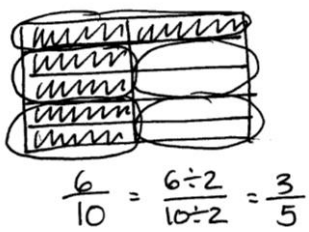
T: 1 and 2 are factors of both. We know then that we can make equal groups of 2. Equal groups of 1 bring us back to the original fraction.

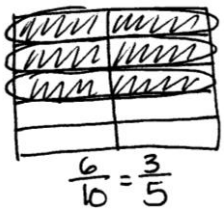
Problem 2: Draw an area model of a number sentence that shows the simplification of a fraction.

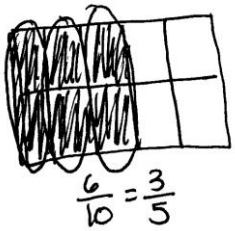
T: (Project $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$.)

T: Draw an area model to show how this number sentence is true.

S: The numerator and denominator are both being divided by 2. I will circle groups of 2. → I know 2 is a factor of 6 and 10, so I could make groups of 2. → There are 3 shaded groups of 2 and 5 total groups of 2. → That's $\frac{3}{5}$.







Problem 3: Simplify a fraction by drawing to find different common factors, and relate it to division.

T: With your partner, draw an area model to represent $\frac{8}{12}$. Rename $\frac{8}{12}$ using larger fractional units. You may talk as you work. (Circulate and listen.)

S: I can circle groups of 2 units. → 2 is a factor of 8 and 12. → There are 6 groups of 2 units. → Four groups are shaded. That's $\frac{4}{6}$.

T: What happens when I use 4 as a common factor instead of 2? Turn and talk.

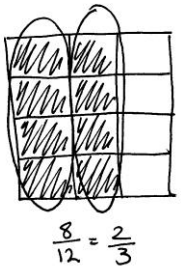
S: Four is a factor of both 8 and 12. It works. → We can make larger units with groups of 4. → Thirds are larger than sixths. $\frac{8}{12} = \frac{2}{3}$. → We have fewer units, but they're bigger.

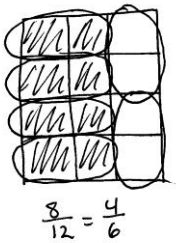
T: Express the equivalent fractions as two division number sentences.

S: (Write $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ and $\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$.)

T: What can you conclude about $\frac{2}{3}$ and $\frac{4}{6}$?

S: They are both equivalent to $\frac{8}{12}$.





T: What is true about dividing the numerator and denominator in $\frac{8}{12}$ by 2 or 4?

S: Two and 4 are both factors of 8 and 12. → The larger the factor used, the larger the new fractional units will be.

T: Interesting. Discuss what your classmate said. "The larger the factor, the larger the new fractional units."

S: When we divided by 2, we got sixths, and when we divided by 4, we got thirds. Thirds are larger. Four is larger than 2. A larger factor gave a larger unit. → When the factor is larger, it means we can make fewer units but larger ones.

Problem 4: Simplify a fraction using the largest possible common factor.

T: Discuss with your partner how to rename $\frac{8}{12}$ with the largest units possible without using an area model.

S: Figure out the greatest number of units that can be placed in equal groups. → Divide the numerator and denominator by the same number, just like we've been doing. → Find a factor of both 8 and 12, and use it to divide the numerator and the denominator.

T: Express the equivalence using a division number sentence.

S: $\frac{8}{12}=\frac{8\div 2}{12\div 2}=\frac{4}{6}$. Four and 6 are still both even, so that wasn't the largest factor. → $\frac{8}{12}=\frac{8\div 4}{12\div 4}=\frac{2}{3}$. The only common factor 2 and 3 have is 1, so 4 must be the largest factor that 8 and 12 have in common.

T: How can we know we expressed an equivalent fraction with the largest units?

S: When we make equal groups, we need to see if we can make larger ones. → When we find the factors of the numerator and denominator, we have to pick the largest factor. Four is larger than 2, so dividing the numerator and denominator by 4 gets us the largest units. → When I found $\frac{4}{6}$, I realized 2 and 4 are both even, so I divided the numerator and denominator again by 2. Two and 3 only have a common factor of 1, so I knew I made the largest unit possible. → Dividing by 2 twice is the same as dividing by 4. Just get it over with faster, and divide by 4.

