Lesson 8: Law of Sines

Student Outcomes

- Students prove the law of sines and use it to solve problems (G-SRT.D.10).

Lesson Notes

In previous lessons, students developed tools for finding a missing side or a missing angle in a right triangle. In particular, students know how to use the Pythagorean theorem to find a missing side of a right triangle, and they know how to use the trigonometric functions to find a missing angle in a right triangle. In this lesson, students extend their knowledge to cover certain oblique triangles. Students develop a proof of the law of sines and apply it to solve problems. At the end of the lesson, students develop a second proof of the law of sines that uses properties of circles.

In the course of the lesson, students draw on their knowledge of the conditions that imply a unique triangle, which students know as the triangle congruence criteria (ASA, SSS, SAS). This topic was addressed in Geometry Module 1 Lessons 22, 24, and 25.

Classwork

Exercises 1–2 (2 minutes)

In these exercises, students apply their prior knowledge of right triangle trigonometry to solve problems.

1. Find the value of $x$ in the figure on the left.

\[
\sin(33^\circ) = \frac{8.4}{x}
\]

\[
x = \frac{8.4}{\sin(33^\circ)} \approx 15.4
\]

2. Find the value of $\alpha$ in the figure on the right.

\[
\sin(\alpha) = \frac{4.2}{9.1}
\]

\[
\alpha \approx 27^\circ
\]
Exploratory Challenge (9 minutes): Oblique Triangles

- In Exercises 1 and 2, what aspects of the triangle were you given, and what aspects did you find?
  - In Exercise 1, we were given a side and an angle in a right triangle. From these measurements, we determined the length of the hypotenuse.
  - In Exercise 2, we were given two sides in a right triangle. From these measurements, we determined the measure of one of the acute angles.

- Now examine the triangle above. What information is provided in this triangle?
  - We are given two angles and a non-included side.

- How is this triangle different from the ones presented in the Exercises 1 and 2?
  - The problems in Exercises 1 and 2 involved right triangles, but this triangle does not have a right angle.

- Do you think you should be able to determine the remaining measurements in this triangle? Why or why not? Think about this for a moment, then explain your reasoning to a neighbor.
  - The missing angle can be found easily using the fact that the sum of the angles is 180°.
  - The AAS criterion for triangle congruence tells us that only one triangle can be formed with the given information so the missing side measurements can be determined by the measurements that are given in the figure.

- See how many of the missing measurements you can determine. Take several minutes to explore this problem with the students around you.

Give students several minutes to work on this problem in groups of 3 or 4. Select one or more students to present their findings to the class.
Give students the opportunity to draw an altitude like the one shown below without being prompted to do so. If students do not see that an auxiliary line is necessary, use the cues in the scaffolding box on the right.

When we draw the altitude to side $\overline{BC}$, we get two right triangles as shown above.

Looking at the yellow triangle, we get $\sin(72°) = \frac{h}{8.4}$. This means that $h = 8.4 \sin(72°)$.

Looking at the green triangle, we get $\sin(41°) = \frac{h}{AC}$. This means that $AC \cdot \sin(41°) = 8.4 \sin(72°)$.

Thus we have $AC \cdot \sin(41°) = 8.4 \sin(72°)$, which means that $AC = \frac{8.4 \sin(72°)}{\sin(41°)} \approx 12.2$.

To make $180°$, the third angle must be $67°$. When we draw in the altitude to side $\overline{BA}$, we get a different pair of right triangles as shown below.

Looking at the blue triangle, we get $\sin(67°) = \frac{k}{12.2}$. This means that $k = 12.2 \sin(67°)$.

Looking at the red triangle, we get $\sin(72°) = \frac{k}{BC}$. This means that $BC = \frac{12.2 \sin(67°)}{\sin(72°)} \approx 11.8$.
Here is a picture of the triangle with the three original measurements, together with the three measurements we calculated above.

- To summarize, we can use what we know about right triangles to learn things about **oblique triangles**, that is, triangles that do not have a right angle.

**Exercise 3 (5 minutes)**

Instruct students to solve the following problem and then to compare their work with a partner. Select two students to write their work on the board to share with the class.

