

Lesson 17: Solving Problems by Function Composition

Classwork

Example 1

Find the domain and range for the following functions:

a. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$

b. $g: \mathbb{R} \rightarrow \mathbb{R}$ given by $g(x) = \sqrt{x - 2}$

c. $f(g(x))$

d. $g(f(x)) = g(x^2) = \sqrt{x^2 - 2}$

Exercise 1

1. Find the domain and range for the following functions:

a. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x + 2$

b. $g: \mathbb{R} \rightarrow \mathbb{R}$ given by $g(x) = \sqrt{x - 1}$

c. $f(g(x))$

d. $g(f(x))$

d. Determine an appropriate domain and range for part (c). Explain why your domain and range are reasonable in this context.

e. How many miles of driving could be generated daily by 20 wind turbines in a day?

Exercises 2–3

2. A product safety commission is studying the effect of rapid temperature changes on the equipment of skydivers as they descend. The commission has collected data on a typical skydiver during the part of the dive when she has reached terminal velocity (maximum speed) to the time the parachute is released. They know that the terminal velocity of a diver is approximately 56 m/s and that, given the altitude of skydivers at terminal velocity, the temperature decreases at an average rate of $6.4 \frac{^{\circ}\text{C}}{\text{km}}$.
- a. Write a function that represents the altitude of a skydiver experiencing terminal velocity if she reaches this speed at a height of 3,000 m.

b. Write a function that represents the relationship between the altitude of the skydiver and the temperature if the temperature at 3,000 m is 5.8°C .

c. Write a function that could be used to determine the temperature, in degrees Celsius, of the air surrounding a skydiver based on the time she has spent descending at terminal velocity. Interpret the equation in context.

- d. Determine an appropriate domain and range for part (c).
- e. How long would it take a skydiver to reach an altitude where the temperature is 8°C ?
3. A department store manager is planning to move some cement spheres that have served as traffic barriers for the front of her store. She is trying to determine the relationship between the mass of the spheres and their diameter in meters. She knows that the density of the cement is approximately $2,500 \text{ kg/m}^3$.
- a. Write a function that represents the relationship between the volume of a sphere and its diameter. Explain how you determined the equation.
- b. Write a function that represents the relationship between the mass and the volume of the sphere. Explain how you determined the function.

- c. Write a function that could be used to determine the mass of one of the cement spheres based on its diameter. Interpret the equation in context.
- d. Determine an appropriate domain and range for part (c).
- e. What is the approximate mass of a sphere with a diameter of 0.9 m?

Problem Set

- Find the domain and range of the following functions:
 - $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = -x^2 + 2$
 - $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = \frac{1}{x+1}$
 - $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = \sqrt{4-x}$
 - $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = |x|$
 - $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = 2^{x+2}$
- Given $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$, for the following, find $f(g(x))$ and $g(f(x))$, and state the domain.
 - $f(x) = x^2 - x$, $g(x) = x - 1$
 - $f(x) = x^2 - x$, $g(x) = \sqrt{x-2}$
 - $f(x) = x^2$, $g(x) = \frac{1}{x-1}$
 - $f(x) = \frac{1}{x+2}$, $g(x) = \frac{1}{x-1}$
 - $f(x) = x - 1$, $g(x) = \log_2(x + 3)$
- A company has developed a new highly efficient solar panel. Each panel can produce 0.75 MW of electricity each day. According to the Los Angeles power authority, all the traffic lights in the city draw 0.5 MW of power per day.
 - Write a function that represents the relationship between the number of solar panels installed and the amount of energy generated per day (in MWh). Define the input and output.
 - Write a function that represents the relationship between the number of days and the energy in megawatts consumed by the traffic lights. (How many days can one megawatt provide?)
 - Write a function that could be used to determine the number of days that the traffic lights stay on based on the number of panels installed.
 - Determine an appropriate domain and range for part (c).
 - How many days can 20 panels power all the lights?
- A water delivery person is trying to determine the relationship between the mass of the cylindrical containers he delivers and their diameter in centimeters. The density of the bottles is 1 g/cm^3 . The height of each bottle is approximately 60 cm.
 - Write a function that represents the relationship between the volume of the cylinder and its diameter.
 - Write a function that represents the relationship between the mass and volume of the cylinder.
 - Write a function that could be used to determine the mass of one cylinder based on its diameter. Interpret the equation in context.
 - Determine an appropriate domain and range for part (c).
 - What is the approximate mass of a cylinder with a diameter of 30 cm?

5. A gold mining company is mining gold in Northern California. Each mining cart carries an average 500 kg of dirt and rocks that contain gold from the tunnel. For each 2 metric tons of material (dirt and rocks), the company can extract an average of 10 g of gold. The average wholesale gold price is \$20/g.
- Write a function that represents the relationship between the mass of the material mined in metric tons and the number of carts. Define the input and output.
 - Write a function that represents the relationship between the amount of gold and the materials. Define the input and output.
 - Write a function that could be used to determine the mass of gold in metric tons as a function of the number of carts coming out from the mine.
 - Determine an appropriate domain and range for part (c).
 - Write a function that could be used to determine the amount of money the gold is worth in dollars and the amount of gold extracted in metric tons.
 - How much gold can 40,000 carts of material produce?
 - How much, in dollars, can 40,000 carts of material produce?
6. Bob operates hot air balloon rides for tourists at the beach. The hot air balloon rises, on average, at 100 feet per minute. At sea level, the atmospheric pressure, measured in inches of mercury, is 29.9 inHg. Using a barometric meter, Bob notices that the pressure decreases by 0.5 inHg for each 500 feet the balloon rises.
- Write a function that represents the relationship between the height of the hot air balloon and the time spent to reach that height.
 - Write a function that represents the relationship between the height of the hot air balloon and the atmospheric pressure being applied to the balloon.
 - Write a function that could be used to determine the pressure on the hot air balloon based on the time it spends rising.
 - Determine an appropriate domain and range for part (c).
 - What is the reading on the barometer 10 minutes after the hot air balloon has left the ground?