

Lesson 7: Expected Value of a Discrete Random Variable

Classwork

Exploratory Challenge 1/Exercises 1–5

A new game, Six Up, involves two players. Each player rolls her die, counting the number of times a six is rolled. The players roll their dice for up to one minute. The first person to roll 15 sixes wins. If no player rolls 15 sixes in the one-minute time limit, then the player who rolls the greatest number of sixes wins that round. The player who wins the most rounds wins the game.

Suppose that your class will play this game. Your teacher poses the following question:

How many sixes would you expect to roll in one round?

1. How would you answer this question?
2. What discrete random variable should you investigate to answer this question?
3. What are the possible values for this discrete random variable?
4. Do you think these possible values are all equally likely to be observed?
5. What might you do to estimate the probability of observing each of the different possible values?

Exploratory Challenge 1/Exercises 6–8

You and your partner will play the Six Up game. Roll the die until the end of one minute or until you or your partner rolls 15 sixes. Count the number of sixes you rolled. Remember to stop rolling if either you or your partner rolls 15 sixes before the end of one minute.

6. Play five rounds. After each round, record the number of sixes that you rolled.

Round 1	Round 2	Round 3	Round 4	Round 5

7. On the board, put a tally mark for the number of sixes rolled in each round.

Number of Sixes Rolled	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Frequency																

8. Using the data summarized in the frequency chart on the board, find the mean number of sixes rolled in a round.

Exploratory Challenge 1/Exercises 9–13

9. Calculate the relative frequency (*proportion*) for each value of the discrete random variable (i.e., the number of sixes rolled) by dividing the frequency for each possible value of the number of sixes rolled by the total number of rounds (the total number of tally marks). (The relative frequencies can be interpreted as estimates of the probabilities of observing the different possible values of the discrete random variable.)

Number of Sixes Rolled	0	1	2	3	4	5	6	7
Relative Frequency								

Number of Sixes Rolled	8	9	10	11	12	13	14	15
Relative Frequency								

10. Multiply each possible value for the number of sixes rolled by the corresponding probability (relative frequency).
11. Find the sum of the calculated values in Exercise 10. This number is called the *expected value* of the discrete random variable.

12. What do you notice about the sum in Exercise 11 and the mean that you calculated in Exercise 8?
13. The *expected value* of a random variable, x , is also called the *mean* of the distribution of that random variable. Why do you think it is called the mean?

Exploratory Challenge 1/Exercise 14

The expected value for a discrete random variable is computed using the following equation:

$$\text{expected value} = \sum (\text{each value of the random variable}(x)) \cdot (\text{the corresponding probability}(p))$$

or

$$\text{expected value} = \sum xp$$

where x is a possible value of the random variable, and p is the corresponding probability.

The following table provides the probability distribution for the number of heads occurring when two coins are flipped.

Number of Heads	0	1	2
Probability	0.25	0.5	0.25

14. If two coins are flipped many times, how many heads would you expect to occur, on average?

Exploratory Challenge 1/Exercises 15–16

15. The estimated expected value for the number of sixes rolled in one round of the Six Up game was 10.4. Write a sentence interpreting this value.
16. Suppose that you plan to change the rules of the Six Up game by increasing the one-minute time limit for a round. You would like to set the time so that most rounds will end by a player reaching 15 sixes. Considering the estimated expected number of sixes rolled in a one-minute round, what would you recommend for the new time limit? Explain your choice.

Exploratory Challenge 2/Exercises 17–19

Suppose that we convert the table on the previous page displaying the discrete distribution for the number of heads occurring when two coins are flipped to two vectors.

Let vector **a** be the number of heads occurring.

Let vector **b** be the corresponding probabilities.

$$\mathbf{a} = \langle 0, 1, 2 \rangle \qquad \mathbf{b} = \langle 0.25, 0.5, 0.25 \rangle$$

17. Find the dot product of these two vectors.
18. Explain how the dot product computed in Exercise 17 compares to the expected value computed in Exercise 14.

19. How do these two processes, finding the expected value of a discrete random variable and finding the dot product of two vectors, compare?

Lesson Summary

The *expected value* of a random variable is the *mean* of the distribution of that random variable.

The expected value of a discrete random variable is the *sum* of the *products* of each possible value (x) and the corresponding probability.

The process of computing the expected value of a discrete random variable is similar to the process of computing the dot product of two vectors.

Problem Set

1. The number of defects observed in the paint of a newly manufactured car is a discrete random variable. The probability distribution of this random variable is shown in the table below.

Number of Defects	0	1	2	3	4	5
Probability	0.02	0.15	0.40	0.35	0.05	0.03

If large numbers of cars were inspected, what would you expect to see for the average number of defects per car?

- 2.
- Interpret the expected value calculated in Problem 1. Be sure to give your interpretation in context.
 - Explain why it is not reasonable to say that every car will have the expected number of defects.
3. Students at a large high school were asked how many books they read over the summer. The number of books read is a discrete random variable. The probability distribution of this random variable is shown in the table below.

Number of Books Read	0	1	2	3
Probability	0.12	0.33	0.48	0.07

If a large number of students were asked how many books they read over the summer, what would you expect to see for the average number of books read?

4. Suppose two dice are rolled. The sum of the two numbers showing is a discrete random variable. The following table displays the probability distribution of this random variable:

Sum Rolled	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

If you rolled two dice a large number of times, what would you expect the average of the sum of the two numbers showing to be?

5. Explain why it is not possible for a random variable whose only possible values are 3, 4, and 5 to have an expected value greater than 6.
6. Consider a discrete random variable with possible values 1, 2, 3, and 4. Create a probability distribution for this variable so that its expected value would be greater than 3 by entering probabilities into the table below. Then, calculate the expected value to verify that it is greater than 3.

Value of Variable	1	2	3	4
Probability				