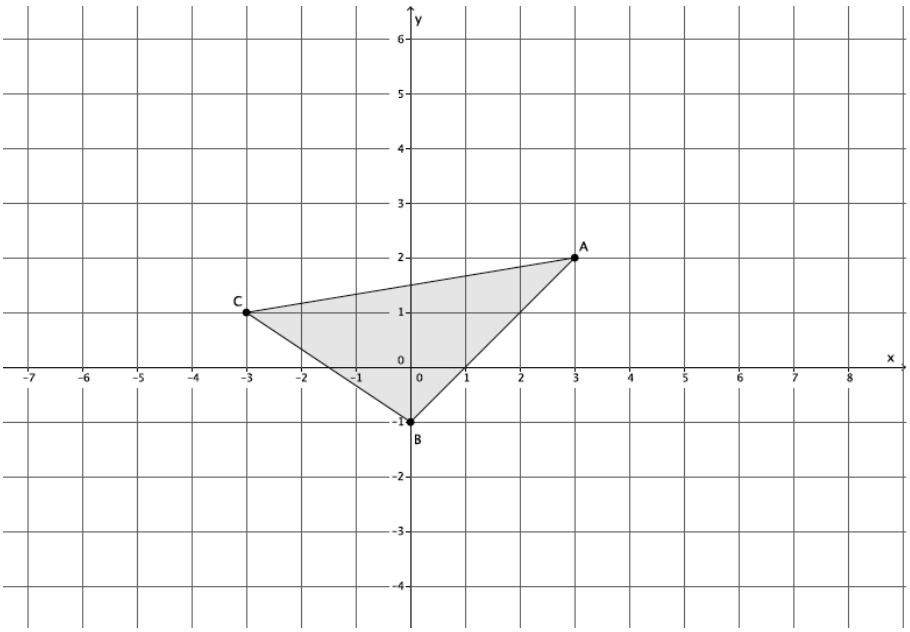
Name _____ Date _____

1. The coordinates of $\triangle ABC$ are shown on the coordinate plane below. $\triangle ABC$ is dilated from the origin by scale factor $r = 2$.



a. Identify the coordinates of the dilated $\triangle A'B'C'$.

b. Is $\triangle A'B'C' \sim \triangle ABC$? Explain.

2. Points A, B, and C are not collinear, forming $\angle BAC$. Extend $\overrightarrow{AB}$ to point P. Line ℓ passes through P and is parallel to segment BC. It meets $\overrightarrow{AC}$ at point Q.

   a. Draw a diagram to represent the situation described.

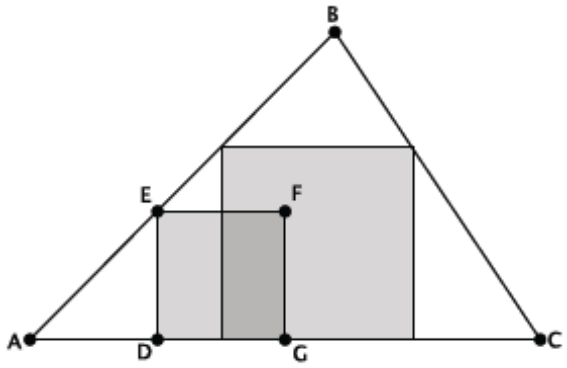
   b. Is $\overline{PQ}$ longer or shorter than $\overline{BC}$?

   c. Prove that $\triangle ABC \sim \triangle APQ$.

   d. What other pairs of segments in this figure have the same ratio of lengths that $\overline{PQ}$ has to $\overline{BC}$?

3. There is a triangular floor space $\triangle ABC$ in a restaurant. Currently, a square portion $DEFG$ is covered with tile. The owner wants to remove the existing tile and then tile the largest square possible within $\triangle ABC$, keeping one edge of the square on $\overline{AC}$.

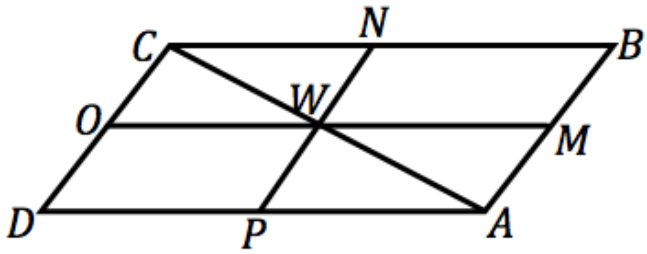
   a. Describe a construction that uses a dilation with center A that can be used to determine the maximum square $D'E'F'G'$ within $\triangle ABC$ with one edge on $\overline{AC}$.



   b. What is the scale factor of $\overline{FG}$ to $\overline{F'G'}$ in terms of the distances $\overline{AF}$ and $\overline{AF'}$?

c. The owner uses the construction in part (a) to mark off where the square would be located. He measures AE to be 15 feet and EE' to be 5 feet. If the original square is 144 square feet, how many square feet of tile does he need for $D'E'F'G'$?

