Lesson 17: Solving Problems by Function Composition

Student Outcomes

- Students write equations that represent functional relationships and use the equations to compose functions.
- Students analyze the domains and ranges of functions and function compositions represented by equations.
- Students solve problems by composing functions (**F-BF.A.1c**).

Lesson Notes

In the previous lesson, students explored the process of function composition in a general setting, interpreting compositions in context and determining when compositions were reasonable. This lesson focuses on composing numerical functions, including those in real-world contexts. Students represent real-world relationships with equations, use the equations to create composite functions, and apply the composite functions to problem solving in both mathematical and real-world contexts.

Classwork

Opening (3 minutes)

At the outset of the previous lesson, students were introduced to free diving, and they explored how function composition could be used to examine the relationship between the atmospheric pressure a diver experiences and the time a free diver has spent in a descent. This lesson provides students with an opportunity to represent the relationships between temperature, depth, and time spent descending using equations to address the same issue, that is, the relationship between atmospheric pressure experienced and the duration of the diver's descent. Pose the following questions after students have examined the tables provided. After a short time of individual reflection, allow students to share their ideas with a partner and then with the class.

> *Scaffolding:*
> Break down the question into parts for struggling students. For instance, ask them, "How could we represent the relationship between the time spent descending and depth?" "How could we represent the relationship between the ocean depth of the diver and atmospheric pressure?" "How could we use these representations to define the relationship between the time a diver has spent descending and the atmospheric pressure experienced by the diver?"

Depth of Free Diver During Descent

s seconds of descent	20	40	60	80	100	120	140	160	180
d depth in meters of diver	14	28	42	56	70	84	98	112	126

Atmospheric Pressure and Ocean Depth

d depth in meters of diver	10	20	30	40	50	60	70	80	90
p pressure in atmosphere on diver	2	3	4	5	6	7	8	9	10

- What patterns do you notice in the relationship between a diver's time descending and her depth and between a diver's depth and the pressure applied to the diver? How could you use this information to model the relationship between the time a diver has spent descending and the atmospheric pressure experienced by the diver?
  - *We could write a linear function representing the relationship between time and depth and another linear function relating depth and pressure and compose them to relate time and pressure.*

Discussion (8 minutes): Functions Represented with Equations

- In the previous lesson, we discussed functions generally, analyzing relationships between sets that were not always numerical. Let's explore numerical functions represented by formulas. For any function, we can define the functional relationship as $f\colon X \to Y$, and the relationship between each input and its corresponding output can be represented as $x \to f(x)$.

MP.4

- How could we use function notation to represent the following relationship? The function f takes a diver's time, in seconds, spent in descent, t, and multiplies it by 0.7 to produce the diver's depth in meters.
  - $f\colon \mathbb{R} \to \mathbb{R}$
    $t \to 0.7t$
- And how could we write an equation that represents the relationship between t and $f(t)$?
  - $f(t) = 0.7t$
- What would a reasonable domain and range be for the function $f(t) = 0.7t$?
  - *Both the domain and range represent nonnegative real numbers, and there is some upper limit given the constraints on the time and depth of a diver's descent.*
- What type of function would represent the relationship between the diver's depth and pressure applied to the diver? Explain.
  - *A linear function would best represent the relationship because the rate of change in the pressure based on a unit increase in depth is constant.*
- And how could we determine a linear equation to represent the relationship between depth and pressure?
  - *Determine the rate of change in atmospheres/meter and extrapolate the y-intercept from the table.*
- What are the rate of change and y-*intercept for the equation?*
  - *The rate of change is 0.1 atmosphere (atm)/meter (m), and the y-intercept is* 1 atm.
- So, what is the equation representing the relationship between depth and pressure?
  - $p(d) = 0.1d + 1$
- What would a reasonable domain and range be for the function $p(d) = 0.1d + 1$?
  - *Both the domain and range represent nonnegative real numbers, and there is some upper limit given the constraints on the depth and pressure applied.*
- How could we use our equations relating time and depth and relating depth and pressure to determine the pressure on a diver 100 seconds (sec.) into the descent?
  - *Substitute $s = 100$ into the equation $f(s) = 0.7s$ to find the diver's depth, and substitute this depth into the equation $p(d) = 0.1d + 1$ to find the pressure applied to the diver.*

