Lesson 2: Calculating Probabilities of Events Using Two-Way Tables

Classwork

Example 1: Building a New High School

The school board of Waldo, a rural town in the Midwest, is considering building a new high school primarily funded by local taxes. They decided to interview eligible voters to determine if the school board should build a new high school facility to replace the current high school building. There is only one high school in the town. Every registered voter in Waldo was interviewed. In addition to asking about support for a new high school, data on gender and age group were also recorded. The data from these interviews are summarized below.

| Age (in years) | Should Our Town Build a New High School? | | | | | |
| | Yes | | No | | No Answer | |
	Male	Female	Male	Female	Male	Female
18–25	29	32	8	6	0	0
26–40	53	60	40	44	2	4
41–65	30	36	44	35	2	2
66 and Older	7	26	24	29	2	0

Exercises 1–8: Building a New High School

1. Based on this survey, do you think the school board should recommend building a new high school? Explain your answer.

2. An eligible voter is picked at random. If this person is 21 years old, do you think he would indicate that the town should build a high school? Why or why not?

3. An eligible voter is picked at random. If this person is 55 years old, do you think she would indicate that the town should build a high school? Why or why not?

4. The school board wondered if the probability of recommending a new high school was different for different age categories. Why do you think the survey classified voters using the age categories 18–25 years old, 26–40 years old, 41–65 years old, and 66 years old and older?

5. It might be helpful to organize the data in a two-way frequency table. Use the given data to complete the following two-way frequency table. Note that the age categories are represented as rows, and the possible responses are represented as columns.

	Yes	No	No Answer	Total
18–25 Years Old				
26–40 Years Old				
41–65 Years Old				
66 Years Old and Older				
Total				

6. A local news service plans to write an article summarizing the survey results. Three possible headlines for this article are provided below. Is each headline accurate or inaccurate? Support your answer using probabilities calculated using the table above.

Headline 1: *Waldo Voters Likely to Support Building a New High School*

Headline 2: *Older Voters Less Likely to Support Building a New High School*

Headline 3: *Younger Voters Not Interested in Building a New High School*

7. The school board decided to put the decision on whether or not to build the high school up for a referendum in the next election. At the last referendum regarding this issue, only 25 of the eligible voters ages 18–25 voted, 110 of the eligible voters ages 26–40 voted, 130 of the eligible voters ages 41–65 voted, and 80 of the eligible voters ages 66 and older voted. If the voters in the next election turn out in similar numbers, do you think this referendum will pass? Justify your answer.

8. Is it possible that your prediction of the election outcome might be incorrect? Explain.

Example 2: Smoking and Asthma

Health officials in Milwaukee, Wisconsin, were concerned about teenagers with asthma. People with asthma often have difficulty with normal breathing. In a local research study, researchers collected data on the incidence of asthma among students enrolled in a Milwaukee public high school.

Students in the high school completed a survey that was used to begin this research. Based on this survey, the probability of a randomly selected student at this high school having asthma was found to be 0.193. Students were also asked if they had at least one family member living in their house who smoked. The probability of a randomly selected student having at least one member in his (or her) household who smoked was reported to be 0.421.

Exercises 9–14

It would be easy to calculate probabilities if the data for the students had been organized into a two-way table like the one used in Exercise 5. But there is no table here, only probability information. One way around this is to think about what the table might have been if there had been 1,000 students at the school when the survey was given. This table is called a *hypothetical 1000 two-way table*.

What if the population of students at this high school was 1,000? The population was probably not exactly 1,000 students, but using an estimate of 1,000 students provides an easier way to understand the given probabilities. Connecting these estimates to the actual population is completed in a later exercise. Place the value of 1,000 in the cell representing the total population. Based on a hypothetical 1000 population, consider the following table:

	No Household Member Smokes	At Least One Household Member Smokes	Total
Student Has Asthma	Cell 1	Cell 2	Cell 3
Student Does Not Have Asthma	Cell 4	Cell 5	Cell 6
Total	Cell 7	Cell 8	1,000

9. The probability that a randomly selected student at this high school has asthma is 0.193. This probability can be used to calculate the value of one of the cells in the table above. Which cell is connected to this probability? Use this probability to calculate the value of that cell.