**3.** Find all of the measurements for the triangle below.

*The measure of \( \angle A \) is 62°. The length of side \( BA \) is 6.4. The length of side \( BC \) is 5.8.*
Discussion (5 minutes): Proving the Law of Sines

- Can you generalize your findings? Try to find an equation that shows the relationship between the quantities in the figure below.

Give students several minutes to work together. Select a student to share his or her work with the class.

First we draw an altitude:

One expression for the altitude is \( h = b \cdot \sin(\alpha) \).
Another expression for the altitude is \( h = a \cdot \sin(\beta) \).
From these observations, it follows that \( a \cdot \sin(\beta) = b \cdot \sin(\alpha) \).
We can use this equation to find \( a \) if we know the value of \( b \) and vice versa.

- The fact that \( a \cdot \sin(\beta) = b \cdot \sin(\alpha) \) is called the law of sines. Since the sine function is a central feature of the formula, this name makes sense. Let’s be explicit about the meaning of the symbols in this equation. How are these quantities related in the figure?
  - The side labeled \( a \) is opposite the angle labeled \( \alpha \), and the side labeled \( b \) is opposite the angle labeled \( \beta \).

- By way of summary, let’s make the reasoning absolutely clear. Why are the expressions \( a \cdot \sin(\beta) \) and \( b \cdot \sin(\alpha) \) equal to each other?
  - Each of these expressions represents the length of an altitude in the triangle. Since they represent the same thing, the expressions must be equal to each other.

- Now let’s get some practice applying the law of sines. Keep in mind that you can always draw an altitude just like you did above, but perhaps it is also worth knowing that the equation \( a \cdot \sin(\beta) = b \cdot \sin(\alpha) \) gives us a shortcut.

- When applying the law of sines, it is often easiest to work with the equation \( \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} \). Can you see that this is equivalent to \( a \cdot \sin(\beta) = b \cdot \sin(\alpha) \)? Explain why these are equivalent.
  - By taking the equation \( a \cdot \sin(\beta) = b \cdot \sin(\alpha) \) and dividing both sides by \( \sin(\alpha) \cdot \sin(\beta) \), you get
    \[
    \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)}
    \]
Exercises 4–5 (4 minutes)

Instruct students to solve the following problems and then to compare their work with a partner. Select two students to write their work on the board to share with the class.

4. Find the length of side $\overline{AC}$ in the triangle below.

$$\frac{AC}{\sin(32^\circ)} = \frac{7.5}{\sin(55^\circ)}$$

which means $AC = \frac{7.5 \cdot \sin(32^\circ)}{\sin(55^\circ)} \approx 4.9$.

5. A hiker at point $C$ is 7.5 kilometers from a hiker at point $B$; a third hiker is at point $A$. Use the angles shown in the diagram below to determine the distance between the hikers at points $C$ and $A$.

In order to apply the law of sines, first we need to find the third angle. The measure of the missing angle is $86^\circ$.

Now we can use the law of sines, which gives $\frac{AC}{\sin(60^\circ)} = \frac{7.5}{\sin(86^\circ)}$, which means $AC = \frac{7.5 \cdot \sin(60^\circ)}{\sin(86^\circ)} \approx 6.5$. Thus, the two hikers at points $C$ and $A$ are about 6.5 km apart.

Discussion (7 minutes): Finding Angles
Examine the triangle on the previous page. What information is given to you in this triangle? How is this situation different from the ones we encountered earlier in the lesson?

- We are given two sides and a non-included angle. In the previous examples, we were given two angles and a side.

Do you think you should be able to determine the remaining measurements in this triangle? Why or why not? Think about this for a moment, then explain your reasoning to a neighbor.

- This is the SSA case. We have to allow for the possibility that there are two triangles with these specific measurements.

Let’s have a look at the two triangles with these measurements:

Notice that both triangles have a side with length 11.5, an angle with measure 32°, and a side with length 6.3 that is opposite the 32° angle.

As you can see in the figure, the angle across from the side with length 11.5 can be either acute or obtuse. What can we say about the relationship between α₁ and α₂? Take a moment to think about this, and then discuss what you notice with a partner.

- The figure contains an isosceles triangle, so its base angles must be congruent.

Now we can see that α₁ + α₂ = 180.

Let’s apply the law of sines and then see if we can determine how to get both of these angles.