- What is the resulting pressure? Explain.
  - $f(100) = 0.7(100) = 70$
    $p(70) = 0.1(70) + 1 = 8$
- Represent this process as a function composition.
  - $p(f(100)) = 8$
- Interpret this result in context.
  - *The pressure on a diver at* 100 sec. *of descent is* 8 atm.
- And how could we find an equation that represents the relationship between time and pressure?
  - *Compose the function equations for time and depth and for depth and pressure.*
- How could we compose the function equations?
  - *Evaluate the function* $p(d) = 0.1d + 1$ *for the input* $f(s) = 0.7s$.
- And why is it possible for us to use $f(s)$ as our input?
  - *It represents the diver's depth, which is equivalent to* d.
- What equation do we get when we compose the functions?
  - $p(f(s)) = p(0.7s) = 0.1(0.7s) + 1 = 0.07s + 1$
- Using this equation, what is the pressure felt by a diver 80 sec. into a descent?
  - $p(f(80)) = 0.07(80) + 1 = 6.6$

MP.2

- Explain the result in context.
  - *There are* 6.6 atm *of pressure applied to a diver* 80 sec. *into the descent.*

Example 1 (5 minutes)

This example addresses numerical functions represented with equations. Students determine the domain and range of functions given the equations. They also compose numerical functions and determine the domain and range of the compositions. This prepares them to analyze real-world relationships that can be represented using functions and compositions of functions. Have students complete parts (a) and (b) in pairs or small groups. Review parts (a) and (b), and then complete part (c) with a teacher-led discussion. Students should then complete part (d) in pairs or small groups and discuss answers in a whole-class setting. Note: It is acceptable, but not necessary, for students to represent the domain and range of the functions using interval notation.

- What is the relationship between the inputs and outputs in part (a)?
  - *Each real number* x *is squared to produce the corresponding output.*
- This function is described as a rule between sets of real numbers. This means that to determine the domain, we must consider all values of x that produce a real number output. What real numbers are included in the domain, and how can you tell?
  - *All real numbers are included because squaring a real number is an example of multiplying two real numbers, and the set of real numbers is closed under multiplication.*
- The range represents the subset of real numbers produced by applying the function rule to the domain. What type of real numbers result when a real number is squared?
  - *Nonnegative real numbers*

- What implications does this have on the range?
  - *The range can only contain elements in the real numbers that are nonnegative.*
- Why is the domain for the function in part (b) different from that in part (a)?
  - *In part (a), any real number could produce an output that is a real number. In the function in part (b), any x-value less than 2 produces an output of a nonreal complex number.*
- And why is this a problem?
  - *The sets X and Y are defined as being real number sets, which do not contain nonreal complex numbers.*
- We have seen how this impacts the domain. What effect does it have on the range?
  - *For all inputs in the domain, the output is the principal square root of a nonnegative real number. This indicates that the range only contains nonnegative real numbers.*
- How do we perform the composition $f\big(g(x)\big)$ in part (c)?
  - *Apply the function g to the inputs x, and then apply the function f to the outputs $g(x)$.*
- If $f\big(g(x)\big) = |x - 2|$, and the expression $|x - 2|$ produces real number outputs for all real number inputs, why is the domain of $f\big(g(x)\big)$ not all real numbers?
  - *If we apply the function rule g to the set of all real numbers, the only values that produce real number outputs are those that are greater than or equal to 2. Therefore, we cannot include values in the domain that are less than 2 because the function g would produce an output that is not a real number, and the composition could not be performed.*
- And what are the restrictions on the range of the function composition? Explain.
  - *The function g produces only nonnegative outputs, which become the inputs when function rule f is applied. Since f has the effect of squaring the input values, the output of the composite function is effectively the squared values of nonnegative real numbers, which are also nonnegative real numbers.*
- Why are the domain and range not the same for the compositions $f\big(g(x)\big)$ and $g\big(f(x)\big)$?
  - *Function composition is not commutative, so the order in which the function rules are applied affect the composite function that results.*