10. The probability that a randomly selected student has at least 1 household member who smokes is 0.421. Which cell is connected to this probability? Use this probability to calculate the value of that cell.

11. In addition to the previously given probabilities, the probability that a randomly selected student has at least one household member who smokes and has asthma is 0.120. Which cell is connected to this probability? Use this probability to calculate the value of that cell.

12. Complete the two-way frequency table by calculating the values of the other cells in the table.

	No Household Member Smokes	At Least One Household Member Smokes	Total
Student Has Asthma			
Student Does Not Have Asthma			
Total			1,000

13. Based on your completed two-way table, estimate the following probabilities as a fraction and also as a decimal (rounded to three decimal places):

   a. A randomly selected student has asthma. What is the probability this student has at least 1 household member who smokes?

   b. A randomly selected student does not have asthma. What is the probability this student has at least one household member who smokes?

   c. A randomly selected student has at least one household member who smokes. What is the probability this student has asthma?

14. Do you think that whether or not a student has asthma is related to whether or not this student has at least one family member who smokes? Explain your answer.

Lesson Summary

Data organized in a two-way frequency table can be used to calculate probabilities.

In certain problems, probabilities that are known can be used to create a hypothetical 1000 two-way table. The hypothetical population of 1,000 can then be used to calculate probabilities.

Probabilities are always interpreted in context.

Problem Set

1. The Waldo School Board asked eligible voters to evaluate the town's library service. Data are summarized in the following table:

	How Would You Rate Our Town's Library Services?							
	Good		**Average**		**Poor**		**Do Not Use Library**	
Age (in years)	Male	Female	Male	Female	Male	Female	Male	Female
18–25	10	8	5	7	5	5	17	18
26–40	30	28	25	30	20	30	20	20
41–65	30	32	26	21	15	10	5	10
66 and Older	21	25	8	15	2	10	2	5

a. What is the probability that a randomly selected person who completed the survey rated the library as good?

b. Imagine talking to a randomly selected male voter who had completed the survey. How do you think this person rated the library services? Explain your answer.

c. Use the given data to construct a two-way table that summarizes the responses on gender and rating of the library services. Use the following template as your guide:

	Good	**Average**	**Poor**	**Do Not Use**	**Total**
Male					
Female					
Total					

d. Based on your table, answer the following:

i. A randomly selected person who completed the survey is male. What is the probability he rates the library services as good?

ii. A randomly selected person who completed the survey is female. What is the probability she rates the library services as good?

e. Based on your table, answer the following:

i. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is male?

ii. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is female?

f. Do you think there is a difference in how male and female voters rated library services? Explain your answer.

2. Obedience School for Dogs is a small franchise that offers obedience classes for dogs. Some people think that larger dogs are easier to train and, therefore, should not be charged as much for the classes. To investigate this claim, dogs enrolled in the classes were classified as large (30 pounds or more) or small (under 30 pounds). The dogs were also classified by whether or not they passed the obedience class offered by the franchise. 45% of the dogs involved in the classes were large. 60% of the dogs passed the class. Records indicate that 40% of the dogs in the classes were small and passed the course.

   a. Complete the following hypothetical 1000 two-way table:

	Passed the Course	Did Not Pass the Course	Total
Large Dogs			
Small Dogs			
Total			

   b. Estimate the probability that a dog selected at random from those enrolled in the classes passed the course.

   c. A dog was randomly selected from the dogs that completed the class. If the selected dog was a large dog, what is the probability this dog passed the course?

   d. A dog was randomly selected from the dogs that completed the class. If the selected dog is a small dog, what is the probability this dog passed the course?

   e. Do you think dog size and whether or not a dog passes the course are related?

   f. Do you think large dogs should get a discount? Explain your answer.