T: It's not wrong to say that $\frac{8}{12}=\frac{4}{6}$. It is true. It's just that, at times, it really is simpler to work with larger units because it means the denominator is a smaller number.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

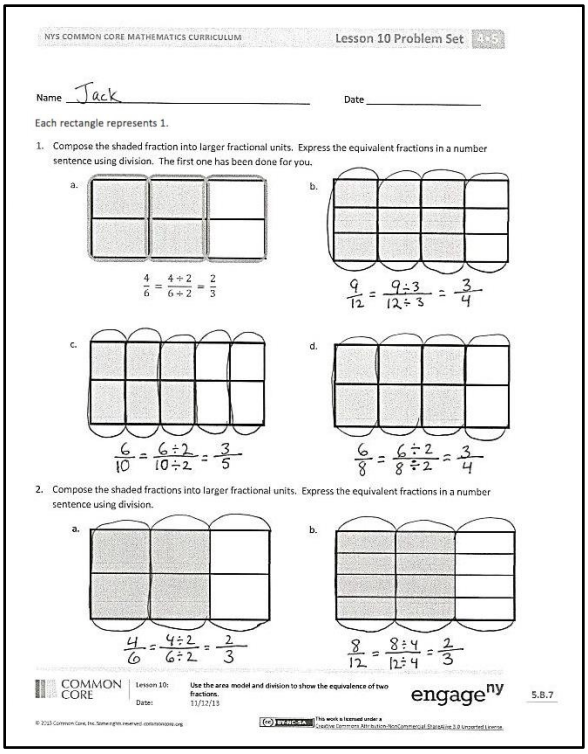
Lesson Objective: Use the area model and division to show the equivalence of two fractions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 2(b), did you compose the same units as your partner? Are both of your answers correct? Why?
- In Problem 4(a–d), how is it helpful to know the common factors for the numerators and denominators?
- In Problem 4, you were asked to use the largest common factor to rename the fraction: $\frac{4}{8}=\frac{1}{2}$. By doing so, you renamed $\frac{4}{8}$ using larger units. How is renaming fractions useful?
- Do fractions always need to be renamed to the largest unit? Explain.
- Why is it important to choose a common factor to make larger units?
- How can you tell that a fraction is composed of the largest possible fractional units?
- When you are drawing an area model and circling equal groups, do all of the groups have to appear the same in shape? How do you know that they still show the same amount?
- Explain how knowing the factors of the numerator and the factors of the denominator can be helpful in identifying equivalent fractions of a larger unit size.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Problem Set 4•5

Name Jack Date

Each rectangle represents 1.

1. Compose the shaded fraction into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

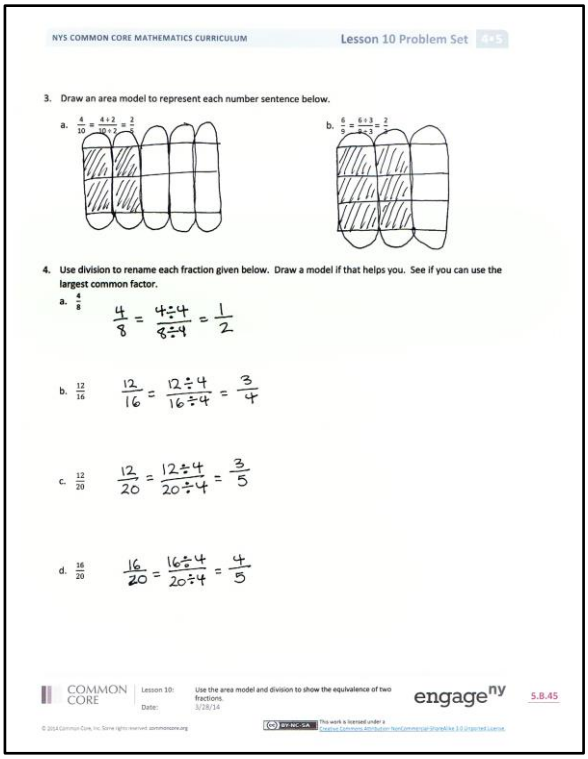
a. $\frac{4}{6}=\frac{4\div2}{6\div2}=\frac{2}{3}$ b. $\frac{9}{12}=\frac{9\div3}{12\div3}=\frac{3}{4}$

c. $\frac{6}{10}=\frac{6\div2}{10\div2}=\frac{3}{5}$ d. $\frac{6}{8}=\frac{6\div2}{8\div2}=\frac{3}{4}$