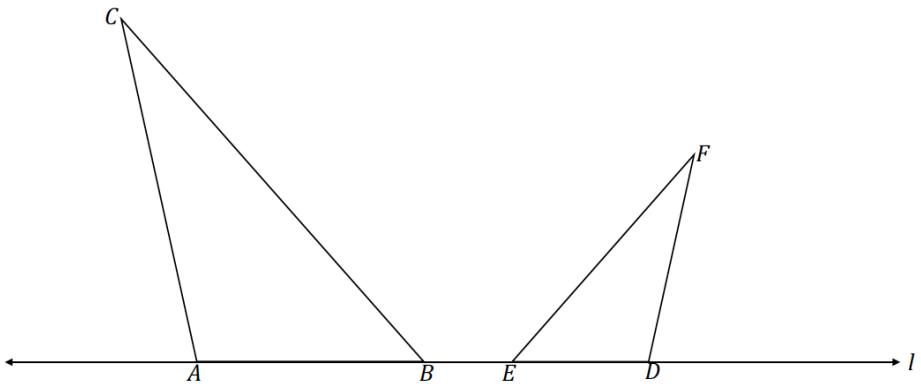
4. $ABCD$ is a parallelogram, with the vertices listed counterclockwise around the figure. Points M, N, O, and P are the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. The segments MO and NP cut the parallelogram into four smaller parallelograms, with the point W in the center of $ABCD$ as a common vertex.



a. Exhibit a sequence of similarity transformations that takes $\triangle AMW$ to $\triangle CDA$. Be specific in describing the parameter of each transformation; for example, if describing a reflection, state the line of reflection.

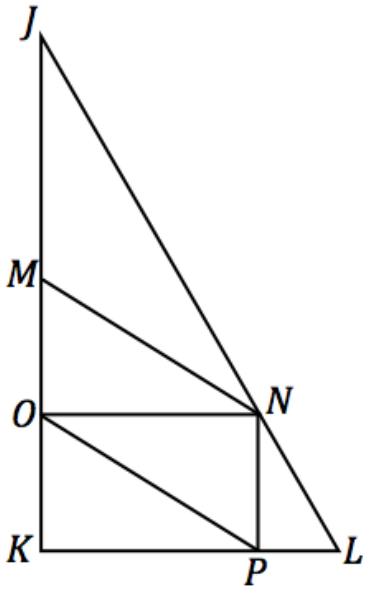
b. Given the correspondence in $\triangle AMW$ similar to $\triangle CDA$, list all corresponding pairs of angles and corresponding pairs of sides. What is the ratio of the corresponding pairs of angles? What is the ratio of the corresponding pairs of sides?

5. Given two triangles, $\triangle ABC$ and $\triangle DEF$, $m\angle CAB = m\angle FDE$, and $m\angle CBA = m\angle FED$. Points A, B, D, and E lie on line l as shown. Describe a sequence of rigid motions and/or dilations to show that $\triangle ABC \sim \triangle DEF$, and sketch an image of the triangles after each transformation.



6. $\triangle JKL$ is a right triangle; $\overline{NP} \perp \overline{KL}, \overline{NO} \perp \overline{JK}, \overline{MN} \parallel \overline{OP}$.

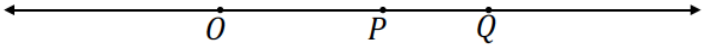
   a. List all sets of similar triangles. Explain how you know.



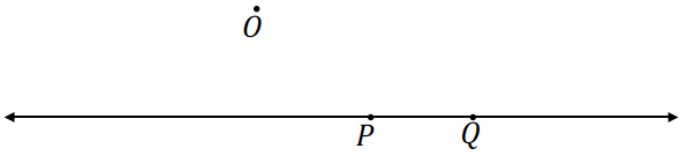
   b. Select any two similar triangles, and show why they are similar.

7.

a. The line PQ contains point O. What happens to $\overleftrightarrow{PQ}$ with a dilation about O and scale factor of $r = 2$? Explain your answer.



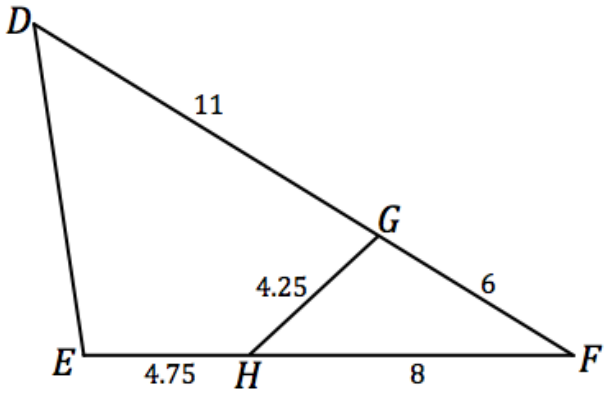
b. The line PQ does not contain point O. What happens to $\overleftrightarrow{PQ}$ with a dilation about O and scale factor of $r = 2$?



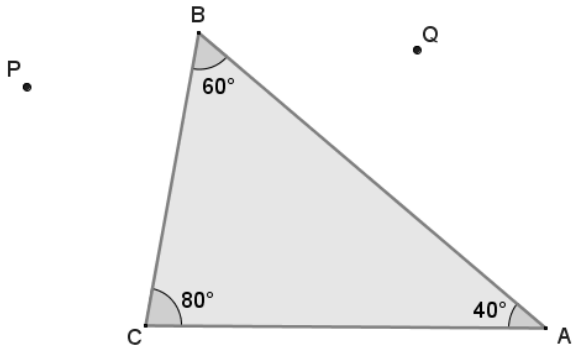
8. Use the diagram below to answer the following questions.

   a. State the pair of similar triangles. Which similarity criterion guarantees their similarity?