Example 1

Find the domain and range for the following functions:

a. $f: \mathbb{R} \to \mathbb{R}$ **given by** $f(x) = x^2$

Domain: All real numbers

Range: All real numbers greater than or equal to 0

b. $g: \mathbb{R} \to \mathbb{R}$ **given by** $g(x) = \sqrt{x-2}$

Domain: Since $x - 2 \geq 0$, $x \geq 2$

Range: $\sqrt{x-2} \geq 0$, *so* $g(x) \geq 0$

c. $f(g(x))$

$f(g(x)) = f\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 = |x-2|$

Domain: $x - 2 \geq 0$, *so* $x \geq 2$

Range: $|x-2| \geq 0$, *so* $f\big(g(x)\big) \geq 0$

Scaffolding:

- Struggling students could graph the functions and their compositions to help them visualize restrictions on the domain and range.
- Advanced students could work together to draw general conclusions about the domains and ranges of function compositions compared to the domains and ranges of the individual functions being composed (e.g., if x is a restricted input value for f or g, it is also a restricted input for the composite function).

d. $g(f(x)) = g(x^2) = \sqrt{x^2 - 2}$

Domain: $x^2 - 2 \geq 0$, *so* $x \leq -\sqrt{2}$ *or* $x \geq \sqrt{2}$

Range: $\sqrt{x^2 - 2} \geq 0$, *so* $g(f(x)) \geq 0$

Exercise 1 (5 minutes)

Students should complete this problem in pairs. They should do the work independently and then verify their responses with their partners. After a few minutes, a few selected volunteers could display their work and share their solving process.

Exercise 1

1. **Find the domain and range for the following functions:**

   a. $f: \mathbb{R} \to \mathbb{R}$ **given by** $f(x) = x + 2$

   Domain: All real numbers

   Range: All real numbers

   b. $g: \mathbb{R} \to \mathbb{R}$ **given by** $g(x) = \sqrt{x - 1}$

   Domain: $x \geq 1$

   Range: $\sqrt{x - 1} \geq 0$, *so* $g(x) \geq 0$

   c. $f(g(x))$

   $f(g(x)) = f(\sqrt{x - 1}) = \sqrt{x - 1} + 2$

   Domain: $x - 1 \geq 0$, *so* $x \geq 1$

   Range: Since $\sqrt{x - 1} \geq 0$, $\sqrt{x - 1} + 2 \geq 2$, *so* $f(g(x)) \geq 2$

   d. $g(f(x))$

   $g(f(x)) = g(x + 2) = \sqrt{(x + 2) - 1} = \sqrt{x + 1}$

   Domain: $\sqrt{x + 1} \geq 0$, *so* $x \geq -1$
   Range: $\sqrt{x + 1} \geq 0$, *so* $g(f(x)) \geq 0$

Example 2 (8 minutes)

This example builds on Example 1 by having students represent real-world relationships using composite functions. Students represent the relationship between variables using equations, which they have done in previous courses. They then apply their understanding of composing functions to write an equation that relates two variables of interest (**F-BF.A.1**). Students then apply the equation to solve a problem. This problem should be completed as part of a teacher-led discussion.

- What relationship is of interest to the individuals in the nonprofit organization?
  - *The relationship of interest is between the energy generated by wind power and the energy used by electric cars.*

- What type of relationship is described between the number of turbines operating and the amount of energy they generate daily?
  - *It is a directly proportional relationship.*
- So, what is the format of the function?
  - $f(x) = kx$, *where* k *represents the constant of proportionality,* $16{,}400$
- Does it matter whether the function is written as $E(t)$ versus $f(x)$?
  - *No. It is just important to remember what the variables represent; for example,* $E(t)$ *represents energy produced in kilowatt-hours (kWh), and* t *represents the number of* 2.5–3 *megawatt* (MW) *turbines operating on the wind farm daily.*
- And how can we interpret the function we wrote in part (a)?
  - *The energy produced, in kilowatt-hours, each day by* t 2.5–3 *MW wind turbines is the product of* $16{,}400$ *and* t.
- What relationship is represented by our function equation in part (b)?
  - *The relationship between the energy in kilowatt-hours expended by an average electric car and miles driven by the car*
- Good. We can see that this relationship is also directly proportional. How was the constant of proportionality found?
  - *By dividing the miles driven by energy expended and rounding to an appropriate number of significant digits*
- And how can we interpret this function equation in context?
  - *An average electric car uses* 1 *kilowatt-hour of energy for every* 2.9 *miles it is driven.*
- Now how can we use the equations we have written to relate the energy generated by the turbines to the miles driven by an average electric car?
  - *Compose the functions: Apply the miles to the energy generated by the number of turbines.*
- Now our composite function equation is in the form of a directly proportional relationship. Why are the domain and range of the composite function not all real numbers?
  - *The domain represents the number of turbines, which can only be a whole number value; the range is the product of a whole number constant and a whole number input, which is also a whole number.*
- Realistically, do the domain and range for the composite function really include all whole numbers?
  - *There is some reasonable upper limit for the number of turbines that is determined by cost, available space, etc.*
- How might individuals from the nonprofit group apply the composite function in their research?
  - *Individuals could apply the composite function to their research to determine whether the energy offset by the turbines outweighs the cost of building them and maintaining them.*