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a. $\frac{4}{6}=\frac{4\div2}{6\div2}=\frac{2}{3}$ b. $\frac{8}{12}=\frac{8\div4}{12\div4}=\frac{2}{3}$

COMMON CORE Lesson 10: Use the area model and division to show the equivalence of two fractions. Date: 11/12/13 engage^ny 5.B.7


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Problem Set 4•5

3. Draw an area model to represent each number sentence below.

a. $\frac{4}{10}=\frac{4\div2}{10\div2}=\frac{2}{5}$ b. $\frac{6}{9}=\frac{6\div3}{9\div3}=\frac{2}{3}$

4. Use division to rename each fraction given below. Draw a model if that helps you. See if you can use the largest common factor.

a. $\frac{4}{8}$ $\frac{4}{8}=\frac{4\div4}{8\div4}=\frac{1}{2}$

b. $\frac{12}{16}$ $\frac{12}{16}=\frac{12\div4}{16\div4}=\frac{3}{4}$

c. $\frac{12}{20}$ $\frac{12}{20}=\frac{12\div4}{20\div4}=\frac{3}{5}$

d. $\frac{16}{20}$ $\frac{16}{20}=\frac{16\div4}{20\div4}=\frac{4}{5}$

COMMON CORE Lesson 10: Use the area model and division to show the equivalence of two fractions. Date: 3/28/14 engage^ny 5.B.45

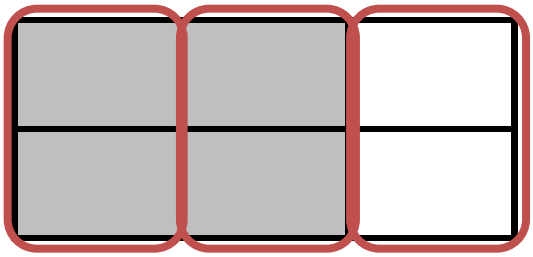
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

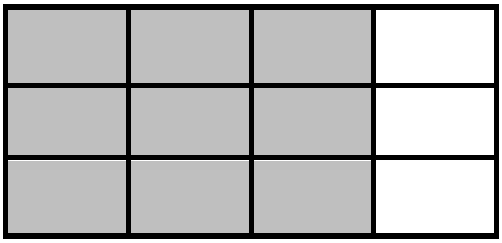
Name _____ Date _____

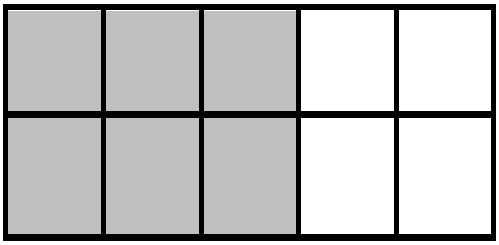
Each rectangle represents 1.

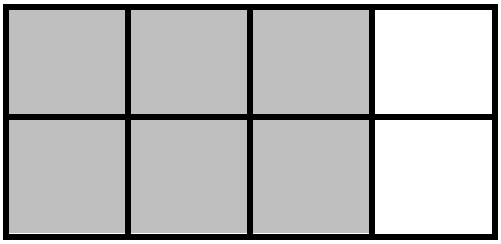
1. Compose the shaded fraction into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a. 

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

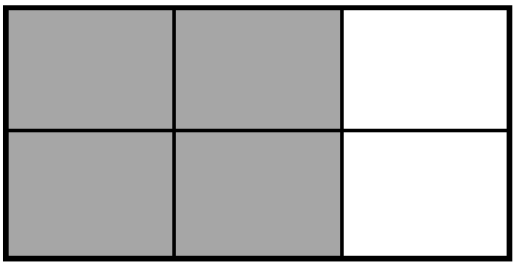
b. 

c. 