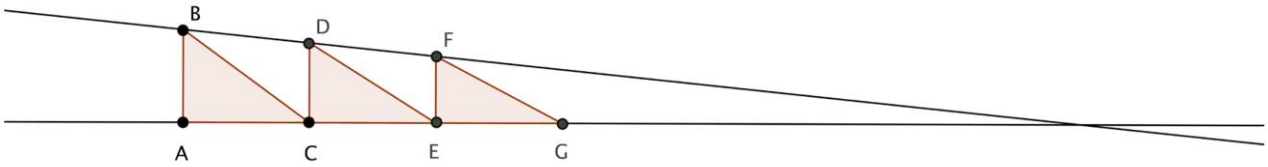
   b. Calculate DE to the hundredths place.



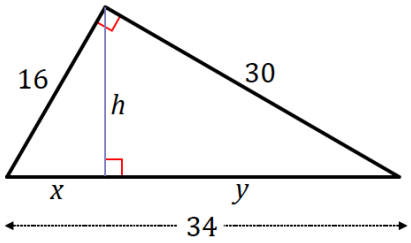
9. In $\triangle ABC$, $m\angle A$ is $40°$, $m\angle B$ is $60°$, and $m\angle C$ is $80°$. The triangle is dilated by a factor of 2 about point P to form $\triangle A'B'C'$. It is also dilated by a factor of 3 about point Q to form $\triangle A''B''C''$. What is the measure of the angle formed by line $A'B'$ and line $B''C''$? Explain how you know.



10. In the diagram below, $|AC| = |CE| = |EG|$, and $\angle BAC$, $\angle DCE$, and $\angle FEG$ are right. The two lines meet at a point to the right. Are the triangles similar? Why or why not?



11. The side lengths of the following right triangle are 16, 30, and 34. An altitude of a right triangle from the right angle splits the hypotenuse into line segments of length x and y.

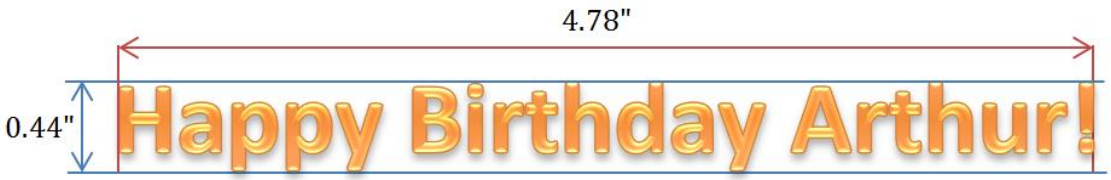


a. What is the relationship between the large triangle and the two sub-triangles? Why?

b. Solve for h, x, and y.

c. **Extension:** Find an expression that gives h in terms of x and y.

12. The sentence below, as shown, is being printed on a large banner for a birthday party. The height of the banner is 18 inches. There must be a minimum 1-inch margin on all sides of the banner. Use the dimensions in the image below to answer each question.



a. Describe a reasonable figure in the plane to model the printed image.

b. Find the scale factor that maximizes the size of the characters within the given constraints.

c. What is the total length of the banner based on your answer to part (a)?

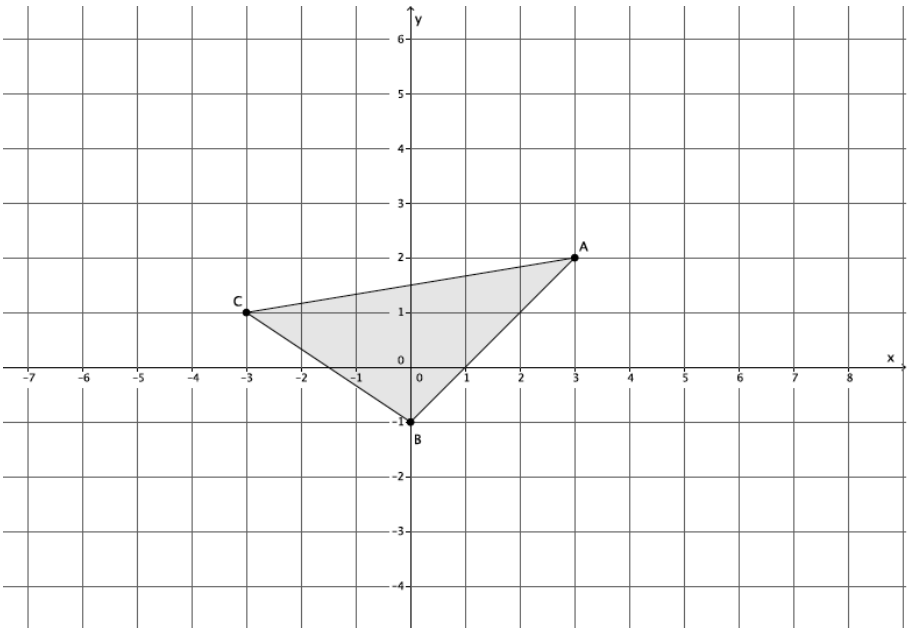
A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a–b** G-SRT.A.2	Student provides at least two incorrect coordinates of part (a), and student does not show clear understanding of why $\triangle A'B'C' \sim \triangle ABC$ for part (b).	Student provides only partially correct and clear answers for both parts (a) and (b).	Student answers part (a) correctly, but part (b) lacks clear explanation regarding the corresponding angles and length measurements. OR Student answers part (b) correctly, and part (a) has errors in the coordinates of the dilated vertices.	Student answers both parts (a) and (b) correctly.
2	**a–d** G-SRT.B.5	Student provides a correct response for part (b), but student shows insufficient understanding or inaccurate answers for parts (a), (c), and (d).	Student incorrectly answers two parts.	Student incorrectly answers one part (e.g., incorrect diagram in part (a) or insufficient evidence provided for (c)).	Student correctly answers all four parts.
3	**a–c** G-SRT.A.1	Student provides a response for part (a) that is missing two or more construction steps, and student incorrectly answers parts (b) and (c).	Student provides a response for part (a) that is missing two or more construction steps, and student incorrectly answers part (b) or part (c).	Student provides a response for part (a) that is missing two or more construction steps but correctly answers parts (b) and (c). OR Student correctly answers part (a) but incorrectly answers part (b) or part (c).	Student correctly answers all three parts.