Example 2

According to the Global Wind Energy Council, a wind turbine can generate about $16,400$ kWh of power each day. According to the Alternative Fuels Data Center, an average electric car can travel approximately 100 miles on 34 kWh of energy. An environmental nonprofit organization is interested in analyzing how wind power could offset the energy use of electric vehicles.

MP.4

a. **Write a function that represents the relationship between the number of wind turbines operating in a wind farm and the amount of energy they generate per day (in kilowatt-hours). Define the input and output.**

$E(t) = 16400t$
t: *number of turbines operating daily*
$E(t)$: *energy (in kilowatt-hours) produced daily by the turbines*

b. **Write a function that represents the relationship between the energy expended by an electric car (in kilowatt-hours) and the number of miles driven.**

The relationship between miles driven and energy used is directly proportional, so $m = kE$.

Since the car drives 100 miles using 34 kWh, $100 = k(34)$ and $k \approx 2.9$.

$m(E) = 2.9E$

E: *amount of energy expended by an average electric car (in kilowatt-hours)*
$m(E)$: *miles driven by the electric car*

c. **Write a function that could be used to determine the number of miles that an electric car could drive based on the number of wind turbines operating daily at a wind farm. Interpret this function in context.**

$m(E(t)) = m(16400t) = 2.9(16400t) = 47560t$

For every turbine operating in a wind farm daily, an average electric car can drive $47,560$ miles.

d. **Determine an appropriate domain and range for part (c). Explain why your domain and range are reasonable in this context.**

Domain: Whole numbers—The domain represents the number of turbines, which can only be represented with whole numbers.

Range: Whole numbers—Given the function $m(E(t)) = 47560t$, the outputs are found by multiplying a whole number by an input that is a whole number, which always produces a whole number. The range represents miles driven, and these values cannot be negative.

e. **How many miles of driving could be generated daily by 20 wind turbines in a day?**

$m(E(20)) = 20 \times 47560 = 951\,200$. $951,200$ *miles of driving can be generated.*

Exercises 2–3 (8 minutes)

Students could complete these exercises in small groups. Each group is assigned to complete one of the exercises. After a few minutes, the groups that completed the same exercise prepare a brief presentation of their solutions, which they could share with the rest of the students.

Exercises 2–3

2. **A product safety commission is studying the effect of rapid temperature changes on the equipment of skydivers as they descend. The commission has collected data on a typical skydiver during the part of the dive when she has reached terminal velocity (maximum speed) to the time the parachute is released. They know that the terminal velocity of a diver is approximately 56 m/s and that, given the altitude of skydivers at terminal velocity, the temperature decreases at an average rate of $6.4\frac{°\text{C}}{\text{km}}$.**

MP.4

a. **Write a function that represents the altitude of a skydiver experiencing terminal velocity if she reaches this speed at a height of $3,000$ m.**

$h(s) = 3000 - 56t$

s: *number of seconds spent descending at terminal velocity*

$h(s)$: *altitude of the skydiver (in meters)*

b. **Write a function that represents the relationship between the altitude of the skydiver and the temperature if the temperature at $3,000$ m is 5.8°C.**

$t(h) - t(h_1) = m(h - h_1)$

$(h_1, t(h_1)) = (3000, 5.8)$

$m = -6.4\frac{°\text{C}}{\text{km}} = -0.0064\frac{°\text{C}}{\text{m}}$

$t(h) - 5.8 = -0.0064(h - 3000)$

$t(h) - 5.8 = -0.0064h + 19.2$

$t(h) = -0.0064h + 25$

h: *altitude of the skydiver (in meters)*

$t(h)$: *temperature corresponding to the altitude of the skydiver*

MP.2

c. **Write a function that could be used to determine the temperature, in degrees Celsius, of the air surrounding a skydiver based on the time she has spent descending at terminal velocity. Interpret the equation in context.**

