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Extending to Three Dimensions

[1]Each lesson is ONE day, and ONE day is considered a 45-minute period.

Geometry • Module 3

Extending to Three Dimensions

OVERVIEW

Module 3, Extending to Three Dimensions, builds on students' understanding of congruence in Module 1 and similarity in Module 2 to prove volume formulas for solids.

Topic A studies informal limit arguments to find the area of a rectangle with an irrational side length and of a disk (**G-GMD.A.1**). It also focuses on properties of area that arise from unions, intersections, and scaling. These topics prepare for understanding limit arguments for volumes of solids.

Topic B begins with a lesson where students experimentally discover properties of three-dimensional space that are necessary to describe three-dimensional solids such as cylinders and prisms, cones and pyramids, and spheres. Cross-sections of these solids are studied and are classified as similar or congruent (**G-GMD.B.4**). A dissection is used to show the volume formula for a right triangular prism after which limit arguments give the volume formula for a general right cylinder (**G-GMD.A.1**).

Lesson 10 uses two-dimensional cross-sections of solids to approximate solids by general right cylindrical slices, leading to an understanding of Cavalieri's principle (**G-GMD.A.1**). Congruent cross-sections for a general (skew) cylinder and Cavalieri's principle lead to the volume formula for a general cylinder.

To find the volume formula of a pyramid, a cube is dissected into six congruent pyramids to find the volume of each. Scaling the given pyramids according to a scaling principle analogous to the one introduced in Topic A gives the volume formula for a right rectangular pyramid. The cone cross-section theorem and Cavalieri's principle are then used to find the volume formula of a general cone (**G-GMD.A.1**, **G-GMD.A.3**).

Cavalieri's principle is used to show that the volume of a right circular cylinder with radius R and height R is the sum of the volume of the hemisphere of radius R and the volume of a right circular cone with radius R and height R. This information leads to the volume formula of a sphere (**G-GMD.A.2**, **G-GMD.A.3**).

Focus Standards

Explain volume formulas and use them to solve problems.[2]

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

[2]The (+) standard on the volume of the sphere is an extension of G-GMD.A.1. It is explained by the teacher in this grade and used by students in G-GMD.A.3. Note: Students are not assessed on proving the volume of a sphere formula until Precalculus.

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Extension Standards

Explain volume formulas and use them to solve problems.

G-GMD.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Foundational Standards

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and the area of a circle.

Understand and apply the Pythagorean theorem.

8.G.B.7 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Focus Standards for Mathematical Practice

MP.6 **Attend to precision**. Students formalize definitions, using explicit language to define terms such as *right rectangular prism* that have been informal and more descriptive in earlier grade levels.

MP.7 **Look for and make use of structure**. The theme of approximation in Module 3 is an interpretation of structure. Students approximate both area and volume (curved two-dimensional shapes and cylinders and cones with curved bases) polyhedral regions. They must understand how and why it is possible to create upper and lower approximations of a figure's area or volume. The derivation of the volume formulas for cylinders, cones, and spheres, and the use of Cavalieri's principle are also based entirely on understanding the structure and sub-structures of these figures.

Terminology

New or Recently Introduced Terms

- **Cavalieri's Principle** (Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.)
- **Cone** (Let B be a region in a plane E, and V be a point not in E. The *cone with base* B *and vertex* V is the union of all segments VP for all points P in B. If the base is a polygonal region, then the cone is usually called a *pyramid*.)
- **General Cylinder** (Let E and E' be two parallel planes, let B be a region in the plane E, and let L be a line which intersects E and E' but not B. At each point P of B, consider the segment PP' parallel to L, joining P to a point P' of the plane E'. The union of all these segments is called a *cylinder with base* B.)
- **Inscribed Polygon** (A polygon is *inscribed in* a circle if all of the vertices of the polygon lie on the circle.)
- **Intersection** (The *intersection* of A and B is the set of all objects that are elements of A and also elements of B. The intersection is denoted $A \cap B$.)
- **Rectangular Pyramid** (Given a rectangular region B in a plane E, and a point V not in E, the *rectangular pyramid with base* B *and vertex* V is the union of all segments VP for points P in B.)
- **Right Rectangular Prism** (Let E and E' be two parallel planes. Let B be a rectangular region in the plane E. At each point P of B, consider the segment PP' perpendicular to E, joining P to a point P' of the plane E'. The union of all these segments is called a *right rectangular prism*.)

- **Solid Sphere or Ball** (Given a point C in the three-dimensional space and a number $r > 0$, the *solid sphere (or ball) with center* C *and radius* r is the set of all points in space whose distance from point C is less than or equal to r.)
- **Sphere** (Given a point C in the three-dimensional space and a number $r > 0$, the *sphere with center* C *and radius* r is the set of all points in space that are distance r from the point C.)
- **Subset** (A set A is a *subset* of a set B if every element of A is also an element of B.)
- **Tangent to a Circle** (A *tangent line to a circle* is a line that intersects a circle in one and only one point.)
- **Union** (The *union* of A and B is the set of all objects that are either elements of A or of B or of both. The union is denoted $A \cup B$.)

Familiar Terms and Symbols[3]

- Disk
- Lateral Edge and Face of a Prism

Suggested Tools and Representations

- Three-dimensional models of rectangular prisms, right circular cylinders, right pyramids
- Deck of cards
- Stack of coins
- Images of sliced figures, such as a loaf of bread or a stack of deli cheese

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	G-GMD.A.1, G-GMD.A.3, G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3

[3]These are terms and symbols students have seen previously.