4	**a–b** **G-SRT.A.2**	Student provides an incomplete or otherwise inaccurate description of a similarity transformation for part (a). Student provides an incomplete list of correspondences for part (b) and makes errors in the ratios of pairs of angles or pairs of sides.	Student provides an incomplete or otherwise inaccurate description of a similarity transformation for part (a). Student provides an incomplete list of correspondences or makes errors in the ratios of pairs of angles or pairs of sides in part (b).	Student provides an incomplete or otherwise inaccurate description of a similarity transformation for part (a) but correctly answers part (b). OR Student provides a correct answer for part (a) but provides an incomplete list of correspondences or makes errors in the ratios of pairs of angles or pairs of sides in part (b).	Student correctly answers both parts (a) and (b).
5	**G-SRT.A.3**	Student describes an incoherent sequence of transformations or provides no description and does not explain parameters to show that one triangle maps to the other.	Student describes a sequence of transformations that maps one triangle to the other but does not provide detail regarding the parameters (e.g., a reflection is cited, but no detail regarding the line of reflection is mentioned).	Student describes a sequence of transformations that maps one triangle to the other but does not provide detail regarding the parameters of one of the transformations, or one additional transformation and its respective parameters are needed to complete a correct sequence of transformations.	Student clearly describes a sequence of appropriate transformations and provides the appropriate parameters for each transformation.
6	**a–b** **G-SRT.B.5**	Student provides a response that is missing three or more similar triangles out of both sets.	Student lists all but two of the similar triangles out of both sets.	Student lists all but one of the similar triangles out of both sets.	Student correctly lists all the similar triangles in each set.
7	**a–b** **G-SRT.B.5**	Student does not show an understanding of the properties of dilations in either part (a) or part (b).	Student correctly answers part (a) but incorrectly answers part (b).	Student correctly answers part (b) but does not provide justification as to why the line maps to itself in part (a).	Student correctly answers parts (a) and (b).

8	**a–b** **G-SRT.B.5**	Student provides incorrect similarity criteria for part (a) and makes more than one conceptual or computational error in part (b).	Student provides incorrect similarity criteria for part (a) and makes one conceptual or computational error in part (b).	Student correctly answers part (a) but makes one conceptual or computational error in part (b). OR Student provides incorrect similarity criteria for part (a) but correctly answers part (b).	Student correctly answers parts (a) and (b).
9	**G-SRT.A.1**	Student shows little or no understanding of why the two dilations lead to an angle of $60°$ formed by $A'B'$ and $B''C''$.	Student includes an attempted diagram but is missing multiple conclusive elements or has an error in the verbal justification.	Student includes an attempted diagram but is missing one conclusive element (e.g., missing one of the dilations or an incorrect dilation) or has an error in the verbal justification.	Student includes an accurate diagram and provides a complete and correct justification.
10	**G-SRT.B.4**	Student shows little or no understanding of why the triangles are not similar.	Student includes a justification that demonstrates why the triangles are not similar but has two conceptual errors.	Student includes a justification that demonstrates why the triangles are not similar but has one conceptual error.	Student includes an accurate justification that demonstrates why the triangles are not similar.
11	**a–c** **G-SRT.B.5**	Student does not provide fully correct answers for parts (a), (b), and (c).	Student provides a fully correct answer for one of the three parts.	Student provides fully correct answers for any two of the three parts.	Student provides correct answers for parts (a), (b), and (c) and clearly explains each.
12	**a–c** **G-MG.A.1** **G-MG.A.3**	Student does not provide fully correct answers for parts (a), (b), and (c).	Student provides a fully correct answer for one of the three parts.	Student correctly computes scaled dimensions for the printed image but fails to consider the required margins.	Student provides correct answers for parts (a), (b), and (c) and clearly explains each.

Name _____ Date _____

1. The coordinates of $\triangle ABC$ are shown on the coordinate plane below. $\triangle ABC$ is dilated from the origin by scale factor $r = 2$.



a. Identify the coordinates of the dilated $\triangle A'B'C'$.