$t(h(s)) = t(3000 - 56t) = -0.0064(3000 - 56t) + 25 \approx 5.8 + 0.36t$

A skydiver begins the portion of her dive at terminal velocity experiencing an air temperature of approximately 5.8°C, and the temperature increases by approximately 0.36°C for each second of descent until she deploys the parachute.

d. **Determine an appropriate domain and range for part (c).**

Domain: Nonnegative real numbers

Range: Real numbers greater than or equal to 5.8

Scaffolding:

- Struggling students could be provided with the formulas for the volume of a sphere and the density of an object relative to its volume and mass.
- Advanced students could complete Exercises 2 and 3 by composing the functions without first being prompted to write the functions in parts (a) and (b).

e. How long would it take a skydiver to reach an altitude where the temperature is $8°\text{C}$?

For a temperature of $8°\text{C}$, $8 = 5.8 + 0.36t$
$0.36t = 2.2$, *so* $t \approx 6.1$ sec.

3. A department store manager is planning to move some cement spheres that have served as traffic barriers for the front of her store. She is trying to determine the relationship between the mass of the spheres and their diameter in meters. She knows that the density of the cement is approximately $2,500\ \text{kg/m}^3$.

MP.4

a. Write a function that represents the relationship between the volume of a sphere and its diameter. Explain how you determined the equation.

$$V(d) = \frac{1}{6}\pi d^3$$

The volume of a sphere is equal to

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{6}\pi d^3$$

b. Write a function that represents the relationship between the mass and the volume of the sphere. Explain how you determined the function.

$m(V) = 2500V$

$\text{Mass} = \text{density} \times \text{volume} = 2500 \times V$

MP.2

c. Write a function that could be used to determine the mass of one of the cement spheres based on its diameter. Interpret the equation in context.

$$m(V(d)) = m\left(\frac{1}{6}\pi d^3\right) = 2500\left(\frac{1}{6}\pi d^3\right) = \frac{2500}{6}\pi d^3$$

The numerical value of the mass of one of the cement spheres is equal to approximately $1,300$ *times the value of the cubed diameter (measured in meters).*

d. Determine an appropriate domain and range for part (c).

Domain: Nonnegative real numbers

Range: Nonnegative real numbers

e. What is the approximate mass of a sphere with a diameter of 0.9 m?

$m(V(0.9)) = \frac{2500}{6}\pi(0.9)^3 \approx 950$. The mass of the sphere is approximately 950 kg.

Closing (3 minutes)

Have students explain how function composition could be applied to represent the relationship between a person's income and interest earned in a savings account if there is a functional relationship between the individual's income and the amount deposited into the savings account. Students should provide a hypothetical example, which they could share with a partner.

Answers will vary. An example of an acceptable response is shown. Let's say that an individual puts a constant percent of her income, p, into a savings account. The relationship between income and the amount placed into the savings account could be defined as $s(d) = pd$, where $s(d)$ is the amount, in dollars, placed in the savings account, and d represents income in dollars. The function $I(s) = rs$ could represent the relationship between interest dollars and the amount in a savings account, where $I(s)$ represents savings interest in dollars, r is the percent interest rate, and s is the amount, in dollars, placed into savings. The relationship between income and savings interest, then, would be represented by the composite function $I\big(s(d)\big)$.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 17: Solving Problems by Function Composition

Exit Ticket

Timmy wants to install a wooden floor in a square room. The cost to install the floor is $\$24$ per 4 square feet.

a. Write a function to find the area of the room as a function of its length.

b. Write a function for the cost to install the floor as a function of its area.

c. Write a function to find the total cost to install the floor.

d. Show how the function in part (c) is the result of a composition of two functions.

e. How much does it cost to install a wood floor in a square room with a side length of 10 feet?