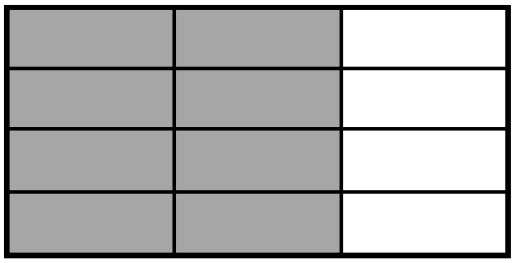
d. 

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.



b.



3. Draw an area model to represent each number sentence below.

a. $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

b. $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

4. Use division to rename each fraction given below. Draw a model if that helps you. See if you can use the largest common factor.

   a. $\frac{4}{8}$

   b. $\frac{12}{16}$

   c. $\frac{12}{20}$

   d. $\frac{16}{20}$

Name _____ Date _____

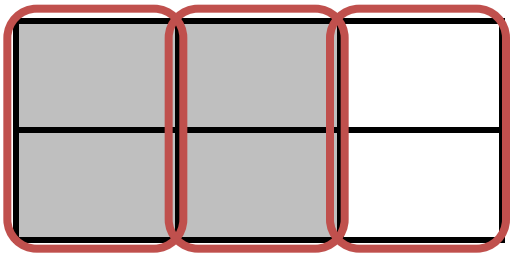
Draw an area model to show why the fractions are equivalent. Show the equivalence in a number sentence using division.

$\frac{4}{10} = \frac{2}{5}$

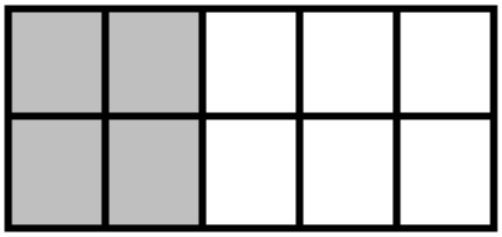
Name _____ Date _____

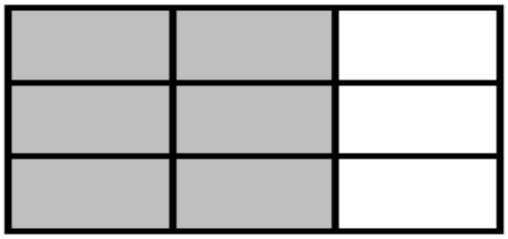
Each rectangle represents 1.

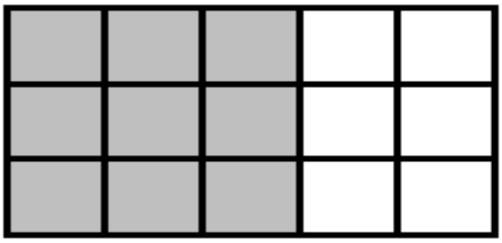
1. Compose the shaded fraction into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a. 

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

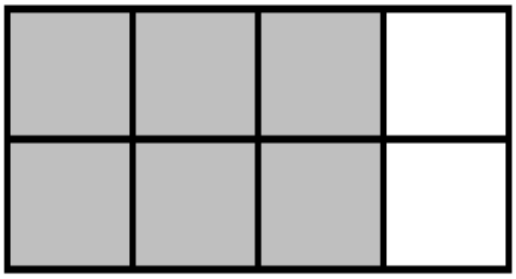
b. 

c. 

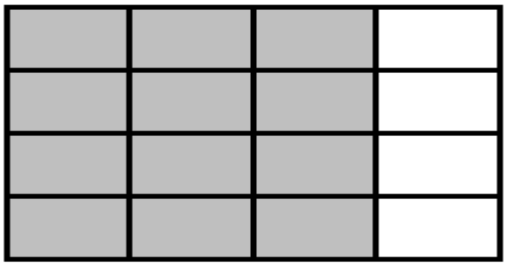
d. 

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.



b.



3. Draw an area model to represent each number sentence below.

a. $\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$

b. $\frac{6}{18} = \frac{6 \div 3}{18 \div 3} = \frac{2}{6}$

4. Use division to rename each fraction given below. Draw a model if that helps you. See if you can use the largest common factor.

a. $\frac{6}{12}$

b. $\frac{4}{12}$

c. $\frac{8}{12}$

d. $\frac{12}{18}$