Point A = (3, 2), then A′ = (2×(3), 2×(2)) = (6, 4)

Point B = (0, −1), then B′ = (2×(0), 2×(−1)) = (0, −2)

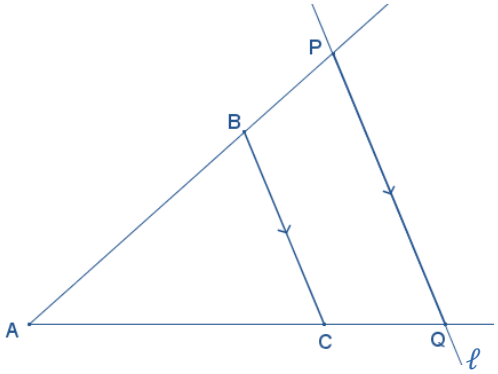
Point C = (−3, 1), then C′ = (2×(−3), 2×(1)) = (−6, 2)

b. Is $\triangle A'B'C' \sim \triangle ABC$? Explain.

Yes. The side lengths of △A′B′C′ are each two times the length of the sides of △ABC, and corresponding sides are proportional in length. Also, the corresponding angles are equal in measurement because dilations preserve the measurements of angles.

2. Points A, B, and C are not collinear, forming $\angle BAC$. Extend $\overrightarrow{AB}$ to point P. Line ℓ passes through P and is parallel to segment BC. It meets ray AC at point Q.

   a. Draw a diagram to represent the situation described.



   b. Is $\overline{PQ}$ longer or shorter than $\overline{BC}$?

   $\overline{PQ}$ is longer than $\overline{BC}$.

   c. Prove that $\triangle ABC \sim \triangle APQ$.

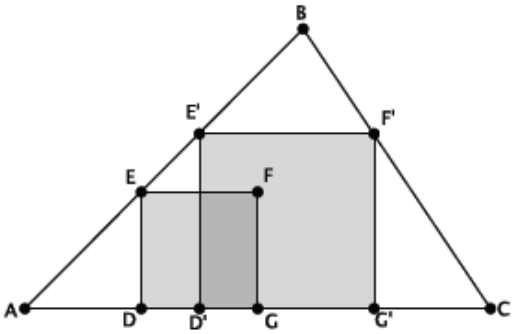
   $\overline{PQ} \parallel \overline{BC}$. Since corresponding angles are equal in measure, then m∠ABC = m∠APQ. Additionally, m∠A = m∠A. Then △ABC ~ △APQ by AA similarity criterion.

   d. What other pairs of segments in this figure have the same ratio of lengths that $\overline{PQ}$ has to $\overline{BC}$?

   PA:BA, QA:CA

3. There is a triangular floor space $\triangle ABC$ in a restaurant. Currently, a square portion $DEFG$ is covered with tile. The owner wants to remove the existing tile and then tile the largest square possible within $\triangle ABC$, keeping one edge of the square on AC.

   a. Describe a construction that uses a dilation with center A that can be used to determine the maximum square $D'E'F'G'$ within $\triangle ABC$ with one edge on $\overline{AC}$.



1. Use A as a center of dilation.
2. Draw $\overrightarrow{AF}$ through BC.
3. Label the intersection of $\overrightarrow{AF}$ and BC as F'.
4. Construct $\overrightarrow{E'F'}$ parallel to EF, where E' is the intersection of AB and the parallel line.
5. Construct $\overrightarrow{F'G'}$ parallel to FG, where G' is the intersection of AC and the parallel line.
6. Construct $\overrightarrow{E'D'}$ parallel to ED, where D' is the intersection of AC and the parallel line.
7. Connect D', E', F', G'.

   b. What is the scale factor of FG to $F'G'$ in terms of the distances AF and AF'?

$\frac{AF'}{AF}$

c. The owner uses the construction in part (a) to mark off where the square would be located. He measures AE to be 15 feet and EE' to be 5 feet. If the original square is 144 square feet, how many square feet of tile does he need for $D'E'F'G'$?

The distance AE' = 20 ft, so the scale factor from DEFG to D'E'F'G' is $\frac{20}{15} = \frac{4}{3}$.
The areas of similar figures are related by the square of the scale factor; therefore,

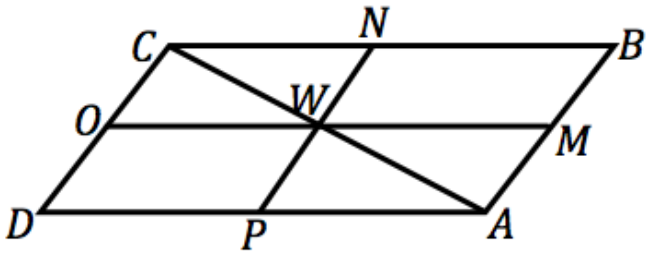
$$\text{Area}(D'E'F'G') = \left(\frac{4}{3}\right)^2 \text{Area}(DEFG)$$

$$\text{Area}(D'E'F'G') = \frac{16}{9}(144)$$

$$\text{Area}(D'E'F'G') = 256$$

The owner needs 256 square feet of tile for D'E'F'G'.

4. $ABCD$ is a parallelogram, with the vertices listed counterclockwise around the figure. Points M, N, O and P are the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. The segments MO and NP cut the parallelogram into four smaller parallelograms, with the point W in the center of $ABCD$ as a common vertex.