Exit Ticket Sample Solutions

Timmy wants to install a wooden floor in a square room. The cost to install the floor is $\$24$ per 4 square feet.

a. **Write a function to find the area of the room.**

$A(x) = x^2$

x: *a side of a square*

$A(x)$: *the area of the square room*

b. **Write a function for the installation cost per square foot.**

$C(A) = 6A$

A: *the area of the floor*

$C(A)$: *installation cost per square foot*

c. **Write the function to find the total cost to install the floor.**

$C(A(x)) = 6x^2$

d. **Show how the function in part (c) is the result of a composition of two functions.**

$A(x) = x^2$

$C(A(x)) = C(x^2) = 6(x^2) = 6x^2$

e. **How much does it cost to install a wood floor in a square room with a side of 10 feet?**

$C(A(10)) = 6(10)^2 = 600$. *The cost is* $\$600.00$.

Problem Set Sample Solutions

Students may use graphing calculators to determine domain and range for Problem 1.

1. **Find the domain and range of the following functions:**

a. $f: \mathbb{R} \to \mathbb{R}$ **by** $f(x) = -x^2 + 2$

Domain: x: *all real numbers*

Range: $f(x) \leq 2$

b. $f: \mathbb{R} \to \mathbb{R}$ **by** $f(x) = \frac{1}{x+1}$

Domain: $x \neq -1$

Range: $f(x) \neq 0$

c. $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = \sqrt{4-x}$

Domain: $x \leq 4$

Range: $f(x) \geq 0$

d. $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = |x|$

Domain: x: *all real numbers*

Range: $f(x) \geq 0$

e. $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = 2^{x+2}$

Domain: x: *all real numbers*

Range: $f(x) > 0$

2. Given $f: \mathbb{R} \to \mathbb{R}$ and: $\mathbb{R} \to \mathbb{R}$, for the following, find $f(g(x))$ and $g(f(x))$, and state the domain.

a. $f(x) = x^2 - x, \ \ g(x) = x - 1$

$f(g(x)) = x^2 - 3x + 2,$ *Domain:* x: *all real numbers*

$g(f(x)) = x^2 - x - 1,$ *Domain:* x: *all real numbers*

b. $f(x) = x^2 - x, \ \ g(x) = \sqrt{x-2}$

$f(g(x)) = x - 2 - \sqrt{x-2},$ *Domain:* $x \geq 2$

$g(f(x)) = \sqrt{x^2 - x - 2},$ *Domain:* $x \leq -1$ *or* $x \geq 2$

c. $f(x) = x^2, \ \ g(x) = \frac{1}{x-1}$

$f(g(x)) = \frac{1}{(x-1)^2},$ *Domain:* $x \neq 1$

$g(f(x)) = \frac{1}{x^2-1},$ *Domain:* $x \neq \pm 1$

d. $f(x) = \frac{1}{x+2}, \ \ g(x) = \frac{1}{x-1}$

$f(g(x)) = \frac{x-1}{2x-1},$ *Domain:* $x \neq 1$ *and* $x \neq \frac{1}{2}$

$g(f(x)) = \frac{x+2}{-x-1},$ *Domain:* $x \neq -2$ *and* $x \neq -1$

e. $f(x) = x - 1, \ \ g(x) = \log_2(x+3)$

$f(g(x)) = \log_2(x+3) - 1,$ *Domain:* $x > -3$

$g(f(x)) = \log_2(x+2),$ *Domain:* $x > -2$

3. A company has developed a new highly efficient solar panel. Each panel can produce 0.75 MW of electricity each day. According to the Los Angeles power authority, all the traffic lights in the city draw 0.5 MW of power per day.

   a. Write a function that represents the relationship between the number of solar panels installed and the amount of energy generated per day (in MWh). Define the input and output.

   $E(n) = 0.75n$

   n is the number of panels operating in one day.

   E is the total energy generated by the n panels.

   b. Write a function that represents the relationship between the number of days and the energy in megawatts consumed by the traffic lights. (How many days can one megawatt provide?)

   $D(E) = \frac{1}{0.5}E = 2E$

   E: the energy in megawatts

   $D(E)$: the number of days per megawatt

   c. *Write a function that could be used to determine the number of days that the traffic lights stay on based on the number of panels installed.*

   $D\big(E(n)\big) = 2E(n) = 2 \times 0.75n = 1.5n$

   d. Determine an appropriate domain and range for part (c).