- We have \( \frac{11.5}{\sin(\alpha)} = \frac{6.3}{\sin(32°)} \) which gives \( \sin(\alpha) = \frac{11.5 \cdot \sin(32°)}{6.3} \approx 0.97. \)
Okay, so now we want to find values of $\alpha$ for which $\sin(\alpha) \approx 0.97$. To see that there are indeed two such values, let’s look at a diagram of the unit circle:

Evidently there are two ways we can rotate the point $(1,0)$ so that its image has a height equal to that of the dotted line.

Can we see that $\alpha_1 + \alpha_2 = 180$ in this diagram as well?

Yes, the two angles labeled $\alpha_1$ are congruent by a reflection across the $y$-axis. Thus, $\alpha_1 + \alpha_2 = 180$.

Okay, now let’s use a calculator to produce the two values for which $\sin(\alpha) = \frac{11.5 \cdot \sin(32^\circ)}{6.3}$.

We find that $\alpha_1 \approx 75.3^\circ$, so $\alpha_2 = 180 - 75.3 = 104.7^\circ$. $\alpha_2 = 104.7^\circ$
Let’s return now to the original problem: What are the two possible values of $\alpha$ in the figure below? Draw the two triangles with the measurements shown in the figure.

We found that $\alpha$ could be $75.3^\circ$, which would give an acute triangle as shown on the left, but $\alpha$ could also be $104.7^\circ$, which would give the obtuse triangle shown on the right.

Exercises 6–7 (3 minutes)

Instruct students to solve the following problems and then to compare their work with a partner. Select two students to write their work on the board to share with the class.

6. Two sides of a triangle have lengths 10.4 and 6.4. The angle opposite 6.4 is 3.6°. What could the angle opposite 10.4 be?

Let $\beta$ represent the angle opposite 10.4. Then, we have \[
\frac{10.4}{\sin(\beta)} = \frac{6.4}{\sin(3.6^\circ)}
\]
which gives $\sin \beta \approx 0.955$ and $\beta = 72.8^\circ$. We could also have $\beta = 107.2^\circ$ because an angle and its supplement have the same sine value.

7. Two sides of a triangle have lengths 9.6 and 11.1. The angle opposite 9.6 is 5.9°. What could the angle opposite 11.1 be?

Let $\beta$ represent the angle opposite 11.1. Then, we have \[
\frac{11.1}{\sin(\beta)} = \frac{9.6}{\sin(5.9^\circ)}
\]
which gives $\beta = 82.3^\circ$. We could also have $\beta = 97.7^\circ$. 
Discussion (Optional): Circumscribed Circles

- One of the real joys of mathematics is that we can look at a problem through different lenses. Sometimes we develop new insights by using a different perspective, and this can be exciting! Let’s return to the problem we solved earlier in the lesson.

- Recall that we were able to calculate the length of side $\overline{AC}$ by drawing the altitude to side $\overline{BC}$. Another way of finding the missing side lengths is to draw the circle that circumscribes the triangle:

- Let’s try to use what we know about circles to find $\overline{AC}$. The key here is to draw in the diameters one at a time.
- Remember, the inscribed angle that includes the diameter must be a right angle, so use that knowledge to create the diameters.

Scaffolding:
- Students may need to be reminded of Thales’ theorem, which says that an angle inscribed in a semicircle must be a right angle.
- Students may need to be reminded that an angle inscribed in a circle has half the measure of its intercepted arc.
Point D is the opposite end of the diameter through point B. What observations can you make about \( \triangle ABD \)? Take a minute to think about this. In particular, can you find the angles in \( \triangle ABD \)?

- Since \( BD \) is a diameter of the circle, it follows that \( \triangle ABD \) has a right angle at \( A \). We can also infer that \( \angle D \) is 41° because it intercepts the same arc as \( \angle C \).

So we’ve distorted \( \triangle ABD \) in such a way that it becomes a right triangle, but the distortion preserves the length of \( AB \) and the measure of \( \angle C \). Clever, isn’t it? Now we can use right triangle trigonometry to describe the length of \( BD \). Can you see how to do this?

- We have \( BD = \frac{8.4}{\sin(41°)} \).

Now let’s run a diameter through point A and conduct a similar analysis.