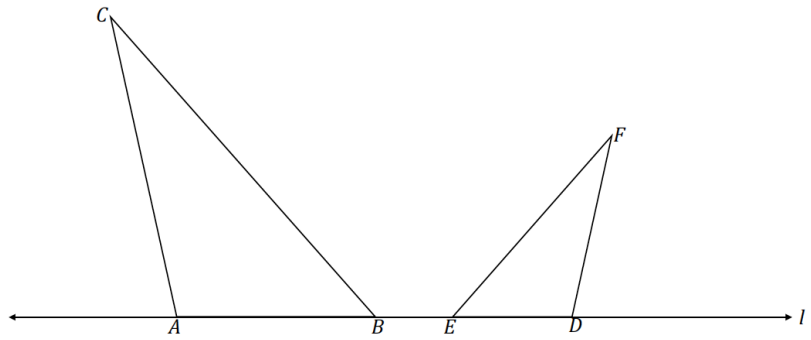
a. Exhibit a sequence of similarity transformations that takes $\triangle AMW$ to $\triangle CDA$. Be specific in describing the parameter of each transformation; for example, if describing a reflection, state the line of reflection.

Answers will vary (e.g., 180° rotation about W; dilation with center of dilation C).

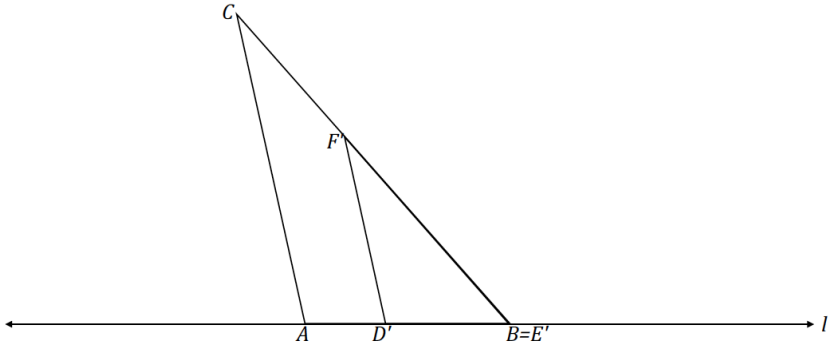
b. Given the correspondence in $\triangle AMW$ similar to $\triangle CDA$, list all corresponding pairs of angles and corresponding pairs of sides. What is the ratio of the corresponding pairs of angles? What is the ratio of the corresponding pairs of sides?

∠MAW corresponds to ∠DCA; ∠AMW corresponds to ∠CDA; ∠AWM corresponds to ∠CAD. $\overline{AM}$ corresponds to $\overline{CD}$; $\overline{MW}$ corresponds to $\overline{DA}$; $\overline{WA}$ corresponds to $\overline{AC}$.
The ratio of corresponding pairs of angles is 1: 1.
The ratio of corresponding pairs of sides is 1: 2.

5. Given two triangles, ABC and DEF, $m\angle CAB = m\angle FDE$, and $m\angle CBA = m\angle FED$. Points A, B, D, and E lie on line l as shown. Describe a sequence of rigid motions and/or dilations to show that $\triangle ABC \sim \triangle DEF$, and sketch an image of the triangles after each transformation.



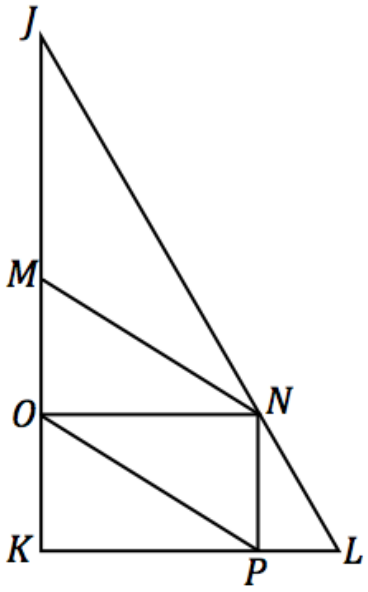
Reflect △DEF over the perpendicular bisector of $\overline{BE}$. The reflection takes E to B and D to a point on line AB. Since angle measures are preserved in rigid motions, F must map to a point on $\overline{BC}$.



- By the hypothesis, m∠A = m∠D; therefore, $\overline{D'F'} \parallel \overline{AC}$ since corresponding angles are equal in measure.
- Since dilations map a segment to a parallel line segment, dilate △D'E'F' about E' and by scale factor $r = \frac{BA}{BD'}$ and that sends D' to A.
- By the dilation theorem, F' goes to C.

6. $\triangle JKL$ is a right triangle; $\overline{NP} \perp \overline{KL}, \overline{NO} \perp \overline{JK}, \overline{MN} \parallel \overline{OP}$.

a. List all sets of similar triangles. Explain how you know.



Set 1

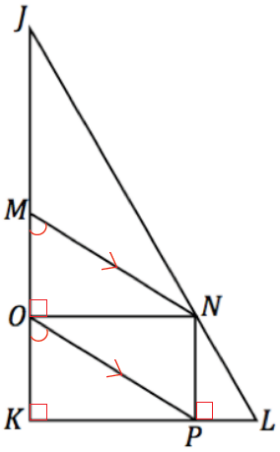
- *△MNO*
- *△PON*
- *△OPK*

Set 2

- *△JON*
- *△JKL*
- *△NPL*

The triangles are similar because of the AA criterion.