   Domain: whole numbers

   Range: whole numbers (whole number multiples of 1.5)

   e. How many days can 20 panels power all the lights?

   $D\big(E(20)\big) = 1.5 \times 20 = 30$. *It takes 30 days.*

4. A water delivery person is trying to determine the relationship between the mass of the cylindrical containers he delivers and their diameter in centimeters. The density of the bottles is $1\ \text{g/cm}^3$. The height of each bottle is approximately 60 cm.

   a. Write a function that represents the relationship between the volume of the cylinder and its diameter.

   $V(d) = 15\pi d^2$

   b. Write a function that represents the relationship between the mass and volume of the cylinder.

   $m(V) = 1V$

   c. Write a function that could be used to determine the mass of one cylinder based on its diameter. Interpret the equation in context.

   $D\big(V(d)\big) = 1 \times 15\pi d^2 = 15\pi d^2$

   The numerical value of the mass of one of the cylindrical water containers is equal to approximately 47.1 times the value of the squared diameter.

d. **Determine an appropriate domain and range for part (c).**

Domain: nonnegative real numbers

Range: nonnegative real numbers

e. **What is the approximate mass of a cylinder with a diameter of 30 cm?**

$D(V(60)) = 15\pi(30)^2 \approx 42411\ g \approx 42.4.$

The mass is approximately 42.4 kg.

5. **A gold mining company is mining gold in Northern California. Each mining cart carries an average 500 kg of dirt and rocks that contain gold from the tunnel. For each 2 metric tons of material (dirt and rocks), the company can extract an average of 10 g of gold. The average wholesale gold price is $\$20/\text{g}$.**

a. **Write a function that represents the relationship between the mass of the material mined in metric tons and the number of carts. Define the input and output.**

$V(n) = 0.5n$

n is the number of carts.

$V(n)$ is the total mass of dirt and rock carried out by the n carts in metric tons.

b. **Write a function that represents the relationship between the amount of gold and the materials. Define the input and output.**

$G(V) = 0.000005V$

V: the amount of material in metric tons

$G(V)$: the mass of gold in metric tons

c. **Write a function that could be used to determine the mass of gold in metric tons as a function of the number of carts coming out from the mine.**

$G(V(n)) = 0.000\,002\,5n$

d. **Determine an appropriate domain and range for part (c).**

Domain: whole numbers

Range: positive real numbers

e. **Write a function that could be used to determine the amount of money the gold is worth in dollars and the amount of gold extracted in metric tons.**

$C(V(n)) = 20\,000\,000 \times 0.000\,002\,5n = 50n$

f. **How much gold can $40{,}000$ carts of material produce?**

$G(V(40000)) = 0.1$ *metric ton*

g. **How much, in dollars, can $40{,}000$ carts of material produce?**

$C(V(40000)) = 50 \times 40000 = 2\,000\,000$ *The cost is* $\$2\,000\,000.$

6. **Bob operates hot air balloon rides for tourists at the beach. The hot air balloon rises, on average, at 100 feet per minute. At sea level, the atmospheric pressure, measured in inches of mercury (inHg), is 29.9 inHg. Using a barometric meter, Bob notices that the pressure decreases by 0.5 inHg for each 500 feet the balloon rises.**

   a. **Write a function that represents the relationship between the height of the hot air balloon and the time spent to reach that height.**

   $H(t) = 100t$

   t is the number of minutes.

   $H(t)$ is the height of the hot air balloon at t time.

   b. **Write a function that represents the relationship between the height of the hot air balloon and the atmospheric pressure being applied to the balloon.**

   $$P(H) = 29.9 - \frac{0.5H}{500}$$

   H: the height of the hot air balloon

   $P(H)$: the reading on the barometer at the height of the hot air balloon

   c. **Write a function that could be used to determine the pressure on the hot air balloon based on the time it spends rising.**

   $$P\big(H(t)\big) = 29.9 - \frac{0.5H(t)}{500} = 29.9 - 0.1t$$

   d. **Determine an appropriate domain and range for part (c).**

   Domain: nonnegative real numbers

   Range: nonnegative real numbers

   e. **What is the reading on the barometer 10 minutes after the hot air balloon has left the ground?**

   $P\big(H(10)\big) = 29.9 - 0.1t = 28.9$ *The reading is* $.9$ inHg.