- Using the same reasoning as before, we conclude that \( \triangle ACE \) is a right triangle and that \( \angle E \) is equal to \( \angle B \). Now we can describe the length of diameter \( AE \) using the sine function: \( \sin(72°) = \frac{AC}{AE} \). This gives us \( AE = \frac{AC}{\sin(72°)} \).

Now let’s put it all together: We know that diameter \( BD = \frac{8.4}{\sin(41°)} \) and that diameter \( AE \) satisfies \( AE = \frac{AC}{\sin(72°)} \). How can we use these observations to compute the length of side \( AC \)? Take a minute to really think about this!

- The length of the diameter of a circle is constant, so we have \( BD = AE \). This means that \( \frac{8.4}{\sin(41°)} = \frac{AC}{\sin(72°)} \), and so \( AC = \frac{8.4 \cdot \sin(72°)}{\sin(41°)} \approx 12.2 \).

Is this consistent with the result we found at the start of the lesson?

- Yes, we got 12.2 in that case also.
Discussion (3 minutes): The Law of Sines via Circles

- Let’s generalize the method involving circles.

- This pair of diagrams shows that the diameter, $d$, of the circle satisfies $\sin(\alpha) = \frac{a}{d}$. This gives $d = \frac{a}{\sin(\alpha)}$.

- What does this pair of diagrams show?
  - These diagrams show that the diameter, $d$, of the circle satisfies $\sin(\beta) = \frac{b}{d}$. This gives $d = \frac{b}{\sin(\beta)}$.

- And what do we get when we combine the two observations about the diameters?
  - Since the diameter of the circle is constant, we have $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)}$.

- In the same way, we can show that the diameter of the circle is described by the expression $\frac{c}{\sin(\gamma)}$.

- Thus, we can express the law of sines in this form: If sides $a$, $b$, and $c$ are opposite angles $\alpha$, $\beta$, and $\gamma$, respectively, then the measurements satisfy the equation $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$. 
Could we also write \( \frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \)? Why or why not?
- Yes. If three fractions are equal, then their reciprocals are also equal.
- It is worth knowing that it is sometimes convenient to set up a problem using this alternate form.

Closing (3 minutes)
- State the law of sines, and explain its uses.
  - The law of sines says that \( \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \).
  - We can use this law to find the measurements of a triangle when we know two angles and one of the opposite sides or when we know two sides and one of the opposite angles.
- If \( b \) and \( c \) are sides of a triangle that are opposite angles \( \beta \) and \( \gamma \), we can write \( b \cdot \sin(\gamma) = c \cdot \sin(\beta) \). What do these two expressions represent geometrically?
  - Each of these expressions represents the length of an altitude of the triangle. Since the expressions represent the same segment, they must be equal to each other.
- If \( b \) and \( c \) are sides of a triangle that are opposite angles \( \beta \) and \( \gamma \), we can write \( \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \). What do these two expressions represent geometrically?
  - Each of these expressions represents the diameter of the circle that circumscribes the triangle. Since the diameter of a circle is constant, the expressions must be equal to each other.

Exit Ticket (4 minutes)
Lesson 8: Law of Sines

Exit Ticket

1. Find the length of side $\overline{AC}$ in the triangle below.

![Diagram of a triangle with sides 11.7, 105°, 12.6, 41°, and 105°]

2. A triangle has sides with lengths 12.6 and 7.9. The angle opposite 7.9 is 37°. What are the possible values of the measure of the angle opposite 12.6?
Exit Ticket Sample Solutions

1. Find the length of side $\overline{AC}$ in the triangle below.

We have

$$\frac{AC}{\sin(105^\circ)} = \frac{11.7}{\sin(41^\circ)},$$

which gives

$$C = \frac{11.7 \cdot \sin(105^\circ)}{\sin(41^\circ)} \approx 17.2.$$  

2. A triangle has sides with lengths 12.6 and 7.9. The angle opposite 7.9 is 37°. What are the possible values of the measure of the angle opposite 12.6?

We have

$$\frac{\sin(x)}{12.6} = \frac{\sin(37^\circ)}{7.9},$$

which means that

$$\sin(x) = \frac{12.6 \cdot \sin(37^\circ)}{7.9}.$$  

The values of $x$ that satisfy this equation are $x \approx 73.7^\circ$ and $x \approx 106.3^\circ$.