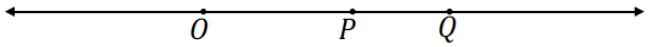
b. Select any two similar triangles, and show why they are similar.



Possible response:
△MNO~△PON by the AA criterion.
∠K is a right angle since △JKL is a right triangle.
∠MON is a right angle since $\overline{NO} \perp \overline{JK}$.
m∠NMO = m∠POK since $\overline{MN} \parallel \overline{OP}$, and $\overline{JK}$ is a transversal that intersects $\overline{MN}$ and $\overline{OP}$; corresponding ∠'s are equal in measure.
Therefore, by the AA criterion, △MNO~△PON.

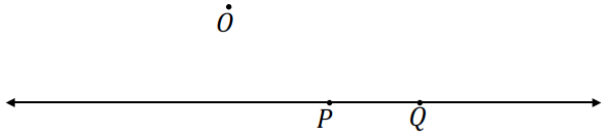
7.

a. The line PQ contains point O. What happens to $\overleftrightarrow{PQ}$ with a dilation about O and scale factor of $r = 2$? Explain your answer.



- *Since the points P and Q are collinear with the center O, then by definition of a dilation, both P′ and Q′ will also be collinear with the center O.*
- *The line PQ maps to itself.*

b. The line $\overleftrightarrow{PQ}$ does not contain point O. What happens to $\overleftrightarrow{PQ}$ with a dilation about O and scale factor of $r = 2$?



The line PQ maps to a parallel line P′Q′.

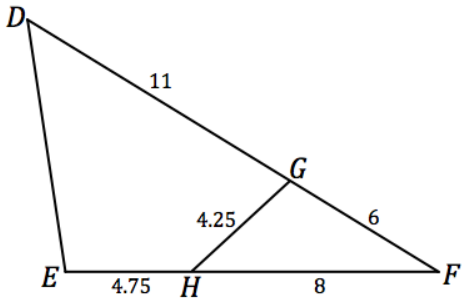
8. Use the diagram below to answer the following questions.

   a. State the pair of similar triangles. Which similarity criterion guarantees their similarity?

   $\triangle DEF \sim \triangle FGH$
   SAS

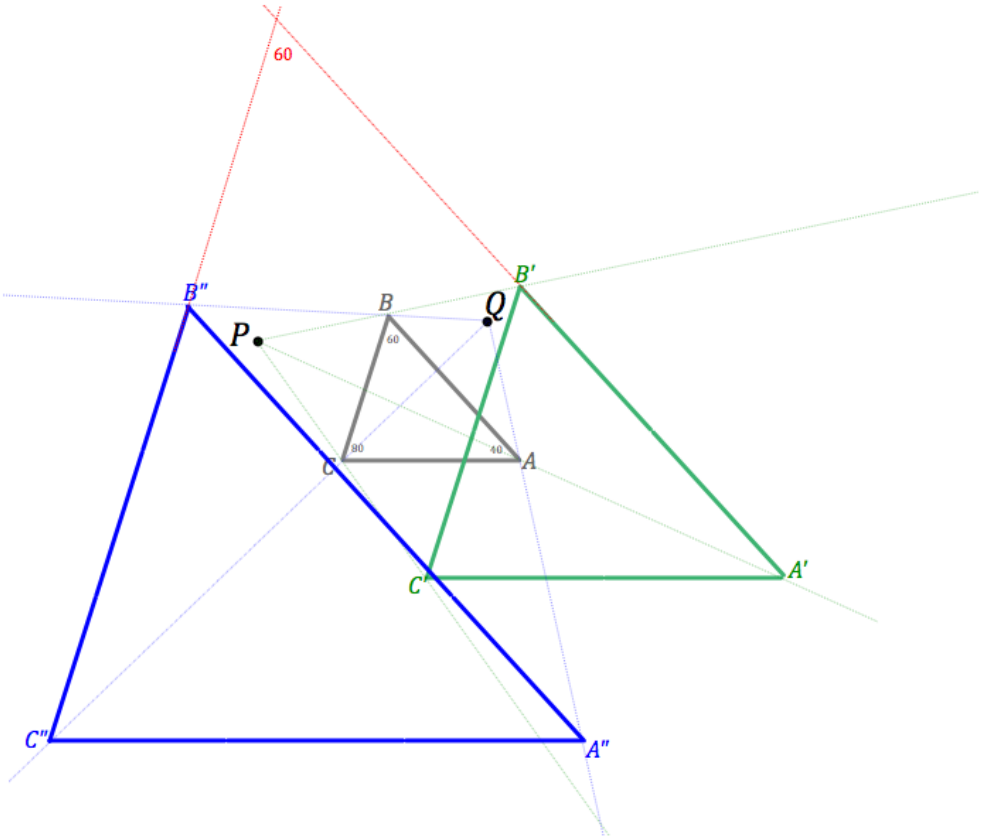
   b. Calculate DE to the hundredths place.

