Lesson 6: Segments That Meet at Right Angles

Classwork

Opening Exercise

Carlos thinks that the segment having endpoints $A(0,0)$ and $B(6,0)$ is perpendicular to the segment with endpoints $A(0,0)$ and $C(-2,0)$. Do you agree? Why or why not?

Working with a partner, given $A(0,0)$ and $B(3,-2)$, find the coordinates of a point $C$ so that $\overline{AC} \perp \overline{AB}$.

Example

Given points $A(2,2), B(10,16), C(-3,1)$, and $D(4,-3)$, are $\overline{AB}$ and $\overline{CD}$ perpendicular? Are the lines containing the segments perpendicular? Explain.
Exercises

1. Given \( A(a_1, a_2), B(b_1, b_2), C(c_1, c_2), \) and \( D(d_1, d_2) \), find a general formula in terms of \( a_1, a_2, b_1, b_2, c_1, c_2, d_1, \) and \( d_2 \) that will let us determine whether \( \overline{AB} \) and \( \overline{CD} \) are perpendicular.

2. Recall the Opening Exercise of Lesson 4 in which a robot is traveling along a linear path given by the equation \( y = 3x - 600 \). The robot hears a ping from a homing beacon when it reaches the point \( F(400,600) \) and turns to travel along a linear path given by the equation \( y - 600 = -\frac{1}{3}(x - 400) \). If the homing beacon lies on the \( x \)-axis, what is its exact location? (Use your own graph paper to visualize the scenario.)

a. If point \( E \) is the \( y \)-intercept of the original equation, what are the coordinates of point \( E \)?

b. What are the endpoints of the original segment of motion?

c. If the beacon lies on the \( x \)-axis, what is the \( y \)-value of this point, \( G \)?

d. Translate point \( F \) to the origin. What are the coordinates of \( E', F', \) and \( G' \)?

e. Use the formula derived in this lesson to determine the coordinates of point \( G \).
3. A triangle in the coordinate plane has vertices $A(0, 10), B(-8, 8)$, and $C(-3, 5)$. Is it a right triangle? If so, at which vertex is the right angle? (Hint: Plot the points, and draw the triangle on a coordinate plane to help you determine which vertex is the best candidate for the right angle.)

4. $A(-7, 1), B(-1, 3), C(5, -5)$, and $D(-5, -5)$ are vertices of a quadrilateral. If $\overline{AC}$ bisects $\overline{BD}$, but $\overline{BD}$ does not bisect $\overline{AC}$, determine whether $ABCD$ is a kite.
Problem Set

1. Are the segments through the origin and the points listed perpendicular? Explain.
   a. \(A(9,10), B(10,9)\)
   b. \(C(9,6), D(4, -6)\)

2. Given \(M(5,2), N(1, -4)\), and \(L\) listed below, are \(LM\) and \(MN\) perpendicular? Translate \(M\) to the origin, write the coordinates of the images of the points, and then explain without using slope.
   a. \(L(-1,6)\)
   b. \(L(11, -2)\)
   c. \(L(9,8)\)

3. Is triangle \(PQR\), where \(P(-7,3), Q(-4,7)\), and \(R(1, -3)\), a right triangle? If so, which angle is the right angle? Justify your answer.

4. A quadrilateral has vertices \((2+\sqrt{2}, -1), (8+\sqrt{2}, 3), (6+\sqrt{2}, 6),\) and \((\sqrt{2}, 2)\). Prove that the quadrilateral is a rectangle.

5. Given points \(G(-4,1), H(3,2)\), and \(I(-2, -3)\), find the \(x\)-coordinate of point \(J\) with \(y\)-coordinate \(4\) so that the \(\overrightarrow{GH}\) and \(\overrightarrow{IJ}\) are perpendicular.

6. A robot begins at position \((-80, 45)\) and moves on a path to \((100, -60)\). It turns 90° counterclockwise.
   a. What point with \(y\)-coordinate \(120\) is on this path?
   b. Write an equation of the line after the turn.
   c. If it stops to charge on the \(x\)-axis, what is the location of the charger?

7. Determine the missing vertex of a right triangle with vertices \((6,2)\) and \((5,5)\) if the third vertex is on the \(y\)-axis. Verify your answer by graphing.

8. Determine the missing vertex for a rectangle with vertices \((3,-2)\), \((5,2)\), and \((-1,5)\), and verify by graphing. Then, answer the questions that follow.
   a. What is the length of the diagonal?
   b. What is a point on both diagonals in the interior of the figure?

9. Leg \(\overline{AB}\) of right triangle \(ABC\) has endpoints \(A(1,3)\) and \(B(6, -1)\). Point \(C(x, y)\) is located in Quadrant IV.
   a. Use the perpendicularity criterion to determine at which vertex the right angle is located. Explain your reasoning.
   b. Determine the range of values that \(x\) is limited to and why.
   c. Find the coordinates of point \(C\) if they are both integers.