Problem Set Sample Solutions

1. Let $\triangle ABC$ be a triangle with the given lengths and angle measurements. Find all possible missing measurements using the law of sines.

   a. $a = 5$, $m \angle A = 43^\circ$, $m \angle B = 80^\circ$.

   $$b \approx 7.22, c \approx 6.15, m \angle C = 57^\circ$$

   b. $a = 3.2$, $m \angle A = 110^\circ$, $m \angle B = 35^\circ$.

   $$b \approx 1.95, c \approx 1.95, m \angle C = 35^\circ$$

   c. $a = 9.1$, $m \angle A = 70^\circ$, $m \angle B = 95^\circ$.

   $$b \approx 9.65, c \approx 2.51, m \angle C = 15^\circ$$
### Lesson 8: Law of Sines

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<td>d.</td>
<td>$a = 3.2$, $\angle B = 30^\circ$, $\angle C = 45^\circ$. $m\angle A = 105^\circ$, $b \approx 1.66$, $c \approx 2.34$</td>
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<td>e.</td>
<td>$a = 12$, $\angle B = 29^\circ$, $\angle C = 31^\circ$. $m\angle A = 120^\circ$, $b \approx 6.72$, $c \approx 7.14$</td>
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<td>f.</td>
<td>$a = 4.7$, $\angle B = 18.8^\circ$, $\angle C = 72^\circ$. $m\angle A = 89.2^\circ$, $b \approx 1.51$, $c \approx 4.47$</td>
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<td>g.</td>
<td>$a = 6$, $b = 3$, $m\angle A = 91^\circ$. $m\angle B \approx 29.99^\circ$, $m\angle C \approx 59.01^\circ$, $c \approx 5.14$</td>
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<td>h.</td>
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<td>i.</td>
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<td>j.</td>
<td>$a = 3.5$, $b = 3.6$, $m\angle A = 37^\circ$. $m\angle B \approx 38.24^\circ$, $m\angle C \approx 104.76^\circ$, $c \approx 5.62$ or $m\angle B = 141.76^\circ$, $m\angle C = 1.24^\circ$, $c = 0.13$</td>
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<td>k.</td>
<td>$a = 9$, $b = 10.1$, $m\angle A = 61^\circ$. $m\angle B \approx 78.97^\circ$, $m\angle C \approx 40.03^\circ$, $c \approx 6.62$ or $m\angle B = 101.03^\circ$, $m\angle C = 17.97^\circ$, $c = 3.17$</td>
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<td>l.</td>
<td>$a = 6$, $b = 8$, $m\angle A = 41.5^\circ$. $m\angle B \approx 62.07^\circ$, $m\angle C \approx 76.43^\circ$, $c \approx 8.8$ or $m\angle B = 117.93^\circ$, $m\angle C = 20.57^\circ$, $c = 3.18$</td>
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2. A surveyor is working at a river that flows north to south. From her starting point, she sees a location across the river that is 20° north of east from her current position, she labels the position $S$. She moves 110 feet north and measures the angle to $S$ from her new position, seeing that it is 32° south of east.

   a. Draw a picture representing this situation.

   b. Find the distance from her starting position to $S$.

   \[
   \frac{\sin(52°)}{110} = \frac{\sin(58°)}{x}
   \]

   \[x \approx 118.4\]

   The distance is about 118 ft.

   c. Explain how you can use the procedure the surveyor used in this problem (called triangulation) to calculate the distance to another object.

   Calculate the angle from the starting point to the object. Travel a distance from the starting point, and again calculate the angle to the object. Create a triangle connecting the starting point, the object, and the new location. Use that distance traveled and the angles to find the distance to the object.
3. Consider the triangle pictured below.

Use the law of sines to prove the generalized angle bisector theorem, that is, \( \frac{BD}{DC} = \frac{c \sin(m\angle BAD)}{b \sin(m\angle CAD)} \). (Although this is called the generalized angle bisector theorem, we do not assume that the bisector of \( \angle BAC \) intersects side \( BC \) at \( D \). In the case that \( AD \) is an angle bisector, then the formula simplifies to \( \frac{BD}{DC} = \frac{c}{b} \)).

a. Use the triangles \( ABD \) and \( ACD \) to express \( \frac{c}{BD} \) and \( \frac{b}{DC} \) as a ratio of sines.

\[
\frac{c}{BD} = \frac{\sin(m\angle BDA)}{\sin(m\angle BAD)}
\]

\[
\frac{b}{DC} = \frac{\sin(m\angle CDA)}{\sin(m\angle CAD)}
\]

b. Note that angles \( BDA \) and \( ADC \) form a linear pair. What does this tell you about the value of the sines of these angles?