   $\frac{DE}{FG} = \frac{DF}{FH}$
   $\frac{DE}{6} = \frac{17}{8}$
   $8DE = 102$
   $DE \approx 12.75$

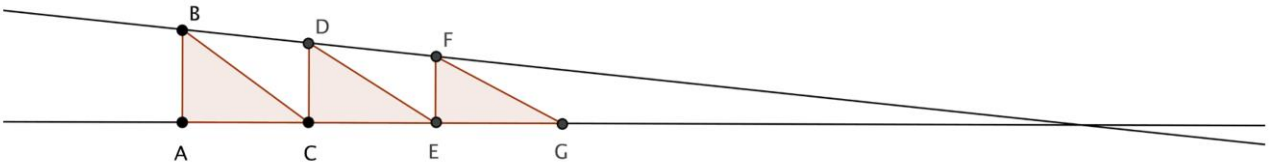


9. In $\triangle ABC$, $m\angle A$ is $40°$, $m\angle B$ is $60°$, and $m\angle C$ is $80°$. The triangle is dilated by a factor of 2 about point P to form $\triangle A'B'C'$. It is also dilated by a factor of 3 about point Q to form $\triangle A''B''C''$. What is the measure of the angle formed by line $A'B'$ and line $B''C''$? Explain how you know.

60°. $\overline{B'C'}$ and $\overline{A'B'}$ meet to form a 60° angle. Since dilations map segments to parallel segments, $\overline{B'C'}||\overline{B''C''}$. Then the angle formed by lines A'B' and B''C'' is a corresponding angle to $\angle B$ and has a measure of 60°.



10. In the diagram below, $AC = CE = EG$, and $\angle BAC$, $\angle DCE$, and $\angle FEG$ are right. The two lines meet at a point to the right. Are the triangles similar? Why or why not?

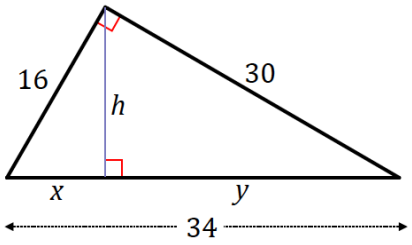


The triangles are not similar.

If they were similar, then there would have to be a similarity transformation taking A to C, B to D, and C to E. But then the dilation factor for this transformation would have to be 1, since AC = CE.

But since the dilation factor is 1 and $\overline{CD}$ is the image of $\overline{AB}$, then it must be true that AB = CD. Additionally, we know that $\overline{AB}$ is parallel to $\overline{CD}$, and since ∠BAC and ∠DCE are right, this implies ACDB is a rectangle. This implies that $\overline{BD}$ is parallel to $\overline{AC}$, but this is contrary to the given.

11. The side lengths of the following right triangle are 16, 30, and 34. An altitude of a right triangle from the right angle splits the hypotenuse into line segments of length x and y.



a. What is the relationship between the large triangle and the two sub-triangles? Why?

An altitude drawn from the vertex of the right angle of a right triangle to the hypotenuse divides the right triangle into two sub-triangles that are similar to the original triangle by the AA criterion.

b. Solve for h, x, and y.

$$\frac{h}{30} = \frac{16}{34} \qquad \frac{x}{16} = \frac{16}{34} \qquad \frac{y}{30} = \frac{30}{34}$$

$$h = \frac{240}{17} \qquad x = \frac{128}{17} \qquad y = \frac{450}{17}$$

c. **Extension.** Find an expression that gives h in terms of x and y.

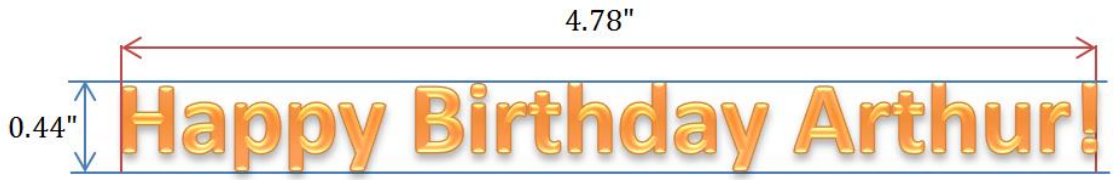
The large triangle is similar to both sub-triangles, and both sub-triangles are, therefore, similar. Then

$$\frac{x}{h} = \frac{h}{y}$$

$$h^2 = xy$$

$$h = \sqrt{xy}.$$

12. The sentence below, as shown, is being printed on a large banner for a birthday party. The height of the banner is 18 inches. There must be a minimum 1-inch margin on all sides of the banner. Use the dimensions in the image below to answer each question.



a. Describe a reasonable figure in the plane to model the printed image.

The sentence can be modeled as a rectangle with dimensions 4.78" × 0.44".

b. Find the scale factor that maximizes the size of the characters within the given constraints.

The scaled height of the rectangle cannot exceed 16" to allow for 1" margins above and below on the banner.
16 = k(0.44), where k represents the scale factor of the banner;

$$k = \frac{16}{0.44} = \frac{400}{11} = 36\frac{4}{11}$$

c. What is the total length of the banner based on your answer to part (a)?

Using the scale factor from part (a),

$$y = \left(\frac{400}{11}\right)\cdot(4.78)$$

$$y = 173\frac{9}{11}$$

The total length of the image in the banner is $173\frac{9}{11}$ inches; however, the banner must have a minimum 1" margin on all sides of the image, so the banner must be at least 2" longer. The total length of the banner must be at least $175\frac{9}{11}$ inches.