Since the angles are supplementary, the sines of these values are equal.

c. Solve each equation in part (a) to be equal to the sine of either \( \angle BDA \) or \( \angle ADC \).

\[
\sin(m\angle BAD) \cdot \frac{c}{BD} = \sin(m\angle BDA)
\]

\[
\sin(m\angle CAD) \cdot \frac{b}{DC} = \sin(m\angle CDA)
\]

d. What do your answers to parts (b) and (c) tell you?

The answers tell me that the two equations written in part (c) are equal to each other.

e. Prove the generalized angle bisector theorem.

From part (d), we have

\[
\sin(m\angle BAD) \cdot \frac{c}{BD} = \sin(m\angle CAD) \cdot \frac{b}{DC}
\]

Dividing both sides by \( \sin(m\angle CAD) \cdot b \), and multiplying by \( BD \), we get

\[
\frac{BD}{DC} = \frac{c \sin(m\angle BAD)}{b \sin(m\angle CAD)}
\]
4. As an experiment, Carrie wants to independently confirm the distance to Alpha Centauri. She knows that if she measures the angle of Alpha Centauri and waits 6 months and measures again, then she will have formed a massive triangle with two angles and the side between them being 2 AU long.
   a. Carrie measures the first angle at $82^\circ 8' 24.5''$ and the second at $97^\circ 51' 34''$. How far away is Alpha Centauri according to Carrie’s measurements?

   *The third angle would be 1.5”.*

   $$\sin\left(82 + \frac{8}{60} + \frac{24.5}{3600}\right) = \frac{\sin\left(\frac{1.5}{3600}\right)}{2}$$

   $$a \approx 272 \text{, } 436 \text{ AU}.$$

   b. Today, astronomers use the same triangulation method on a much larger scale by finding the distance between different spacecraft using radio signals and then measuring the angles to stars. Voyager 1 is about 122 AU away from Earth. What fraction of the distance from Earth to Alpha Centauri is this? Do you think that measurements found in this manner are very precise?

   *Voyager 1 is about $\frac{122}{276.364}$ or $0.004$ the distance of Earth to Alpha Centauri. Depending on how far away the object being measured is, the distances are fairly precise on an astronomical scale. One AU is almost 93 million miles, which is not very precise.*

5. A triangular room has sides of length 3.8 m, 5.1 m, and 5.1 m. What is the area of the room?

   *Since the room is isosceles, the height bisects the side of length 3.8 at a right angle. We get $\cos(\theta) = \frac{1.9}{5.1}$, therefore, $\theta \approx 68.127^\circ$.*

   $$\frac{1}{2} \cdot 3.8 \cdot 5.1 \cdot \sin(68.127^\circ) \approx 8.99$$

   *The area of the room is approximately 8.99 m$^2$. 

![Diagram of a triangular room with sides labeled 3.8, 5.1, and 5.1, and height labeled 5.1. The angle between the 3.8 and 5.1 sides is labeled 68.127°.]
6. Sara and Paul are on opposite sides of a building that a telephone pole fell on. The pole is leaning away from Paul at an angle of 59° and towards Sara. Sara measures the angle of elevation to the top of the telephone pole to be 22°, and Paul measures the angle of elevation to be 34°. Knowing that the telephone pole is about 35 ft. tall, answer the following questions.

a. Draw a diagram of the situation.

b. How far apart are Sara and Paul?

We can use law of sines to find the shared side between the two triangles and then again with the larger triangle to find the distance:

\[
\frac{\sin(22°)}{35} = \frac{\sin(59°)}{r} \\
r \approx 80.086 
\]

Then we use this value in the larger triangle:

\[
\frac{\sin(124°)}{x} = \frac{\sin(34°)}{80.086} \\
x \approx 118.733 
\]

They are about 118.7 ft. away from each other.

c. If we assume the building is still standing, how tall is the building?

\[
\sin(59°) = \frac{y}{35} \\
y \approx 30.001 
\]

The building is about 30 ft. tall.